Electromagnetism and Light
(PHY104)

Sébastien Corde
sebastien.corde@polytechnique.edu

Arnaud Couairon
arnaud.couairon@polytechnique.edu

Alexandre Lazarescu
alexandre.lazarescu@polytechnique.edu

Siyi Yu
siyi.yu@polytechnique.edu

Before starting the physics, we'll review:

- Who are your teachers
- How the course is organized
- Why learning physics and electromagnetism
- The outline of the course
- Project on special relativity
- Grading system and expectations

Who are your teachers for this course?

Siyi Yu and Sébastien Corde, Laboratory for Applied Optics (LOA) at Ecole Polytechnique

Research on plasma-based particle acceleration in extreme conditions.

Alexandre Lazarescu and Arnaud Couairon, Center for Theoretical Physics (CPHT) at Ecole Polytechnique

Research on mathematical physics (statistical physics, field theory) and on theory of laser-plasma interaction.

Do not hesitate to contact us by email or come by in our office if you have questions! We'll reply as best as we can. For questions of interest to everyone, I will present them and reply at the start of the next lecture.

Plasma-based particle acceleration, in a few words?

Plasma: 4th state of matter

We use a fully ionized gas:
medium with ions and free electrons

Extreme conditions

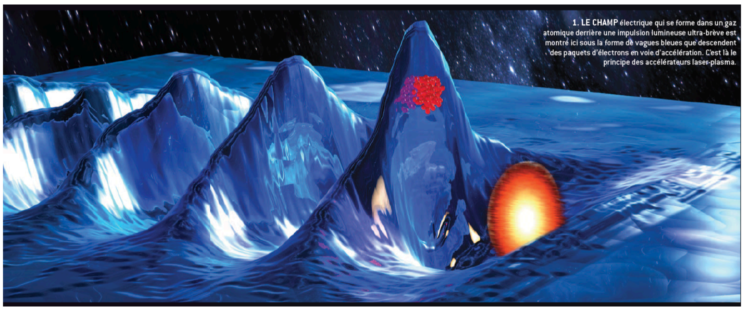
We send a very intense laser pulse or very dense particle bunch in the plasma. Waves of electron density in the wake of the bunch support extremely high fields

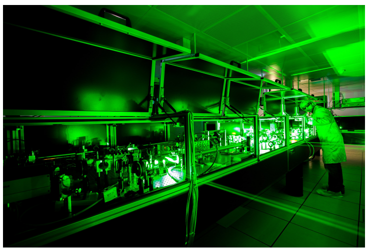
Particle acceleration

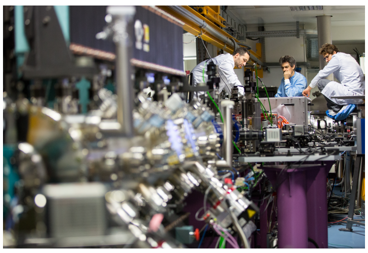
Particles are accelerated to very high energies in a much shorter distance than in conventional accelerators

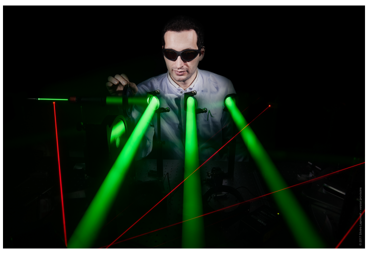
Lab visit

Possible to visit our lab and see what kind of science we're doing! Please ask at any time and we'll organize it (ideally in the second part of the semester)









PHY104 course organization

- Every Thursdays 13:15-15:15: PHY104 lecture in Amphi Gay-Lussac (except for 05/04/18, 12/04/18, 31/05/18 in Amphi Cauchy, check on ENEX NG)
- Every Thursdays 15:30-17:30: PHY104 tutorials, exercices and problem solving
- Every 2-3 weeks: one homework assignment (mandatory and individual)

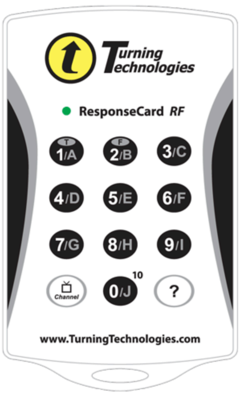
Textbook, as a complement to the course (optional):

The Feynman Lectures on Physics, Volume II

http://www.feynmanlectures.caltech.edu/II_toc.html

- Every Wednesdays 13:15-15:15 or 15:30-17:30: Mathematical Methods for Physics, PHY105 joint lecture & tutorials
- Physics tutoring sessions also available every Fridays 18:00-20:00; please sign in on the form

Use of clickers during lectures



- Always bring your clicker with you for the lectures!
- We'll use them for a few quizzes in each lecture
- Quizzes are anonymous, no record of your individual response is kept
- Play the game and participate
- We use channel 41 (default), see user manual here:

https://www.turningtechnologies.com/pdf/UserGuides/ResponseCard_RF_LCD_UG.pdf

Moodle: Polytechnique course catalogue

- Numerical versions of lectures, tutorials, homework assignments and project are available here:

https://moodle.polytechnique.fr/course/view.php?id=5205

Need to be logged in with your Polytechnique credentials.

Why physics and electromagnetism

- Physics is fascinating and its history is an incredible human adventure, which is still ongoing! From the most fundamental laws of nature and of the universe to common everyday experience (why the sky is blue?) and objects (how your GPS works?).

- Physics is an experimental science: it aims at describing reality with high accuracy using laws formulated with mathematics. From empirical observations we build universal theories with very wide prediction power, which then need experimental confirmation. Physics is not a dictionary of effects and behaviors.

- New physics most often comes with more advanced mathematics and with revolutionizing concepts, which are defying our common-sense understanding.

Why physics and electromagnetism

Physics examples beyond common-sense:

✳ Existence of waves propagating in vacuum, without support or medium, and whose speed is the same in all inertial frames

✳ At high speeds (near speed of light *c*), composition of velocities

In rest frame of particle 1, particle 2 is not moving at 1.8 c, but at 0.99 c !

✳ Time is not absolute: *watch or re-watch Interstellar!*

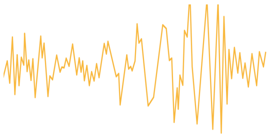
✳ At small scales, quantum mechanics: abandon determinism and trajectories!

Cannot know simultaneously the position and momentum of a particle with arbitrary precision

Why physics and electromagnetism

Recent news in physics:

The Nobel Prize in Physics 2017: Rainer Weiss, Barry C. Barish and Kip S. Thorne "*for decisive contributions to the LIGO detector and the observation of gravitational waves*".



produced by black hole merging; GW predicted by Einstein a century ago (1916)

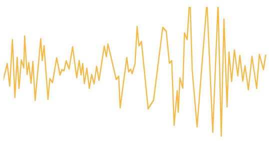
August 17, 2017

- *First cosmic event observed with both gravitational waves and light (radio, visible, gamma-rays)*
- *Proved that heavy elements, such as lead and gold, are created in these events*

Why physics and electromagnetism

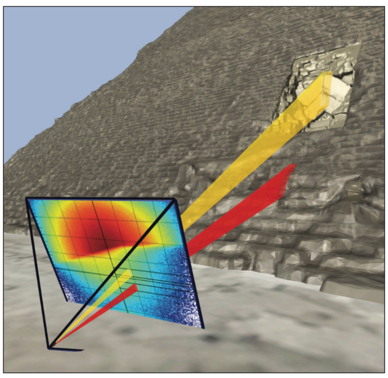
Recent news in physics:

The Nobel Prize in Physics 2017: Rainer Weiss, Barry C. Barish and Kip S. Thorne "*for decisive contributions to the LIGO detector and the observation of gravitational waves*".



produced by black hole merging; GW predicted by Einstein a century ago (1916)

Discovery of a large void in Great Pyramid of Giza using muon radiography



Nature 552, 386-390 (21 December 2017)

- *Muon is an elementary particle with identical properties than electron but with higher mass (x207)*
- *Atmospheric muons are created when cosmic rays (high-energy particles from the outer space) impact our atmosphere*
- *These muons are used for radiography of a very thick block of stones… here a pyramid.*

→ Use of modern physics to study world's archaeological heritage

Why physics and electromagnetism

Electromagnetism: everywhere around us

* ✴ We « see » using light
* ✴ When we think, neurons are exchanging electrical signals; our nervous system works with electricity; muscle contraction and heartbeat requires electricity
* ✴ Electrical power: 20% of world final energy consumption is in electrical form; solar energy comes to earth in the form of light
* ✴ Lightnings and sparks
* ✴ Old and modern communications: electrical signals in telegraph, landline phone and internet; electromagnetic waves in radio, Wifi, bluetooth, 4G, GPS, etc.; light in optical fibers in modern internet and submarine cables; radars
* ✴ Electromagnetism is what make atoms and molecules hold together, it is the fundamental force at the origin of contact force and all effective macroscopic forces

Why physics and electromagnetism

Electromagnetism: one of the four fundamental interactions (or forces) of nature

Gravitation	Acts on mass
Electromagnetism	Acts on electrical charge, what makes atoms and molecules hold together
Weak interaction	Responsible for radioactivity (beta decays)
Strong interaction	What makes nucleus hold together

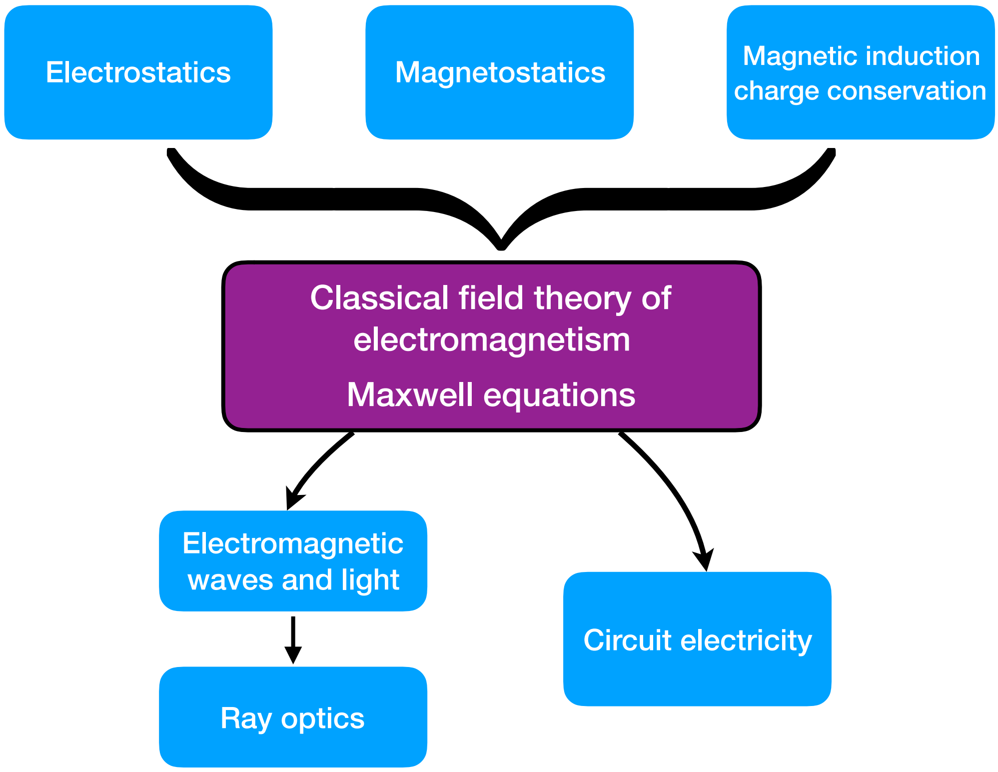
Electromagnetism governed by Maxwell equations:

$$\text{div } \vec{E} = \frac{\rho}{\epsilon_0} \qquad \text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{div } \vec{B} = 0 \qquad \text{curl } \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

By the end of the course, you'll master Maxwell equations and understand how they describe electrical, magnetic and light phenomena

Outline of the lecture



Project on special relativity

Practical considerations:

- Project (longer homework assignment) distributed in April
- You'll have 2 months to work on it, project to be submitted in June
- You can work in groups of up to 4 students, only one project to submit per group

Topics explored in the project:

- Time dilatation, twin thought experiment
- Causality, relativity of simultaneity
- Mass-energy equivalence ($E=mc^2$)
- Particle acceleration to near-speed-of-light velocities

Few specificities about the project:

- You won't be applying concepts seen in class to solve problems; instead you'll have to learn new physics from the literature, outside the classroom
- The project will include numerical analysis using Jupyter and Python

Grading system

- Your final grade for PHY104 will be composed of:

Homework assignments	Midterm exam	Project	Final exam
20%	20%	20%	40%

↳ (Homework assignments) Mostly based on participation (75%): if you try but your result is not correct, you'll have at least 75% of the points

Numeric grades:	Letter grades:
0 to 20	A+, A, A-, B+, B, B-, C, D, E, F

A+, A, A-: ≥16   B+, B, B-: ≥13 & <16   C: ≥10 & <13

D: ≥7 & <10   E: <7   F: 0

- In the midterm and final exams, and in the project, qualitative questions will count for a significant number of points.
- For the midterm and final exams, no documents and no phone allowed, just you and your brain.

Expectations

By the end of the course, we expect you to:

- Have a general understanding of the theory of electromagnetism, and how it can describe in a unified way various physical phenomena, including electricity, magnetism and light; see the bigger picture
- Be able to apply the theory to specific problems, understand « how it works »
- Master the mathematical tools used to describe the physical state of the system and its evolution in the framework of electromagnetism
- Have acquired a new intuition to understand and guess the behavior of fields and light, the motion of particles in fields, etc.
- Be able to learn new physics and science and solve problems analytically and numerically outside the classroom (with the project on special relativity), closer to real life conditions

What we don't want:

- Being able to make correct calculations without understanding the physics behind it

Electrostatics

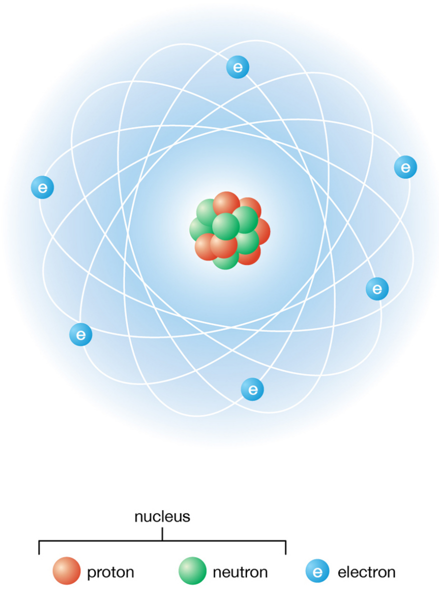
Coulomb's law, the electric field and the principle of superposition

Feynman Vol. II Chapter 4

1. Electric charge

- Ordinary matter is made of atoms, which are made of electrons, protons and neutrons. These constituents have a fundamental property: their electric charge q.

- In contrast to mass, always positive ($m \geq 0$), charge can be positive or negative. The force between two charged particles is either attractive or repulsive depending on the relative sign of their charge.



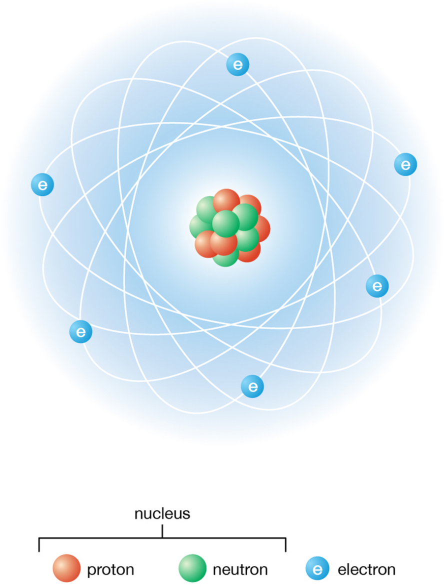
1. Electric charge

- In SI units (International System of Units), the electric charge is expressed in coulomb (C), and we have:

$$q_e = -e = -1.6 \times 10^{-19}\ \mathrm{C}$$

$$q_p = e = 1.6 \times 10^{-19}\ \mathrm{C}$$

$$q_n = 0$$



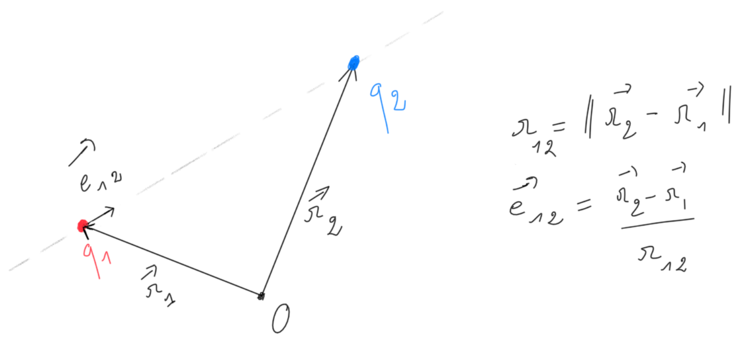
- Electrically-charged macroscopic objects have either an excess or a lack of electrons:

$$Q = ne, \quad n \in \mathbb{Z}$$

- Conservation of charge: the total charge of an isolated system is constant.

2. Electric force

- We consider two point charges at rest, separated by a distance r_{12}:



- Coulomb established his famous law by observing that the electric force F_2 experienced by q_2 due to q_1 verifies

$$\left.\begin{array}{r} ||\vec{F}_2|| \propto q_1 \\ ||\vec{F}_2|| \propto q_2 \\ ||\vec{F}_2|| \propto 1/r_{12}^2 \\ \vec{F}_2 \parallel \vec{e}_{12} \end{array}\right\} \quad \vec{F}_2 = k_e \frac{q_1 q_2}{r_{12}^2}\, \vec{e}_{12} = -\vec{F}_1$$

2. Electric force

- In SI units, the Coulomb constant k_e is written: $$k_e = \frac{1}{4\pi\epsilon_0}$$

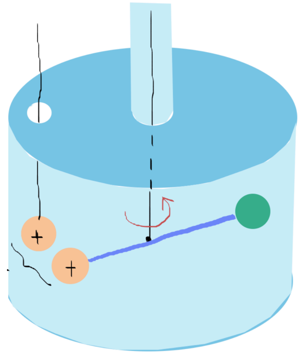
where ε_0 is called the permittivity of free space, or vacuum permittivity.

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\,\vec{e}_{12}$$

- The values of these constants are:

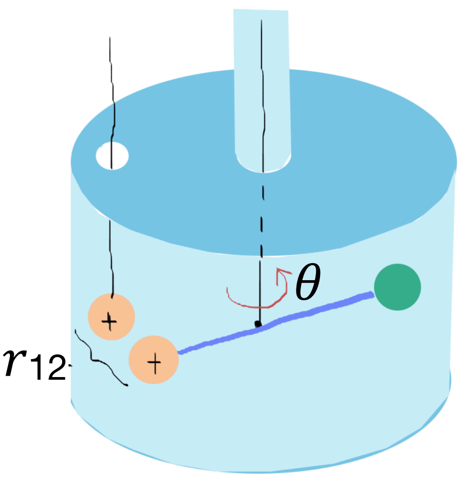
$$k_e = \frac{1}{4\pi\epsilon_0} = 9.0\times10^{9}\ \mathrm{N\,m^2\,C^{-2}}$$
$$\epsilon_0 = 8.9\times10^{-12}\ \mathrm{C^2\,m^{-2}\,N^{-1}}$$

- Historically, Coulomb used a torsion balance to experimentally prove his inverse-square law



2. Electric force

Principle of the torsion balance experiment



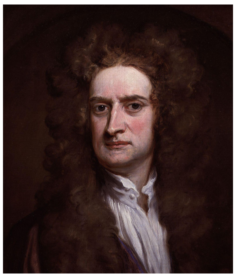
- When the central wire is twisted, a torsion restoring force (linear in the rotation angle θ) tends to bring it back to its equilibrium position (θ=0).
- A charged metallic sphere is approached to another charged metallic sphere at the extremity of a bar suspended by the torsion wire. Once a new equilibrium is reached, the Coulomb force equals the torsion force, which is directly proportional to the angle of twist θ.
- One can then verify than θ scales as $1/(r_{12})^2$. The apparatus is sensitive to very weak forces, because the torsion force itself is very small.

2. Electric force

Charles-Augustin Coulomb
1736-1806

$$\vec{F}_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\,\vec{e}_{12}$$

Coulomb's law
1785



Issac Newton
1643-1727

$$\vec{F}_g = -G\,\frac{m_1 m_2}{r_{12}^2}\,\vec{e}_{12}$$

Newton's law of universal gravitation
1687

→ Strong analogy between electrical and gravitational forces

3. Electric field

- Instead of considering the force only, one can introduce a new quantity called the electric field, by the following relationship:

$$\vec{F}_e = q\vec{E}$$

- The electric field is the force per unit of charge, due to all other charges.

- In the case of two point charges q_1 and q_2, the field at q_2 reads:

$$\vec{F}_2 = q_2\vec{E}(\vec{r}_2)$$

$$\vec{E}(\vec{r}_2) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{12}^2}\,\vec{e}_{12} = \frac{q_1}{4\pi\epsilon_0}\frac{\vec{r}_2-\vec{r}_1}{\|\vec{r}_2-\vec{r}_1\|^3}$$

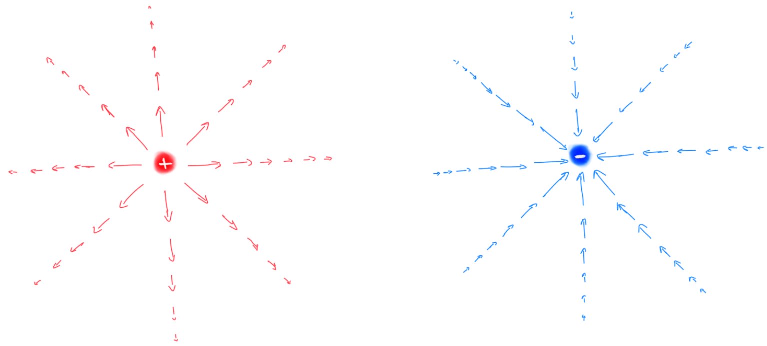
- This field is no longer dependent on the charge of the particle on which the force is applied (here q_2). At this stage, using the field is mainly for convenience, but one can also say that the field exists regardless of the presence of the charge q_2, which will become more evident in the following of the course.

3. Electric field

- The electric field is a vector field. First it's a vector, and second it's a quantity that takes a value at each point in space. The electric field produced by a single charge q_1 reads

$$\vec{E}(\vec{r}) = \frac{q_1}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_1}{\|\vec{r}-\vec{r}_1\|^3}$$

- One can think of it as the force per unit charge that a test particle would experience if placed at the position $\vec{r}$.
- The electric field generated either by a positive or negative charge can be represented as follows:



4. Principle of superposition

What about the electric field from a distribution of N point charges?

- In this case, the electric field from the N point charges is obtained by taking the vector sum of the electric fields from each individual charges:

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N}\frac{q_i}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_i}{\|\vec{r}-\vec{r}_i\|^3}$$

- This also means that the force experienced by a charge q at position $\vec{r}$ due to N point charges is the sum of the individual forces from each charge:

$$\vec{F}_e = q\vec{E}(\vec{r}) = q\sum_{i=1}^{N}\frac{q_i}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_i}{\|\vec{r}-\vec{r}_i\|^3} = \sum_{i=1}^{N}\frac{qq_i}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_i}{\|\vec{r}-\vec{r}_i\|^3} = \sum_{i=1}^{N}\vec{F}_{e,i}$$

- This result of great importance is called the *principle of superposition*.

5. Charge distribution

- So far, we have considered discrete charge distributions, with either 2 or N point charges. The discrete charge distribution is characterized by the set:

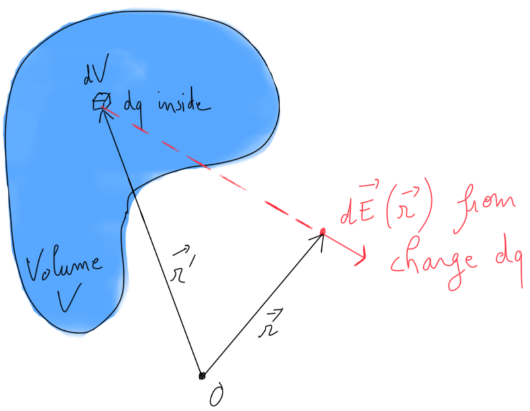
$$\{q_i, \vec{r}_i\}, \quad i = 1, ..., N$$

- One can also consider continuous charge distributions, to describe charged continuous media, in which the charge is spread out in a continuous smear. This is acceptable (and very convenient) if there is a large number of discrete charges in a volume of the smallest scale of interest. Doing so, we discard what's happening on a very small scale. The continuous charge distribution is characterized by the « charge density », the charge per unit volume in C m^{-3}:

$$\rho(\vec{r}), \quad \vec{r} \in \mathbb{R}^3$$

5. Charge distribution

- We define the charge density at the position $\vec{r}\,'$ by:



Charge density (in C m^{-3})

$$dq(\vec{r}\,') = \rho(\vec{r}\,')\, dV$$

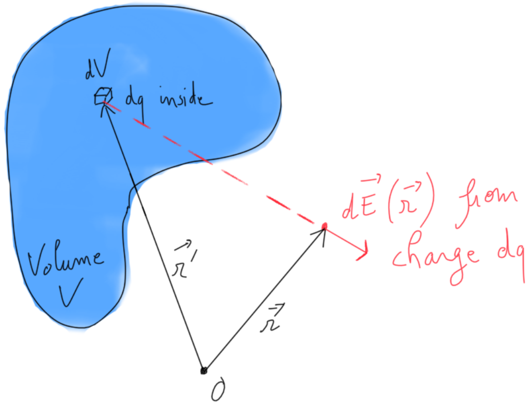
Infinitesimal charge in dV

Infinitesimal volume

- This infinitesimal charge generates the following electric field at the position $\vec{r}$:

$$d\vec{E}(\vec{r}) = \frac{dq}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r}\,'}{\|\vec{r} - \vec{r}\,'\|^3} = \frac{\rho(\vec{r}\,')}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r}\,'}{\|\vec{r} - \vec{r}\,'\|^3}\, dV$$

5. Charge distribution



$$d\vec{E}(\vec{r}) = \frac{\rho(\vec{r}\,')}{4\pi\epsilon_0}\,\frac{\vec{r}-\vec{r}\,'}{\|\vec{r}-\vec{r}\,'\|^3}\,dV$$

To obtain the electric field from all the charge in the volume V, we apply the principle of superposition and sum over the contributions from all infinitesimal volumes :

$$\vec{E}(\vec{r}) = \iiint_V d\vec{E}(\vec{r}) = \iiint_V \frac{\rho(\vec{r}\,')}{4\pi\epsilon_0}\,\frac{\vec{r}-\vec{r}\,'}{\|\vec{r}-\vec{r}\,'\|^3}\,dV = \frac{1}{4\pi\epsilon_0}\iiint_V \rho(\vec{r}\,')\,\frac{\vec{r}-\vec{r}\,'}{\|\vec{r}-\vec{r}\,'\|^3}\,d^3\vec{r}\,'$$

with $dV = d^3\vec{r}\,' = dx'dy'dz'$

Summary

Coulomb's law: $$\vec{F}_e = \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_2}{r_{12}^2}\,\vec{e}_{12}$$

Principle of superposition:

Discrete charge distribution

$$\{q_i, \vec{r}_i\}, \quad i = 1, \ldots, N$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N} q_i\,\frac{\vec{r}-\vec{r}_i}{\|\vec{r}-\vec{r}_i\|^3}$$

discrete sum

Continuous charge distribution

$$\rho(\vec{r}), \quad \vec{r} \in \mathbb{R}^3$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\iiint \rho(\vec{r}\,')\,\frac{\vec{r}-\vec{r}\,'}{\|\vec{r}-\vec{r}\,'\|^3}\,d^3\vec{r}\,'$$

continuous sum

The force experienced by a charge q at the position $\vec{r}$:

$$\vec{F}_e = q\vec{E}(\vec{r})$$