Problem 5 - Wind of Change

Using data obtained by a LIDAR tool, we estimate the wind speed at a given altitude above ground level.

Physical Model

In general, the reflection of a wave on a moving target leads to a frequency shift, due to the Doppler effect. Therefore, by quantifying this frequency shift, we can exploit the Doppler effect in order to measure the velocity of one or multiple particles following a collective movement. When applied to wind speed measurement, the emission of a laser beam is followed by the analysis of the frequency of the backscattered light, to compare it with that of the emitted light. The Doppler effect gives a relation between the Doppler shifted frequency and the velocity of the particle along the laser beam.

The emitted and backscattered light frequencies lie in the optical range, so they are very high and thus difficult to measure directly. Furthermore, their relative difference is very small. To circumvent these drawbacks, we examine the coherent superposition of the emitted and backscatered waves, in order to exploit the phenomenon of beating in superposed waves of slightly different frequencies. The beating frequencies, lying in the MHz range, are easily detectable with conventional tools.

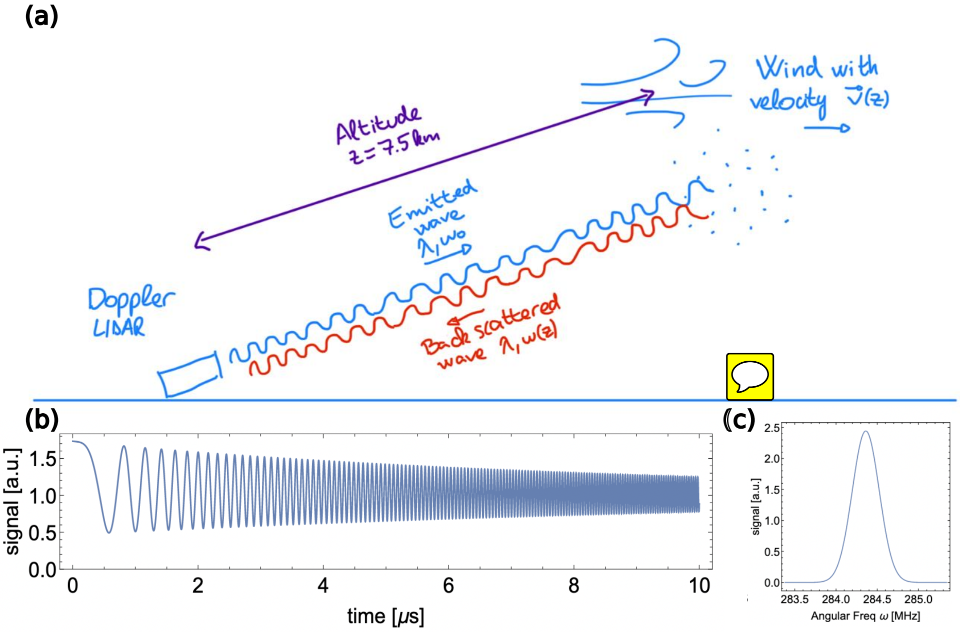

Figure 1: (a) Schematic representation of the wave emitted by the Doppler LIDAR interacting with the wind particles at an altitude $z = 7.5\,\text{km}$ (b) Recorder signal due to the superposed light waves, as a function of time (c) Spectrum of the signal around $t = 50\,\mu\text{s}$, corresponding to scattering at altitude $z = 7.5\,\text{km}$. The peak frequency corresponds to the beating frequency between the incident and scattered waves.

Relevant Quantities

- Given
  - Laser wavelength $\lambda = 2\,\mu\text{m}$.
  - Altitude of interest $z = 7.5\,\text{km}$.
  - Doppler frequency shift $\Delta\omega(z) = 284.35\,\text{MHz}$. (see Figure 1(c) and explanation below).

- Intermediate
  - Propagation time t
  - Laser frequency ω_0
  - Laser frequency in wind's frame ω'
  - Reflected laser frequency $\omega(z)$
- Unknown
  - Wind speed $v(z)$

Laws at stake

- Link between the altitude of interest z and corresponding detection time t

$$z = \frac{ct}{2} \tag{1}$$

- Laser frequency

$$\omega_0 = \frac{2\pi c}{\lambda} \tag{2}$$

- Doppler shift upon change to reference frame moving at speed v

$$\omega' = \frac{\omega_0}{1 \pm \frac{v}{c}} \tag{3}$$

  The plus sign corresponds to frame movement opposite to wave propagation, and the minus sign corresponds to frame movement along the direction of wave propagation.

- Frequency matching condition after scattering, in the frame moving with the wind, for the case of wind moving away from the laser source

$$\frac{\omega(z)}{1 - \frac{v}{c}} = \frac{\omega_0}{1 + \frac{v}{c}} \tag{4}$$

  The signs in the denominators are reversed in the case of wind moving toward the source.

- Doppler shifted frequency after reflection, under the assumption of nonrelativistic wind speed, oriented away from the laser source

$$\omega(z) = \omega_0 \left(1 - \frac{2v(z)}{c}\right) \tag{5}$$

  The result follows directly from equation 4 by performing a Taylor expansion, under the assumption $\frac{v}{c} \ll 1$. The minus sign turns into a plus when considering opposite wind direction.

- Frequency shift after reflection at altitude z

$$\Delta\omega = |\omega(z) - \omega_0| = \omega_0 \frac{2v(z)}{c} \tag{6}$$

Solution

In the experiment, the signal is the superposition of the emitted and backscattered light. The signal at time t gives us information about the particles at a certain altitude z, as described by (1). Indeed, as time increases, we get the superposition with light scattered by particles further away, as the light takes more time to travel the distance z.

To measure the speed of a particle at a distance z, we perform a spectral analysis of the signal for a small time step around $t = \frac{2z}{c}$. The peak frequency corresponds to the beating frequency and thus the change in frequency $\Delta\omega$. Combining equations 5 and 6 gives the formula for the velocity

$$v = \frac{\Delta\omega c}{2\omega_0} = \frac{\Delta\omega\lambda}{4\pi} \tag{7}$$

Numerically evaluating our expression, we find that we are interested in the frequency shift at the time

$$t = \frac{2 \times 7500\,\mathrm{m}}{3 \times 10^8\,\mathrm{m/s}} = 50\,\mu\mathrm{s}$$

Reading of the peak frequency off the spectrum in figure 1(c), centered around $t = 50\,\mu\mathrm{s}$, we find $\Delta\omega = 284.3\,\mathrm{MHz}$ which by equation 7 corresponds to the speed

$$v = \frac{284.3 \times 10^6\,\mathrm{Hz} \times 10 \times 10^{-6}\,\mathrm{m}}{4\pi} = 45.3\,\mathrm{m/s} = 163\,\mathrm{km/h}$$

Discussion

First, we try to give an intuitive justification that this value is reasonable. Notice that the 7.5 km altitude we considered is close to typical cruising altitudes of commercial airplanes. The winds that these airplanes encounter reach values of up to 200 km/h[1]. The wind speed that we measured is therefore in line with typical expected values.

There are, however, several limitations for the use of this method. One downside of this method is the inability to tell the direction of the velocity of the wind and its transverse component. This is due to the fact that the beating frequency only gives us the absolute value of the frequency change, not its sign. Therefore, we cannot tell if it corresponds to an increase or a decrease in frequency, ie. if the wind is moving away or towards the light source. This can be overcome, however, by measurements over an extended volume of the atmosphere and by requiring that the wind flow satisfies the laws of fluid mechanics.

Another drawback of the method is the measurement distance, which is limited by the use of backscattered light. Indeed, light is scattered in all directions, but we only detect a small part of the solid angle. Due to this, the intensity of the backscattered light drops as the inverse square of distance. Taking into account other dissipative processes such as atmospheric absorption, we reach an intrinsic limitation of this method.

[1] according to `https://turbli.com/wind-during-flights/`.