Master in Economics
Paris Saclay
Macroeconomics 1: Economic Growth
Examen. December 2016
2 hours. No document authorized. Answers can be either in English or in French.

Exercise 1 (10 points)

We consider endogenous growth with expanding variety of intermediate products as in Paul Romer (1990). There are 3 types of good: labor, supplied by households; the final good (numeraire): consumed, invested; and intermediate products used to produce the final good. The size of the population, equal to L, is constant over time. Households are identical and live forever. Their preferences are represented by the welfare function

$$\int_0^\infty \frac{c(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} \mathrm{d}t, \sigma \geq 0, \sigma \neq 1$$

where $c(t)$ denotes the quantity of final good consumed at date t. Each household supplies one unit of labor per unit of time at no cost. Households work in the final output industry

The final output industry is perfectly competitive. The final output is produced in quantity $Y(t)$ with a continuum of intermediate products, defined over the interval $[0, A(t)]$, and labor, with the production function

$$Y(t) = L(t)^{1-\alpha} \int_0^{A(t)} x_i(t)^\alpha \mathrm{d}i, 0 < \alpha < 1,$$

where $x_i(t)$ denotes the quantity of intermediate product i, $L(t)$ the quantity of labor and $A(t)$ is the number of varieties of products. The labor market is perfectly competitive.

Let us denote by $p_i(t)$ the unit price of intermediate product i and by $w(t)$ the wage.

1. Write the profit of the final output industry

Answer:

$$Y(t) - w(t)L(t) - \int_0^{A(t)} p_i(t) x_i(t)^\alpha \mathrm{d}i$$

2. Compute the demand for the intermediate product i

Answer:

$$L^{1-\alpha} \alpha x_i(t)^{\alpha-1} = p_i(t)$$

Intermediate products are produced with the final good: one unit of final good produces one unit of intermediate product. Each intermediate product is produced by a monopoly. Producers of intermediate products get a subsidy $s \in (0, 1)$ for each unit of production of intermediate product.

3. Write the profit of the monopoly which produces the intermediate product i

Answer

$$\pi_i(t) = [p_i(t) + s] x_i(t) - x_i(t)$$

4. Determine the production of intermediate product i and the profit of the producer of this product

Answer: we solve

$$\max_{x_i(t)} \pi_i(t) = L^{1-\alpha} \alpha x_i(t)^\alpha + s x_i(t) - x_i(t) \tag{1}$$

The first-order condition yields

$$x_i(t) = (1-s)^{\frac{1}{\alpha-1}} L \alpha^{\frac{2}{1-\alpha}}, \ i \in [0, A(t)], t \geq 0 \tag{2}$$

Substituting the expression for $x_i(t)$ (equation (2)) into equation (1), we get the equilibrium profit flow for every firm in the intermediate product sector

$$\pi = \frac{1-\alpha}{\alpha}(1-s)^{\frac{\alpha}{\alpha-1}} L\alpha^{\frac{2}{1-\alpha}} \tag{3}$$

5. What is the impact of an increase in the subvention s on the production of intermediate products? On the profit of intermediate products producers?

Answer: the subvention increases profit and production.

It is assumed that the number of new varieties depends on the amount $R(t)$ of final good used in research

$$\dot{A}(t) = \lambda R(t) \tag{4}$$

where $\lambda > 0$ is an indicator of productivity of research. There is free entry in the research sector which is perfectly competitive. The firm that discovers a blueprint for a new intermediate product receives a fully enforced perpetual patent on this input variety. Let $V(t)$ denotes the present discounted value of an innovation at date t

6. Write the zero profit condition in the research sector

Answer:

$$V(t)\lambda R(t) = R(t)$$

The left hand side is the number of new blueprints discovered at date t times the value of a blueprint . The right hand side is the cost of research

7. Define $V(t)$ as function of the interest rate r and the profit flow, denoted by π, of the producer of an intermediate product.

Answer:

$$V(t) = \int_t^{\infty} \pi e^{-r(\tau-t)} \mathrm{d}\tau = \frac{\pi}{r}$$

8. Deduce from the previous questions the equilibrium value of the interest rate as function of λ and π.

Answer: we know from the zero profit condition in the research sector that $V(t) = 1/\lambda$ at equilibrium. Thus we get:

$$r = \lambda\pi$$

9. Use the expression for π computed question 4 to provide the equilibrium value of the interest rate as function of parameters λ, s, α, L.

Answer:

$$r = \lambda\pi = \lambda\frac{1-\alpha}{\alpha}(1-s)^{\frac{\alpha}{\alpha-1}} L\alpha^{\frac{2}{1-\alpha}} \tag{5}$$

10. Use the Keynes-Ramsey rule

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}(r-\rho) \tag{6}$$

to determine the rate of growth of the gross domestic product $GDP(t)$ on a balanced growth path.

Answer: the Gross Domestic Product, $GDP(t)$, equal to final output $Y(t)$ minus the amount used in intermediate production is

$$GDP(t) = Y(t) - X(t) = C(t) + R(t)$$

Using the equilibrium value of $X(t) = xA(t)$ and the production function

$$Y(t) = L^{1-\alpha} A(t)^{1-\alpha} X(t)^{\alpha} = A(t) L^{1-\alpha} x^{\alpha}$$

it appears that output $Y(t)$, the production of intermediate products $X(t)$ and then $GDP(t)$ grow at the same rate as $A(t)$. On a balanced growth path, consumption and research expenditure grow at the same rate as $A(t)$. Thus, using equations (5) and (6) we get the rate of growth of GDP

$$g = \frac{\lambda\pi - \rho}{\sigma} = \frac{\lambda \frac{1-\alpha}{\alpha} (1-s)^{\frac{\alpha}{\alpha-1}} L \alpha^{\frac{2}{1-\alpha}} - \rho}{\sigma} \tag{7}$$

11. What is the impact of an increase in the subsidy s on the growth rate of GDP? Explain.

Answer: the subsidy increases the growth rate because it raises the profit of producers of intermediate products and then the value of innovations.

Exercise 2 (10 points) The standard growth model emphasizes the role of disembodied technological progress. However, it is commonly believed that much of the productivity growth is driven by technological progress that is embodied in new types of capital equipments. In this exercise, technological progress is "investment specific": it improves the quality of investment goods that becomes embodied in the productive capital stock.

Consider an economy with identical consumers (the population size is normalized to 1). The representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \tag{8}$$

The production function of the final good is

$$y_t = A \left(k_t^e\right)^{\alpha} n_t^{1-\alpha}$$

where $n_t \leq 1$ is the labor input and k_t^e is the amount of equipments used in the final good production. A is the neutral technological progress (for tractability, this is kept constant and equal to 1). Since leisure is not in the utility function, one can set $n_t = 1$

The resources constraint is

$$y_t = i_t^e + c_t \tag{9}$$

That is, the final good can be used for consumption and for investment in equipments Equipments k_t^e follow the law of motion

$$k_{t+1}^e = k_t^e (1-\delta) + i_t^e q_t \tag{10}$$

That is, the current productive capital stock is equal to undepreciated capital from the previous period plus net investment $i_t^e q_t$.

Note that physical investment i_t^e in consumption units is enhanced by a factor q_t. Investment in efficiency units is $i_t^e q_t$. The factor q_t represents the state of technology to produce equipments at t. In fact, q_t represents the quantity of equipments that is obtained if you use one unit of the final good. Changes to q_t represent changes in the technology that is specific to investment. γ_q is the exogenous rate of growth of investment-specific technological change. Let $q_0 = 1$, then

$$q_t = \left(1 + \gamma_q\right)^t$$

1. (1 point) What does σ represent in (8)? Explain.

   Answer: $1/\sigma$ is the EIS.

2. (2 points) Solve the problem of the social planner who maximizes (8) subject to (9) and (10). Suppose that the solution is interior, take the derivative with respect to k_{t+1}^e and find the Euler equation.

   Answer: The resource constraint can be written as

$$A\left(k_t^e\right)^\alpha - \frac{k_{t+1}^e}{q_t} + \frac{k_t^e}{q_t}(1-\delta) - c_t = 0$$

Then, the foc is

$$\frac{c_t^{-\sigma}}{q_t} = \beta c_{t+1}^{-\sigma}\left[\alpha A\left(k_{t+1}^e\right)^{\alpha-1} + \frac{(1-\delta)}{q_{t+1}}\right]$$

3 (1 point) Write the transversality condition.

$$\lim_{t\to\infty}\beta^t u'(c_t)k_t^e[\alpha A\left(k_t^e\right)^{\alpha-1} - \frac{(1-\delta)}{q_t}] = 0$$

4. (1 point) Provide the definition of "balanced growth path" (BGP).

   Answer: The rates of growth of all variables are constant (not necessarily the same)

5. (2 point) Using the Euler equation, show that along a BGP, it is needed that for all t

$$\frac{y_t q_t}{k_t} \tag{11}$$

   is constant.

   Answer: Write

$$\frac{c_t^{-\sigma}}{q_t} = \beta\frac{c_{t+1}^{-\sigma}}{q_{t+1}}\left[\alpha\frac{q_{t+1}y_{t+1}}{k_{t+1}} + 1 - \delta\right]$$

or

$$\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \beta\frac{1}{1+g_q}\left[\alpha\frac{q_{t+1}y_{t+1}}{k_{t+1}} + 1 - \delta\right]$$

Then, BGP, which requires that $\frac{c_{t+1}}{c_t}$ is constant, implies that $\frac{q_{t+1}y_{t+1}}{k_{t+1}}$ is constant. If so, the left hand side is constant.

6. (2 point) Using (9) find out whether on a BGP y_t, i_t^e and c_t grow at the same rate. Denote by γ_y the rate of growth of y. Show that along a BGP the ratio γ_y/γ_q is constant and determine this ratio.

   Answer: Take the production function

$$y_t = A\left(k_t^e\right)^\alpha$$

Take logs:

$$ln(y_t) = \ln(A) + \alpha\ln(k_t^e)$$

Differentiate respect to time t and get

$$\gamma_y = \alpha\gamma_k$$

Since $\frac{q_{t+1}y_{t+1}}{k_{t+1}}$ *has to be equal to a constant,*

$$\gamma_y + \gamma_q = \gamma_k.$$

From the resource constraint,

$$y = c + i$$

Then the three variables have to grow at the same rate: $\gamma_y = \gamma_c = \gamma_i$
One obtains, $\gamma_y = \alpha(\gamma_y + \gamma_q)$ *or*

$$\gamma_y = \frac{\alpha}{1-\alpha}\gamma_q$$

7. (1 point) Suppose that the final good is the numeraire. Along a BGP, will the price of the final good and the price of equipment coincide as in the standard growth model? Discuss.

   Answer: The price of equipments is 1/q. It is the cost of producing an equipment in terms of final output. Since 1/q is decreasing over time, equipments become less expensive relative to the final good.