Part 1 – Motion Planning
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Part 2 – Crowd Simulation

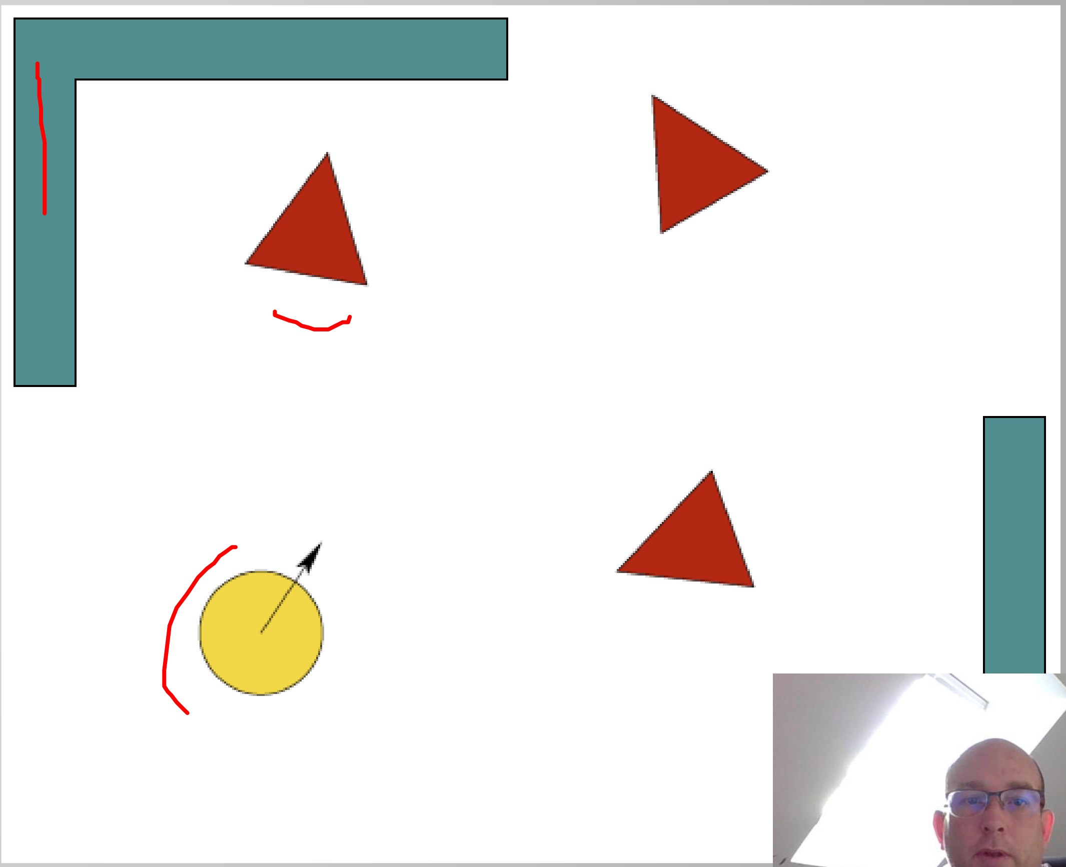
Motion Planning in Dynamic Environments

Dynamic environments

A robot moves among static and moving obstacles

How to model this?

The time axis $T = [0, t_F]$ becomes critical, unlike before.

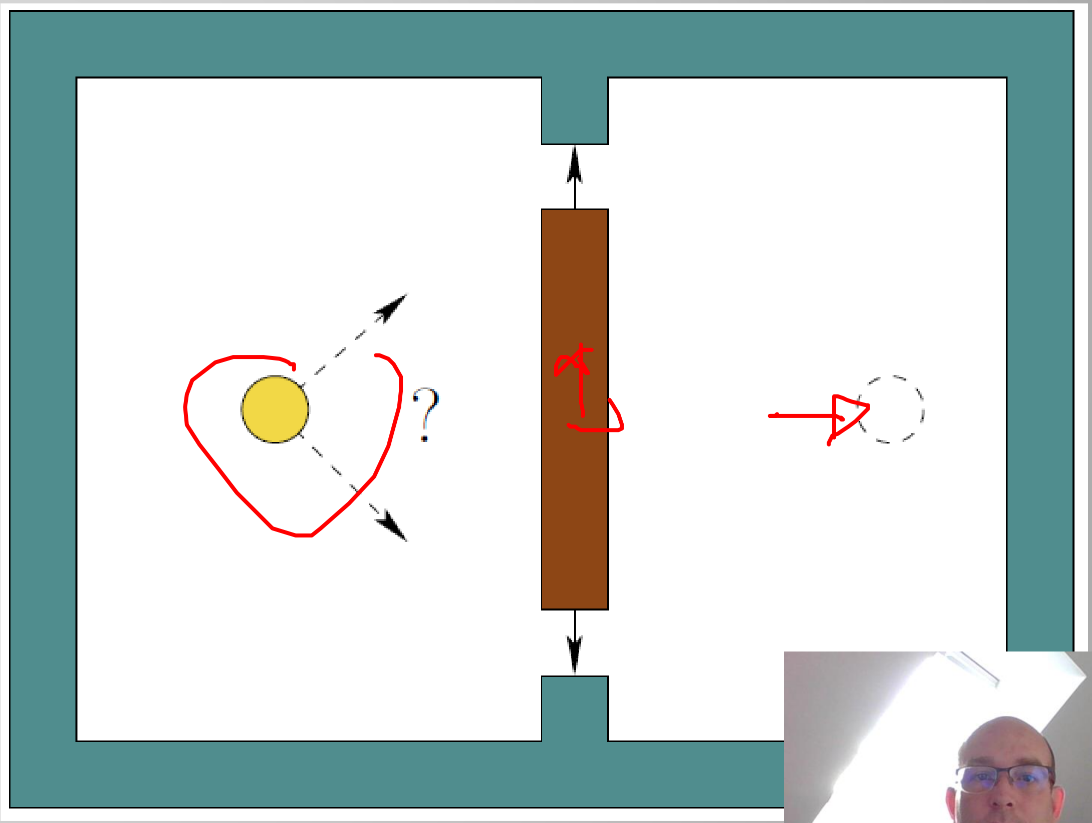


FUNDAMENTAL LIMITATIONS

A THOUGHT EXPERIMENT

THERE IS AN AUTONOMOUS SLIDING DOOR.

WHERE SHOULD THE ROBOT MOVE?

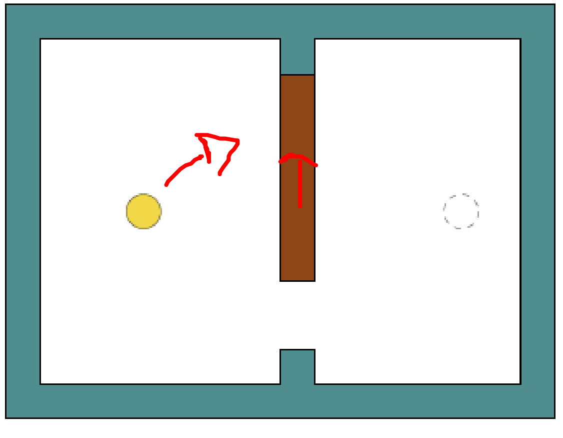


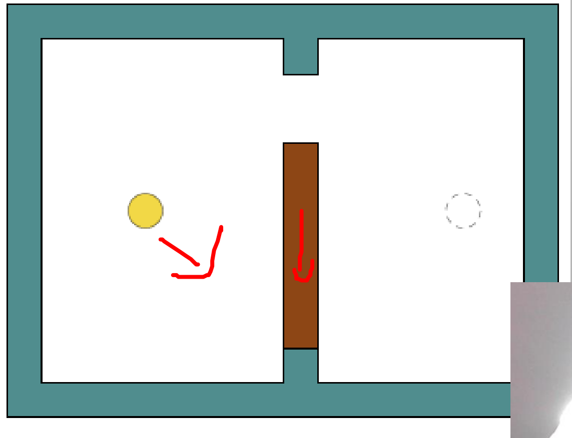
Fundamental limitations

There is no solution if an adversarial "demon" moves the wall.

Iterative replanning leads to oscillation.

If wall position follows a Markov chain, then wait by either potential opening.


Mode 1


Mode 2

Predictable Obstacles

Let $T = [0, t_F]$ denote a time interval of interest.

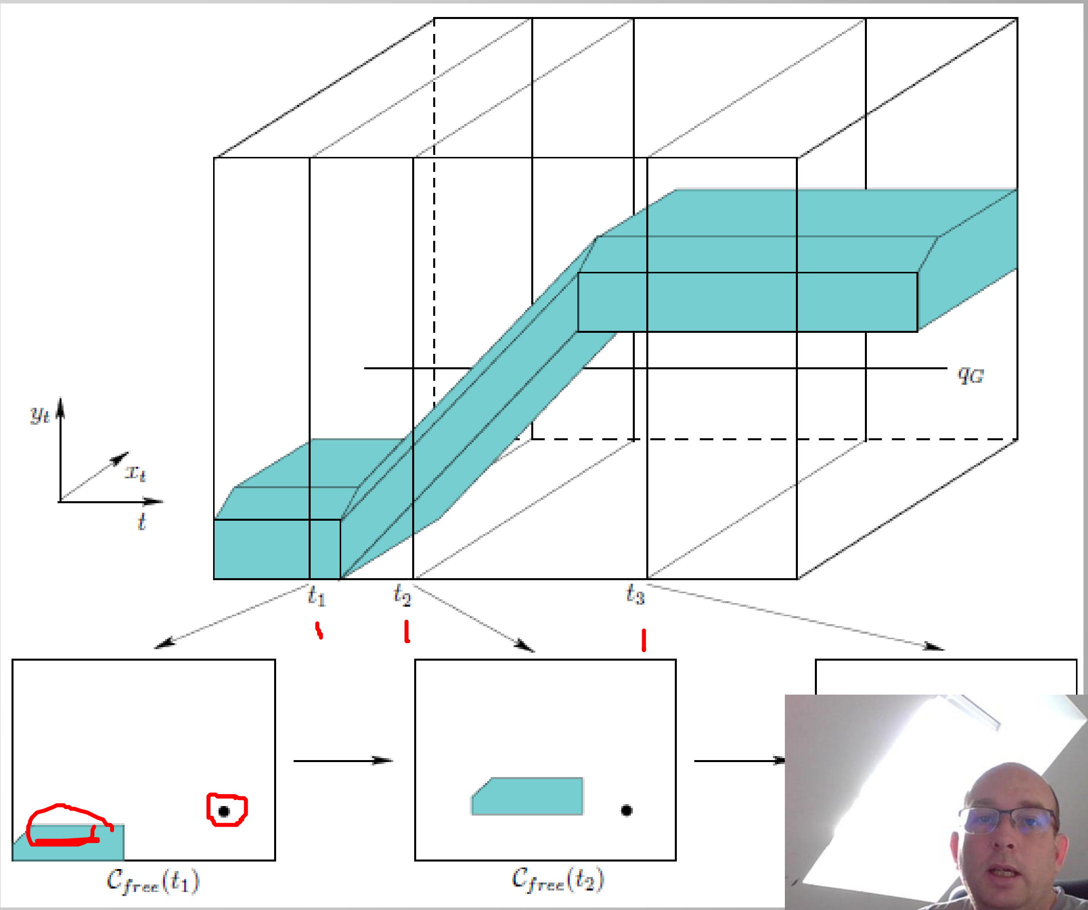
Let $O(t) \subset W$ denote the obstacle at time $t \in T$.

Assume $O(t)$ is given for all $t \in T$.

Let $Z = C \times T$ denote the *configuration-time* space.

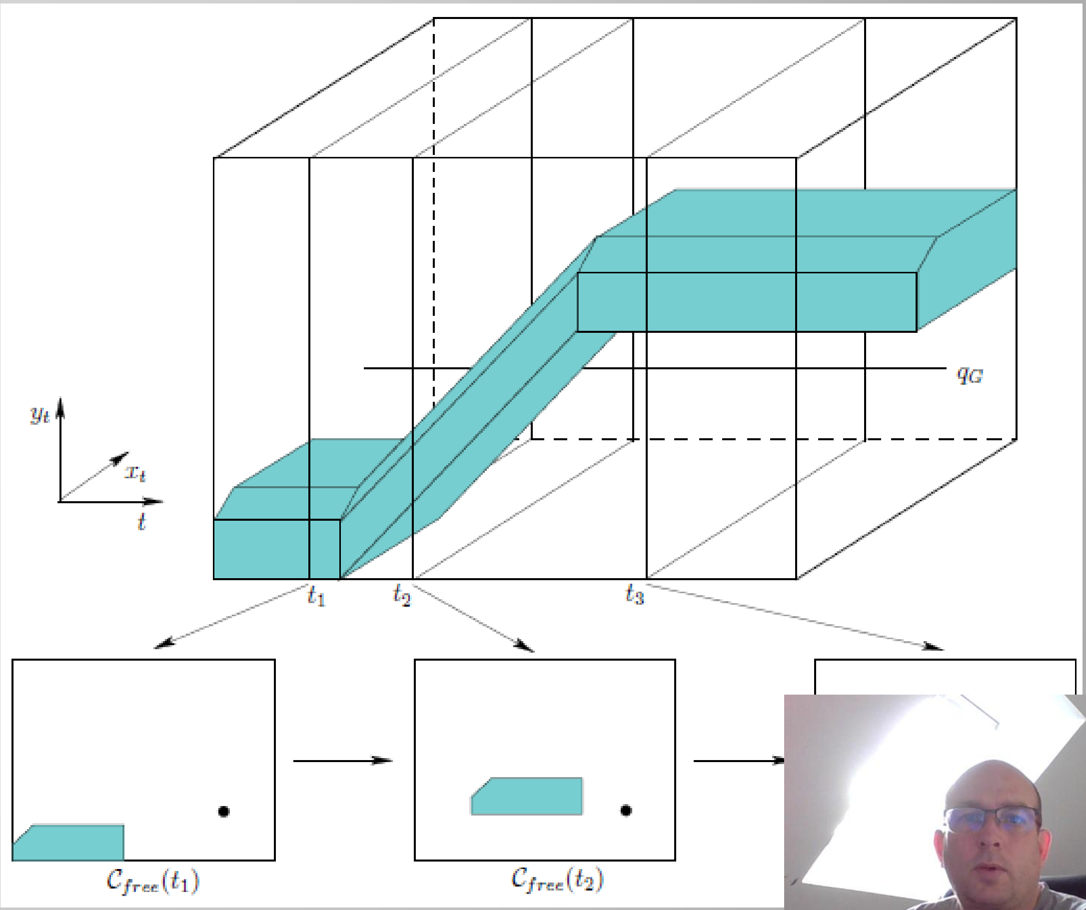
Each $(q, t) \in Z$ specifies both $A(q)$ and $O(t)$.

→ At each time slice $t \in T$, we must avoid
$C_{obs}(t) = \{q \in C \mid A(q) \cap O(t) \neq \emptyset\}$



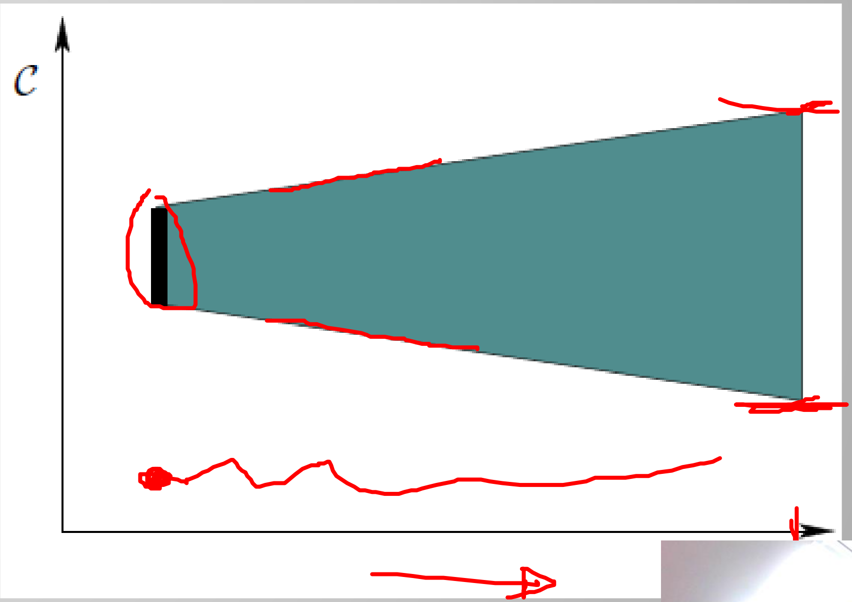
Finding a Collision-Free Path

- Let: $C_{obs} = \{(q, t) \in X \mid A(q) \cap O(t) \neq \emptyset\}$,
- and $C_{free} = C \setminus C_{obs}$.
- Initial state: $q_{init} = (q_I, 0)$.
- Goal region $q_{goal}(t) \subset C_{free}(t)$ (a combination of time and configuration).
- Problem: Compute a continuous *trajectory*
- $\tau : T \rightarrow C_{free}$
- so that $\tau(0) = q_{init}$ and $\tau(t) \in q_{goal}$ for some $t \in T$.
- Note: A trajectory is a time-parametrized path.
- More challenging case: The robot has a maximum speed bound
- Even more challenging: Robot motion is specified as a nonlinear system



Bounded Uncertainty Models

- Let one moving obstacle be called a *body*.
- The body moves with a maximum speed bound:
  $||vk|| \leq c$.
- Using bounded uncertainty models, we once again reason in configuration-time space Z.
- This is called a *reachable set* computation.
- Determine a safe $q \in Cfree(t)$ for every future t.
- Find a trajectory : $T \rightarrow Zfree$.

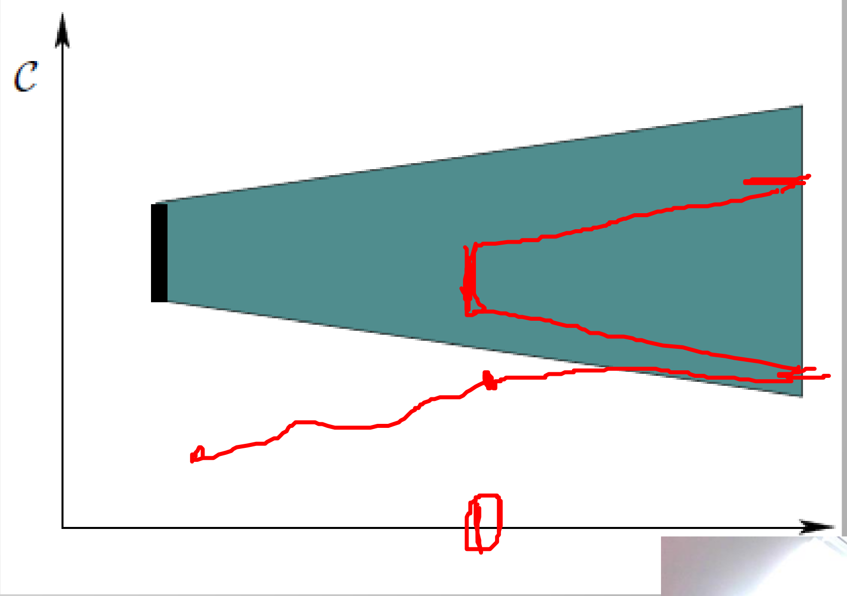


Bounded Uncertainty Models

Could over-approximate Cobs(t): Conservative bounds fine, but lose completeness.

What should happen if sensors can tell current obstacle locations during execution?

- If there was a solution from the initial time, then on-line information is not necessary.
- The problem may initially appear unsolvable, but on-line information could make it solvable.
- It is tempting to try a replanning approach.



Probabilistic Models

- Rather than bounded uncertainty, suppose that a density

$$p(x'b \mid xb)$$

  is known.
- xb is the body state at time t
- $x'b$ is the body state at time $t + \Delta t$
- Where might the body go next?
  - Simple diffusion models
  - Brownian motions
  - Could calculate with particle filters

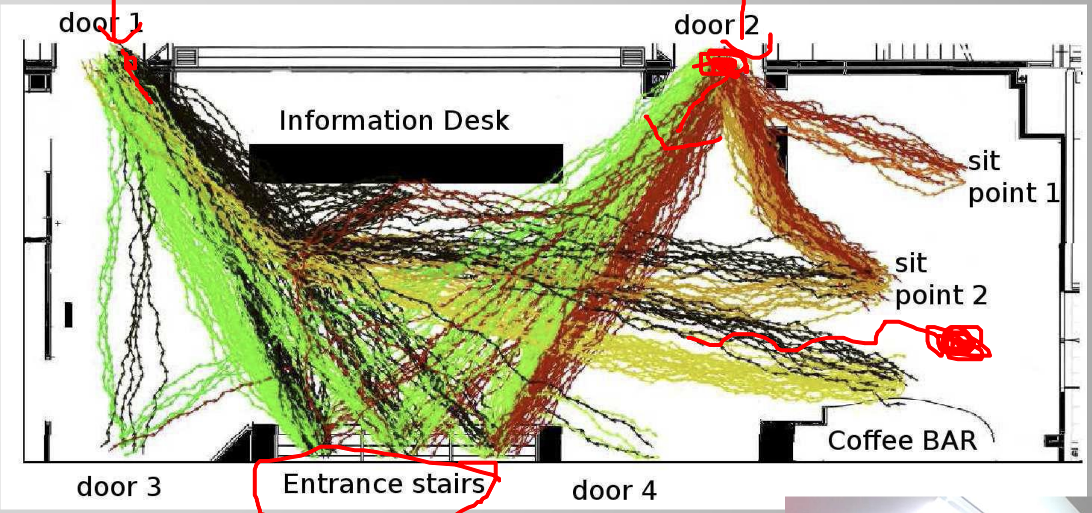
Probabilistic Models

Perhaps a model can be learned from data.

Intentions become important to reduce model complexity.

Could learn a Hidden Markov Model (HMM) that captures positions, velocities, and intentions of obstacles.

Could develop sampling-based (particle) representations of future obstacle trajectories.



Velocity Obstacle

Two rigid bodies A and B moving in R2.

They have constant velocities vA and vB.

If vB is constant, what values of vA cause collision?