Report 6

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1 Statement of the Problem:

A gaussian laser beam is tightly focused around a group of cold potassium atoms. Upon measuring the oscillation frequency of the atoms in the transverse plane around the focal point, we seek to find the waist of the beam at the focal point.

2 Quantities and variables:

We know the following quantities:

- The wavelength of the laser is ($\lambda_L = 1064nm$)
- Mass of a single potassium atom is ($m = 39.96u$)
- We consider a group of $n = 40$ potassium atoms
- The closest optical transition of the atoms has the following properties:
  - Wavelength $\lambda_0 = 770.1nm$
  - Linewidth $\Gamma = 6MHz$
  - Saturation intensity $I_{sat} = 1.75mW/cm^2$
- The angular frequencies of oscillation of the atoms in the transverse plane around the focal point is $\omega_{\perp}$.
- The laser beam's power is P.

- The waist of the laser is w_0.

These last two quantities are related through the table in figure 2:

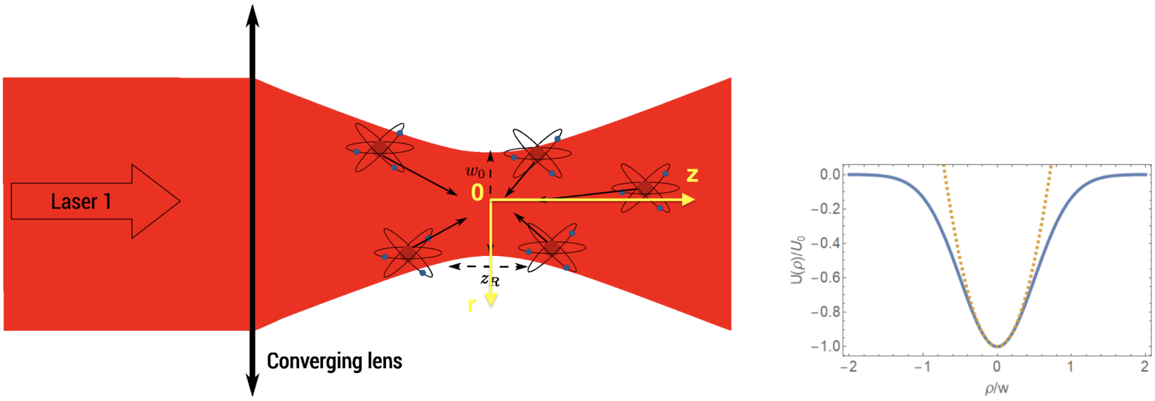

Figure 1: Transverse trapping potential of an optical dipole trap.

Laser power (mW)	15.5	25	29	34	40	75	193
Oscillation frequency (Hz)	115	160	145	180	177	256	386

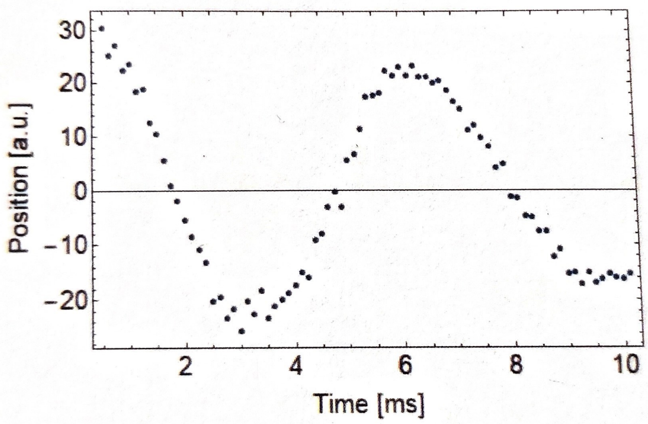

Figure 2: Above: table showing an angular frequency ($\omega_\perp$) as a function of (P) and below: an example of the oscillation of a particle at ($P = 30mW$)

We will also use the following variables:

- Detuning $\delta = \omega_L - \omega_0' = \frac{2\pi c}{\lambda_L} - \frac{2\pi c}{\lambda_0}$

- Radius from the central axis of the laser is ρ, coordinate on the central axis is z
- The intensity of the laser is $I(\rho, z)$
- The intensity of the laser at it's waist is I_0
- The depth of the trap potential is U_0

We want to find the waist of the beam ω_0

3 Model:

We consider the gaussian beam being focused by a lens onto a group of potassium atoms. To solve this problem, we use the semi-classical approach, in which the atoms are described by the quantum model and light is described classically. Hence, both the radiation pressure and the dipole force are acting on the atoms. Let us compute the detuning:

$$\delta = -6.76 \cdot 10^{14}$$

Due to $|\delta_0|$ being this large compared to our other values, we can neglect the radiation pressure due to the fact that:

$$\langle \vec{F}_{PR} \rangle = \frac{I \sigma_0}{c(1 + 4(\frac{\delta - \vec{k}_L \cdot \vec{v}}{\Gamma})^2)} \approx 0$$

We thus only have the dipole force and since $\delta < 0$, the resulting potential is attractive. Therefore, the atoms will experience a force opposed to their direction of motion at all instances so that all atoms are attracted to the center, hence the oscillations in the transversal plane.

Due to the frequency of light being close to its resonance, we also assume that the detuning is smaller than the width of the peak of the resonance curve. Since the atoms are cold, we assume that their kinetic energy is low enough to be trapped in the region surrounding the focal plane. Moreover, we observe that due to δ_0 being so large, we also have:

$$s = \frac{I}{I_{sat}} \frac{1}{1 + (\frac{2\delta}{\Gamma})^2} = \frac{I}{I_{sat}} 1.9 \cdot 10^{-17} << 1$$

4 Problem solving:

Since we have $s \ll 1$,

$$U_{Dip} = \frac{\hbar}{8} \frac{\Gamma^2}{I_{sat}} \frac{I}{\delta}, \tag{1}$$

Now let's write down the intensity profile of a gaussian beam with power P:

$$I(\rho, z) = \frac{2P}{\pi w_0^2} \cdot \frac{1}{1 + (\frac{z}{z_R})^2} \cdot e^{-2\frac{\rho^2}{w_0^2(1+(\frac{z}{z_R})^2)}},$$

here $\frac{2P}{\pi w_0^2} = I_0$, with 2 in the numerator coming from the integration across the waist. Using the fact that $\frac{\rho}{w_0} \ll 1$ near enough to the center, let's Taylor-expand this function to the first order of $\frac{\rho^2}{w_0^2}$ at $z = 0$:

$$I(\rho, z = 0) \approx \frac{2P}{\pi w_0^2}(1 - \frac{2\rho^2}{w_0^2}), \tag{2}$$

meaning that the intensity profile near the center can be approximated as a parabola.

Plugging expression (2) into the into expression (1):

$$U_{Dip} = \frac{\hbar}{8} \frac{\Gamma^2}{I_{sat}} \frac{\frac{2P}{\pi w_0^2}(1 - \frac{2\rho^2}{w_0^2})}{\delta}, \tag{3}$$

Around the bottom of the trap, the potential can be approximated by a harmonic potential, as we saw that it can be approximated by a parabola. So we can write:

$$U(\rho \ll w_0) \approx -U_0 + \frac{1}{2} m w_\perp^2 \rho^2 + \frac{1}{2} m w_\parallel^2 z^2, \tag{4}$$

where U_0 refers to the depth of the potential well and it controls how fast atoms can still be trapped, since atoms with a kinetic energy higher than U_0 can overcome the potential trap.

As the oscillation takes place only in the transverse plane with respect to the propagation direction of the laser beam, we just take into consideration the kinetic energy with the factor ρ^2. Using (4) and (3) this gives us:

$$\frac{1}{2}mw_{\perp}^2\rho^2 = -\frac{2P}{\pi w_0^2} \cdot \frac{2\rho^2}{w_0^2} \cdot \frac{\hbar}{8} \cdot \frac{\Gamma^2}{I_{sat}} \cdot \frac{1}{\delta}$$

Developing,

$$w_{\perp}^2 = \frac{P\Gamma^2\hbar}{\pi w_0^4 I_{sat}|\delta|m}$$

5 Numerical values

We plot the square of the oscillation angular frequency as a function of laser power (figure 3).

Here we plotted $w_{\perp}^2 = (2\pi \cdot f)^2$, where f is the oscillation frequency v.s. the laser power P, as we expect a square relation.

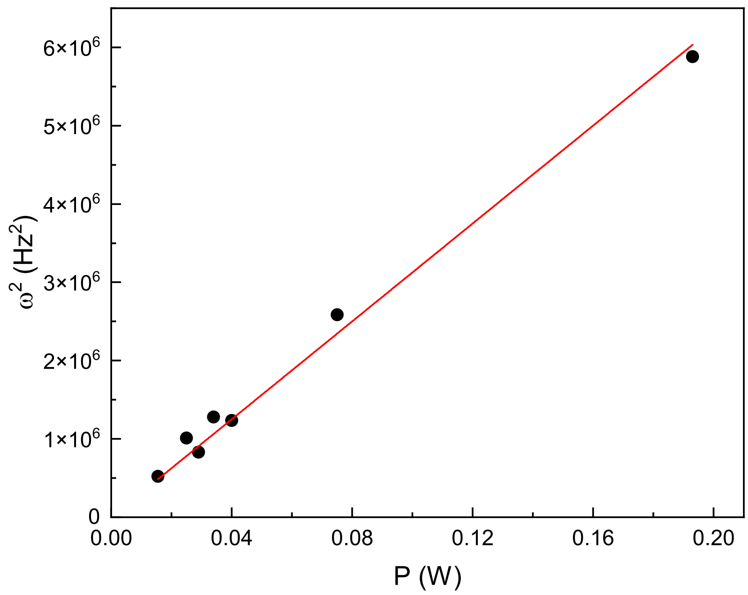

Figure 3: Square of the transverse oscillations angular frequency of the atoms as a function of laser power (black dots) and the corresponding linear fit with fixed intercept at 0(red).

From the slope of the graph $\frac{P}{\omega_{\perp}^2} = 29 \cdot 10^{-9}[\frac{W}{Hz^2}]$, obtained with MSE method, we deduce the waist of the laser beam to be

$$w_0 = (\frac{\Gamma^2\hbar}{\pi I_{sat}|\delta|m} \cdot \frac{P}{w_\perp^2})^{\frac{1}{4}} = (23 \pm 3)\mu m$$

6 Conclusion

As a result we get that the size of the waist is $23 \pm 3\mu m$ which is a reasonable order of magnitude, but the value is considered to be very low since in the previous report the problem in which we were supposed to find minimum size of waist to penetrate the wood desk we obtained a waist of $250\mu m$. Also obtained waist size is much bigger than the wavelength, as it should be.

The first notable source of error comes from the power and oscillating frequency of the laser since in our calculation of the waist we used the MSE method and getting more points on the graph would make the slope calculation more precise.

Second source of error stems from our numerous assumptions made to derive our model. An example of one of them is the harmonic oscillation model assumed. The point is that the harmonica oscillation assumption holds for atoms of $\frac{\rho}{\omega}$ close to zero, which we might not have if the amplitude of the atoms is high enough. In our model the values of the delta is $\delta = -6.76 \cdot 10^{14}$. Although the values were given but in the certain situation the assumption could be broken.