The Riddle - PHY208

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1 Introduction and description of the phenomenon

In this problem, we tried to estimate the waist of a laser beam at the focal point, by analyzing the oscillations in the transverse plane of potassium 40 atoms on which the laser is focused. Let us analyze and focus on the phenomenon. When a laser beam is focused on an atom (or a cloud of atoms as in this case), different phenomena can occur depending on the laser's wavelength. If its wavelength is within the interval an optical transition can happen, then photons will be absorbed. This is not our case. Indeed we have that $|\delta| = |2\pi c(\frac{1}{\lambda_L} - \frac{1}{\lambda})| \approx 2\pi \times 1.08 \times 10^8 MHz > \Delta\omega$, where λ is the wavelength of the closest optical transition, λ_L the wavelength of the laser, and $\Delta\omega$ the linewidth of this transition. We can then conclude that no absorption is involved. We can then conclude that we only need to consider radiative forces: radiation pressure and dipole forces. As seen in the lecture, we have the radiation pressure $F_{RP} \propto \frac{1}{\delta^2}$ and the dipole force $F_{DP} \propto \frac{1}{\delta}$. As δ is in the order of magnitude of 10^{14}, we can ignore the radiation pressure in the problem. As seen in the lecture, we can define a potential associated with the dipole force. To study the movement of these atoms, we need to study the gradient of this potential. In particular, from the lecture, we know that if a Gaussian laser beam focused on a cloud of atoms is red-detuned (meaning $\lambda_L > \lambda$ or $\delta < 0$), the atoms will be attracted to regions with higher intensity. Since the beam is Gaussian, this happens in the center of the beam. In particular, we only consider the oscillations in the transverse plane, meaning that, assuming the laser beam propagates in the z-direction, we consider oscillations in the planes $z = cte$. In particular, we will consider the focus of the beam to be at $z = 0$ and oscillations around the origin. Finally, reminding that any potential can be approximated to a harmonic potential around a minimum (in this case it is at $\rho = 0$), we will compare the potential due to this phenomenon to the one of a harmonic oscillator to find the waist of the beam. Finally, we will run a linear regression between the values of the laser power and the oscillation frequency to find the value of the waist.

2 Assumptions and simplifications

2.1 Variables

- λ: wavelength
- w_0: waist
- P: power
- I_L: intensity
- U_{sat}: saturation intensity
- Γ: linewidth

Firstly we realize that $\lambda_L > \lambda_{sat}$ (meaning the laser is red-detuned), thus $s << 1$ where s is the ratio between I_L and I_{sat}.Taking this into account in the equation for the potential equation we end up with:

$$U_{dip} = \frac{\hbar}{8}\frac{\Gamma^2}{I_{sat}}\frac{I_L}{\delta} \tag{1}$$

The expression for the radiation pressure is: $F_{PR} \propto \frac{I_L}{\delta^2}$. The intensity profile of a Gaussian beam with power P is:

$$I(\rho, z) = \frac{2P}{\pi w_0^2}\frac{1}{1+(\frac{z}{z_R})^2}exp(-2\frac{\rho^2}{w_0^2(1+(\frac{z}{z_R})^2})) \tag{2}$$

where the variable $z_R = \frac{\pi w}{\lambda}$

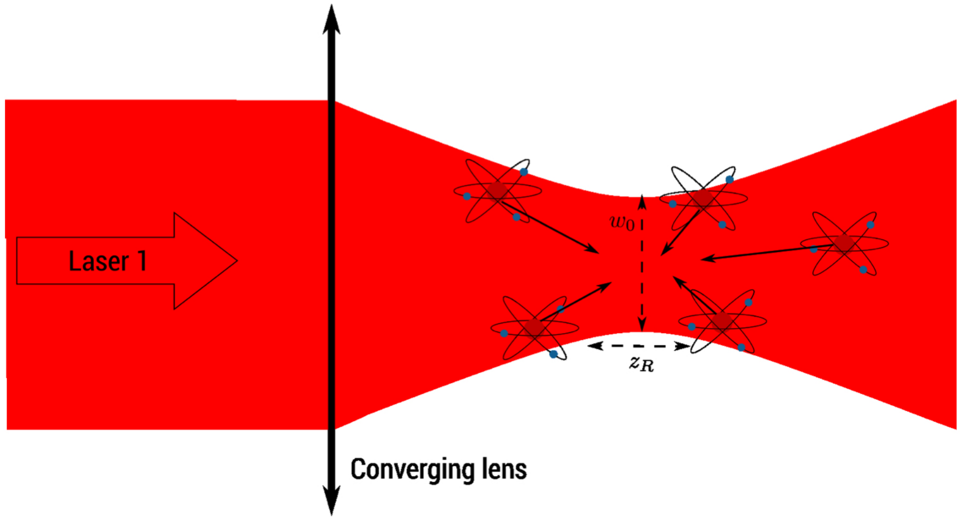

Figure 1: Simplification of our system (taken from the textbook)

With this final expression, let us compute the Taylor Expansion of the intensity around $\rho = 0$. In particular, as mentioned in the previous section, we set $z = 0$. We obtain:

$$I(\rho, z) = \frac{2P}{\pi w_0^2}(1 - \frac{2\rho^2}{w_0^2}) \tag{3}$$

Around the bottom of the trap, the optical trap can be approximated by a harmonic potential following the formula:

$$\frac{1}{2}m\omega^2\rho^2 = \frac{6c^2}{\omega_0^3}\frac{\Gamma}{\delta}\frac{P}{w_0^4}\rho^2 \tag{4}$$

with δ defined as in the previous section.

We finally obtain $\omega^2 = \frac{12c^2}{m\omega_0^3}\frac{\Gamma}{\delta}\frac{P}{w_0^4}$ and from this:

$$w_0 = \sqrt[4]{\frac{12c^2\Gamma P}{m\omega_0^3\delta\omega^2}} \tag{5}$$

3 Calculations

We are finally ready to compute the final value of the laser's waist length. Indeed we all the data required to compute the right hand part of Equation 5.

- The celerity of light
$$c = 2.99 * 10^8 m/s$$
- The line-width of the laser
$$\Gamma = 2\pi * 6 * 10^6 = 3.77 * 10^7 Hz$$
- Mass of the potassium 40 atoms
$$m = 39.96u = 6.63 * 10^{-26} kg$$
- The angular frequency of the laser
$$\omega_0 = \frac{2\pi * c}{\lambda_0} = \frac{2\pi * 2.99 * 10^8}{7.70 * 10^{-7}} = 2.43 * 10^{15} rad/s$$
- Delta
$$\delta = 2\pi c(\frac{1}{\lambda_L} - \frac{1}{\lambda}) = 6.72 * 10^{14} Hz$$

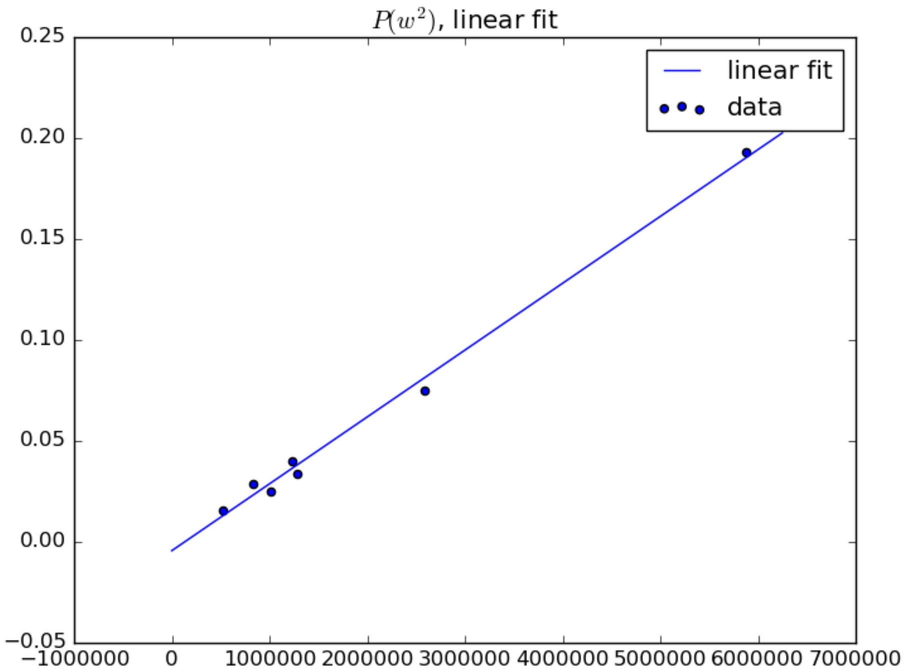

Figure 2: Plot of P vs ω^2, $k = 3 \cdot 10^{-8}$

The only value require to obtain w_0 that is left would be the ratio between the laser power and the oscillation frequency squared : $\frac{P}{\omega^2}$. Thankfully we are experimental data of the oscillation frequency for varying laser power. It suffices then to plot P as a function of ω^2 and to apply a linear fit to it. The slope k of the fit would be an experimental estimate of $\frac{P}{\omega^2}$.

Therefore we have the final value of the waist length :

$$\begin{aligned} w_0 &= \sqrt[4]{\frac{12c^2\Gamma k}{m\omega_0^3\delta}} \\ &= \sqrt[4]{\frac{12 * (2.99 * 10^8)^2 * 3.77 * 10^7 * k}{6.63 * 10^{-26} * (2.43 * 10^{15})^3 * 6.72 * 10^{14}}} \\ &= 3.80 * 10^{-5} m \\ &= 38.0 \mu m \end{aligned}$$

4 Conclusion

The order of magnitude of this result is correct. Collimated laser beams are typically smaller than a millimeter in diameter. Thus it is feasible to have it focussed to the scale of a thirtieth of a millimeter. On the other hand, it is 100x the magnitude of the laser's wavelength which respects the fact that one cannot focus a laser to a smaller scale than of its wavelength.

We have seen a way to measure a Gaussian's beam waist by analysing the oscillation frequency of atoms around the beam's waist on the transverse plane. We could extend this problem to obtain the shape of the overall shape of the Gaussian beam by measuring the atoms' oscillation frequency along the z-axis.