THE RIDDLE

PHY208

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1 Problem statement

A 1064nm Gaussian laser is tightly focused on a cloud of cold potassium 40 atoms (atomic mass: 39.96u). The closest optical transition has the following properties:

- Wavelength: 770.1 nm
- Linewidth: $2\pi \times 6$ MHz
- Saturation intensity: 1.75 mW/cm^2

The oscillation frequency of the atoms in the transverse plane around the focal point is measured as a function fo the laser power:

Laser power (mW)	15.5	25	29	34	40	75	193
Oscillation frequency (Hz)	115	160	145	180	177	256	386

Table 1: Power and oscillation frequency dependency.

Question: What is the radius of the laser beam at its waist?

2 Variables

2.1 Known data and constants

From the problem statement we can extract the following data:

	Quantity	Variable	Value
1	Laser wavelength	λ_L	1064 nm
2	Relevant Transition wavelength of ^{40}K	λ_0	770.1 nm
3	Atomic mass of ^{40}K	m	39.96 amu
4	Linewidth of optical transition	$\Delta\nu_0$	6 MHz
5	Saturation intensity of transition	I_{sat}	1.75 mW/cm^2
6	Laser power	P	Table 1
7	Measured transverse oscillation angular frequency	$\omega_\perp$	Table 1 (frequencies $\frac{\omega_\perp}{2\pi}$)
	Physical Constant	Label	Value
1	Reduced Planck's constant	$\hbar$	1.055×10^{-34}J/s
2	Boltzmann constant	k_B	$1.381 \times 10^{-23}\mathrm{m^2\,kg\,s^{-2}\,K^{-1}}$
3	Speed of light	c	$2.998 \times 10^8\mathrm{m\,s^{-1}}$

Table 2: Known data and constants

2.2 Sought after variable

The variable we are looking for is:

- w_0 - radius of the laser beam at the waist

3 Model

In our considerations we use the semi-classical model. A sketch of the problem can be seen in Figure 1. Since the laser is red-detuned ($\delta = \frac{2\pi c}{\lambda_L} - \frac{2\pi c}{\lambda_0} = \omega_L - \omega_0 < 0$), the dipole force creates an attractive potential about the center of the beam that traps atoms. With the given parameters, we model the dipole potential as a harmonic oscillator potential and use the frequency of oscillations from the data to find its gradient. The gradient depends on several known or calculable quantities and one unknown that is the beam size at the waist which can be found. Although the radiation pressure creates the opposite effect as the dipole force, it can be neglected since it scales with $\frac{1}{\delta^2}$ while dipole force scales with $\frac{1}{\delta}$ and we have that the frequency of the laser ($\omega_L = 1.779 \cdot 10^{15}\,\mathrm{Hz}$) is far from the resonant frequency of the relevant optical transition ($\omega_0 = 2.446 \cdot 10^{15}\,\mathrm{Hz}$), by a value much greater than the linewidth ($2\pi \cdot 6\,\mathrm{MHz}$).

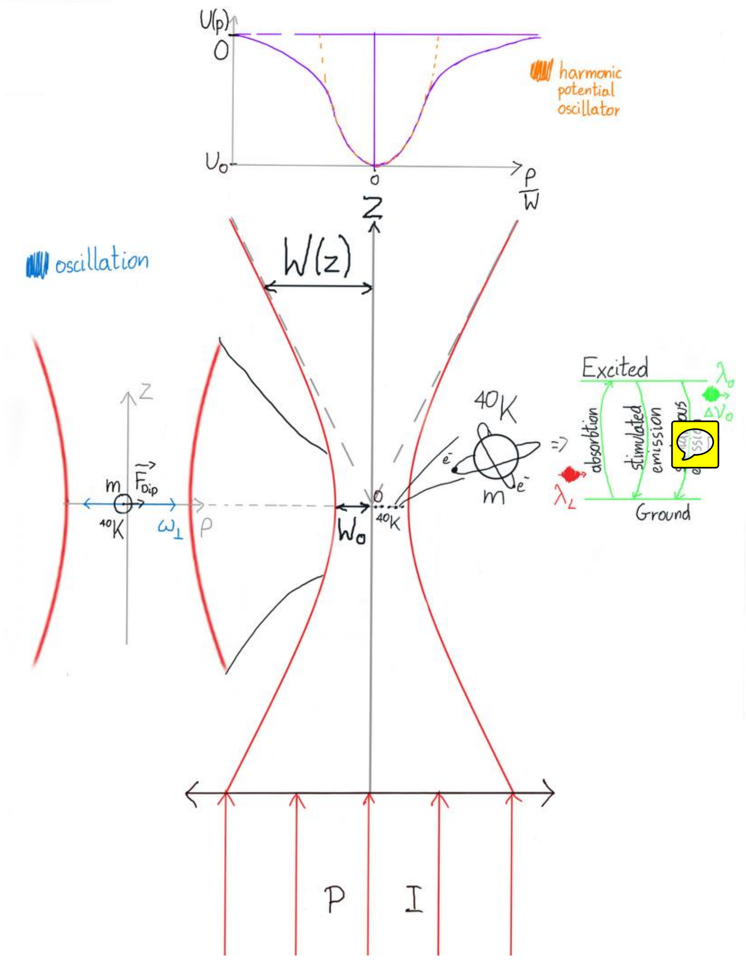

Figure 1: Sketch of the problem

Since the experiment used a Gaussian laser, we model it with a Gaussian beam profile. The intensity of the Gaussian laser beam has the general expression,

$$I(\rho, z) = I_0 \frac{1}{1 + \left(\frac{z}{z_R}\right)^2} \exp\left(-\frac{2\rho^2}{w_0^2\left(1 + \left(\frac{z}{z_R}\right)^2\right)}\right), \tag{1}$$

where I_0 is the peak intensity at the center of the waist, w_0 is the radius at the waist, z_R is the Rayleigh length whose precise expression will not matter, z is the position along the axis parallel to the laser beam and ρ the radial position (see figure 1). Since the beam is focused to the location of the trap and the atoms oscillate in the transverse plane, we can set $z = 0$ and the intensity reduces to,

$$I(\rho, 0) = I_0 e^{-\frac{2\rho^2}{w_0^2}}. \tag{2}$$

To relate I_0 to the power of the laser P, we need to integrate the laser intensity over the cross section at $z = 0$,

$$P = \int_0^\infty I(\rho, 0) 2\pi\rho \, d\rho = I_0 \int_0^\infty e^{-\frac{2\rho^2}{w_0^2}} 2\pi\rho \, d\rho = 2\pi I_0 \frac{1}{2}\frac{w_0^2}{2}\left[-e^{-\frac{2\rho^2}{w_0^2}}\right]_0^\infty = \frac{\pi I_0 w_0^2}{2}. \tag{3}$$

Thus, we have $I_0 = \frac{2P}{\pi w_0^2}$. From the lecture slides and textbook, the potential in the transverse plane U_{dip} corresponding to the dipoles force is,

$$U_{dip}(\rho, 0) = \frac{\hbar\Gamma^2 I(\rho, 0)}{8 I_{sat}\delta} = \frac{\hbar\Gamma^2}{8 I_{sat}\delta}\frac{2P}{\pi w_0^2} e^{-\frac{2\rho^2}{w_0^2}}. \tag{4}$$

Here I_{sat} is the saturation intensity of the transition and $\Gamma = 2\pi\Delta\nu_0$ is the spontaneous decay rate of the transition derived from the Heisenberg uncertainty principle. Assuming small ρ compared to w_0, we perform a Taylor expansion around the origin and obtain a quadratic potential corresponding to a harmonic oscillator. Taking into consideration that the laser is red-detuned with respect to the optical transition,

$$U_{dip}(\rho, 0) = \frac{\hbar\Gamma^2 P}{4 I_{sat}\delta\pi w_0^2}\left(1 - \frac{2\rho^2}{w_0^2}\right) = -\frac{\hbar\Gamma^2 P}{4\pi w_0^2 |\delta| I_{sat}} + \frac{\hbar\Gamma^2 P}{2|\delta| I_{sat}\pi w_0^4}\rho^2. \tag{5}$$

Since this resembles a harmonic oscillator of the form $U_{dip}(\rho) = U_0 + \omega_\perp \rho^2$, we have the following expression for the frequency of oscillations,

$$\omega_\perp^2 = \frac{\hbar\Gamma^2 P}{\pi(\omega - \omega_0) m I_{sat} w_0^4} = \alpha P, \tag{6}$$

where α replaces the proportionality constant between $\omega_\perp^2$ and P. Thus, the beam radius at the waist w_0 is given by the formula,

$$w_0 = \left(\frac{\hbar\Gamma^2}{\pi|\omega - \omega_0| m I_{sat}\alpha}\right)^{\frac{1}{4}}. \tag{7}$$

With all other parameters of Equation 7 already specified, we turn to the data to extract the value of α. Figure 2 shows the oscillation frequency squared as a function of the laser power. We found a

Linear Regression fit matches the data with parameter $R^2 = 0.9315$, and since Equation 6 has $\omega_\perp^2$ as linear in P, the model is validated.

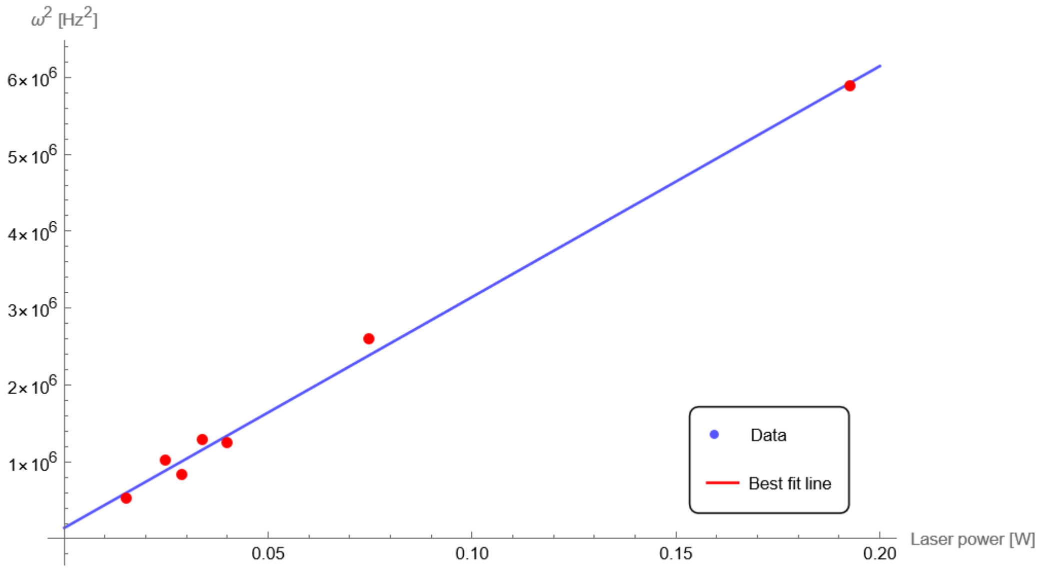

Figure 2: Linear fit of the data. Line of best fit: $\omega_\perp^2 = 1416 + 3.003 \times 10^7 P$

From Figure 2, we find that $\alpha = 3.003 \times 10^7$ via the linear regression fit. Thus, we finally find that,

$$w_0 = 37.68\,\mu\text{m}. \tag{8}$$

4 Conclusion and extension

We have seen that it is possible to measure the radius at the waist of the laser beam using the oscillation frequencies of atoms trapped in the dipole potential generated by this laser. The value we obtained for the radius of the beam at waist, $w_0 = 53.28\,\mu\text{m}$ is reasonable as it is much larger than the wavelength of the laser $\lambda_L = 1.064\,\mu\text{m}$ and we know that the laser wavelength places a lower bound on the focusing size of a beam of light due to diffraction. At the same time, the value is of microscopic dimensions so that it makes sense to use this method to determine the radius at the beam waist instead of simply measuring it using traditional optical techniques.

To perform the Taylor expansion in Equation 5, we have assumed that the Potassium atoms oscillate near the center of the laser beam, where we could approximate the potential profile by a harmonic (quadratic) one. However, if the temperature of the gas of atoms does not have a low enough temperature, some atoms might be able to get to the edges of the traps where the potential tappers off towards 0 at infinity. We can thus estimate the temperature that the ensemble of Potassium atoms needs to have in order to be trapped by imposing the condition that the average energy of one atom is smaller than the depth of the potential well. If we work in two-dimensions (only transverse motion is considered), the avergae thermal kinetic energy associated to two degrees of freedom is,

$$E_{th} = 2 \cdot \frac{1}{2} k_B T. \tag{9}$$

Equating the binding energy of the potential to the kinetic energy of the atom, we have,

$$|U_0| = k_B T_{max}. \tag{10}$$

Here, T_{max} is the maximum allowed temperature of the ensemble of atoms. The depth of the potential well is,

$$|U_0| = \frac{\hbar\Gamma^2 P}{4\pi w_0^2 |\delta| I_{sat}} \tag{11}$$

and this gives the maximum temperature of potassium atoms that can be trapped:

$$T_{max} = \frac{\hbar\Gamma^2 P}{4\pi w_0^2 |\delta| I_{sat} k_B} \tag{12}$$

For the highest power in the problem ($P = 193\,\mu\text{W}$) we get,

$$T_{max} = 10\,\mu\text{K}. \tag{13}$$

This value is much smaller than room temperature. Moreover it is also an order of magnitude smaller than the temperature that can be achieved with optical molasses (formula extracted from the lectures): $\frac{\hbar\Gamma}{2k_B} = 144\,\mu\text{K}$. Since only atoms with energies smaller than $k_B T_{max}$ will remain inside the optical dipole trap, we see another mechanism to cool down atoms. If we lower the laser power enough we can achieve much lower temperatures than those achieved with optical molasses. However, there is a trade-off: while we can achieve very low temperatures, we will lose atoms that have energies larger than $k_B T_{max}$.