École Polytechnique
Bachelor of Science

Wind of change

PHY208: Problem 5 report

Lukas Bjorkland, Emilis Strazdas, Léopold Rousseau and Tianli Xu

14th May 2023

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CHAPTER 1

OVERVIEW OF THE PROBLEM (OR LIDAR TECH)

This week, we were tasked to analyze the Doppler LIDAR - a device which will allow us to estimate the speed of wind at different altitudes.

1.0.1 BASICS OF A LIDAR

LIDAR is an acronym that stands for "laser imaging, detection, and ranging". It works by emitting a laser pulse towards a target, detects the light that has been scattered or reflected by our target and then computes the distance between it and the device. This technique is visualized in figure 1.1.
LIDARs are usually used for determining distances between two objects or mapping an area.

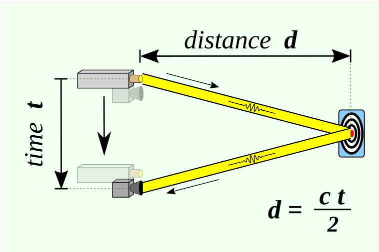


FIGURE 1.1: Simplified version of a LIDAR, [1].

1.0.2 BASICS OF A DOPPLER LIDAR

A Doppler LIDAR is a slightly modified version of a simple LIDAR. Instead of just noting down the time it took for the pulse to come back, it superimposes the back-scattered / reflected signal with the original one from the laser. This is then sent to a photo-diode which, produces a voltage proportional to the beam intensity.

1.0.3 FORMULATION OF THE PROBLEM

In our case, a Doppler LIDAR sends a pulse of $\lambda = 2\mu m$ towards the sky. It then outputs the photo-diode readings. Spectral analysis on short-time windows is then performed. Knowing the the spectral analyses at $t = 10, 50, 75\mu m$, we will estimate the wind speed (along the line of sight) at an altitude of $7.5km$.

CHAPTER 2

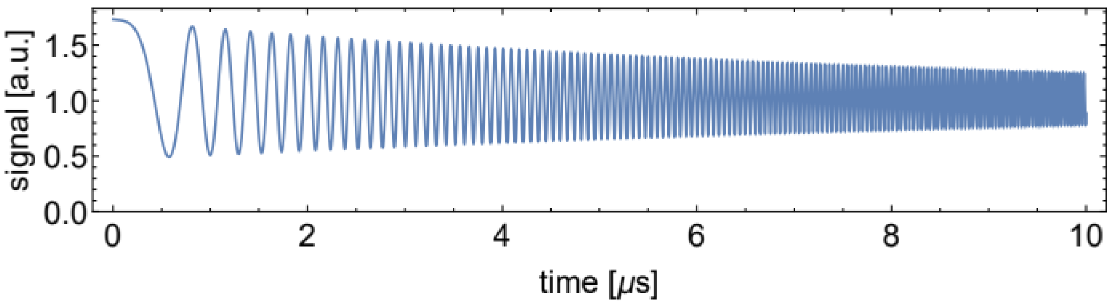
OUR INTERPRETATION OF THE PROBLEM

2.0.1 ASSUMPTIONS

In order to solve this problem, we must first make the following assumptions:

- The laser pulse is shot straight upwards with no horizontal component

As we saw before, the laser pulse that was sent up and reflected down is then superposed with the original laser. Since the frequency of the returning pulse is slightly altered, it no longer resonates with the original laser beam and generates a beating. This beating is represented in the following data figure :



The two lasers have almost equal frequency but not exactly the same. Thus the beating between them feature a slowly oscillating envelope and an extremely fast oscillation within that envelope. Since the photo-diode cannot detect high frequencies, it only reads the shape of the envelope, which is what we see. When looking at the figure we notice two things : Firstly, the intensity of the signal is decreasing. This can be explained by the fact that later time correspond to reflections of the pulse that took longer to arrive. Thus reflections of the pulse that arrive later have travelled a longer distance in the sky and thus have been more absorbed by the air. The second thing we notice is that the frequency of the beating's envelope is increasing. This is actually an important detail since it is directly related to the wind speed. Indeed, a beating with higher frequency corresponds to a larger difference between the frequency of the original laser beam and of the reflected laser pulse. Thus laser pulse reflections that arrive later have been more altered than earlier ones. This is explained by the fact that wind speeds get greater as altitude increases which a consequence of fluid mechanics and decreasing air density as altitude increases.

However, only using this figure makes measuring the frequency of the beating at our desired time quite difficult. This is why we are provided three extra figures.

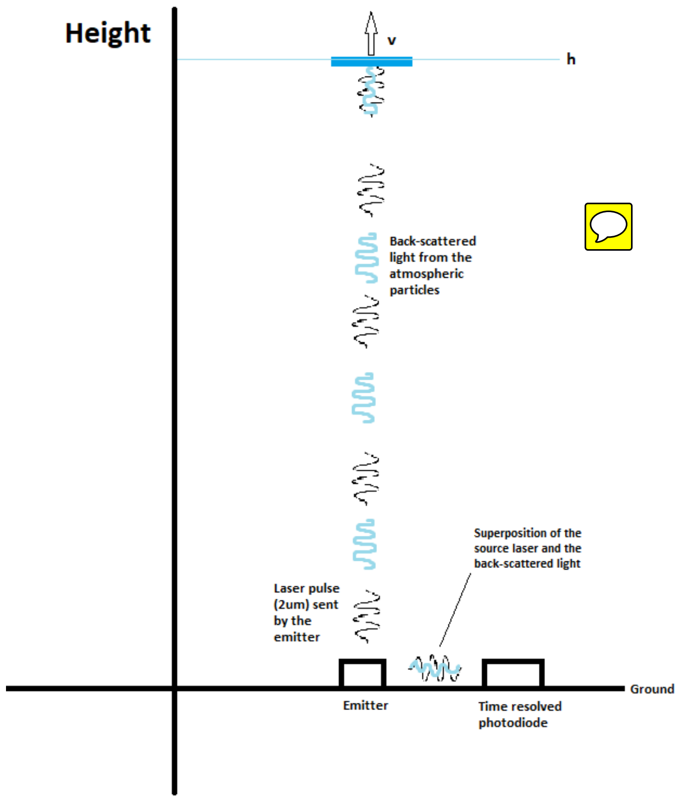
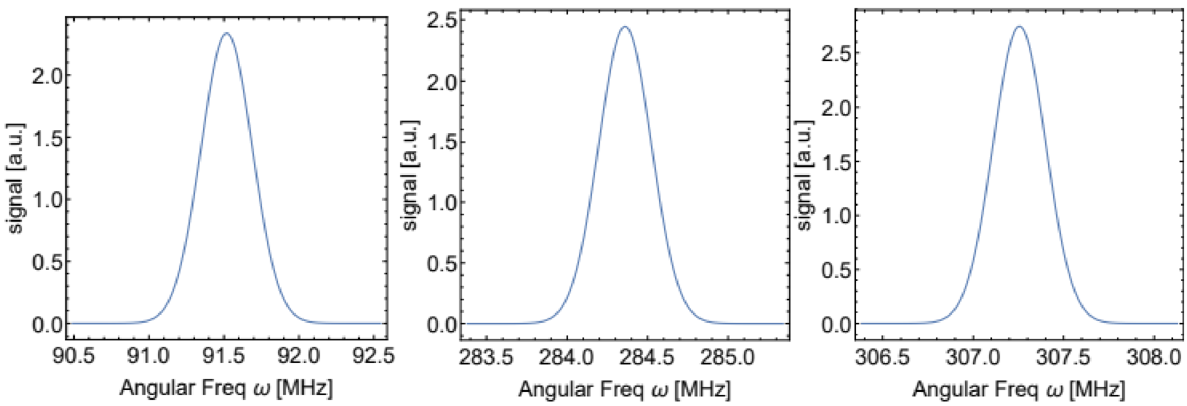


FIGURE 2.1: Drawing of our model



These three figures are the result of applying a Fourier transform around t = 10, 50 and 70 µs. The frequency at such times are given by the peak of the Fourier transform which makes the measuring of the frequency trivial.

We now have every elements to compute the final value.

CHAPTER 3

CALCULATIONS OF THE FINAL VALUE

By sending waves of short duration, we are able to receive the scattered waves by waiting for a certain time. We consider that each molecule in the atmosphere scatters back another electromagnetic wave of equal wavelength. However, since said molecules are not necessarily at rest, we must consider the Doppler shift of the emitted wavelength. Therefore, for $v << c$, we obtain the frequency of our laser pulse as perceived by the moving particle at height h:

$$\omega_p(h) = \frac{\omega_0}{1 + \frac{v(h)}{c}}$$

Furthermore, the scattered wave is also Doppler-shifted, since the particle is moving. Therefore, the frequency of the back-scattered wave received by the receiver writes:

$$\omega_m(h) = \frac{\omega_p(h)}{1 + \frac{v(h)}{c}} = \frac{\omega_0}{(1 + \frac{v(h)}{c})^2}$$

Since $\frac{v(h)}{c}$ is very small, we can approximate this formula via a Taylor expansion, obtaining:

$$\omega_m(h) = \omega_0(1 - 2\frac{v(h)}{c})$$

The receiver therefore picks up the superposition of our emitted laser pulse and the back-scattered ray, both of which have very similar frequencies. Therefore, the signal should look like:

$$E_0(\cos(\omega_0 t) + \cos(\omega_m(h)t)) = 2E_0 \cos(\frac{\omega_0 + \omega_m(h)}{2}t)\cos(\frac{\omega_0 - \omega_m(h)}{2}t)$$

We observe that the term $\cos(\frac{\omega_0 - \omega_m(h)}{2}t)$ is the envelope, $|\frac{\omega_0 - \omega_m(h)}{2}|$ is the beating frequency. We are therefore interested in finding this value. It is important to note that the beating frequency is an absolute value, we thus do not have any information on the direction of the wind speed. Since the intensity is proportional to the square of the electric field, from the trigonometric identities, we obtain:

$$I \propto 4E_0^2 \cos(\frac{\omega_0 + \omega_m(h)}{2}t)\frac{\cos((\omega_0 - \omega_m(h))t) + 1}{2}$$

We therefore aim to find the value of $|\omega_0 - \omega_m(h)|$. Since we want to calculate the speed of the wind at $h = 7.5\text{km}$, we can calculate the amount of time needed for the short laser pulse to reach said height and back:

$$t = \frac{2h}{c} = \frac{2 \cdot 7.5 \cdot 10^3\text{m}}{3 \cdot 10^8\text{m/s}} = 50\mu\text{s}$$

Since we have the spectral analysis of the signal after 50µs, we can obtain an approximate value of $\omega_0 - \omega_m(h)$ by the angular frequency at which the intensity reaches its peak, which is 284.4MHz. Since $\omega_0 = \frac{2\pi c}{\lambda_0}$, from the Doppler effect equation, we obtain the velocity of the wind along the line of sight at altitude h:

$$v(h) = \frac{\lambda_0}{4\pi}(\omega_0 - \omega_m(h)) = \frac{2 \cdot 10^{-}6\text{m}}{4\pi}(284.4\text{MHz}) = 45.3\text{m/s}$$

Chapter 4

Comparisons and some remarks

Note that the usual wind speed lies in the magnitude of 10^1 to 10^2 m/s, which indicates that our result seems reasonable. However, we remark that due to the fact that the spectral pattern we have generates a finite width, our result carries a potential uncertainty. More precisely, we could read that the width in the figure before is approximately 0.4 MHz. Since the wind speed is proportionally correlated to the peak angular frequency obtained around 50 μs, the uncertainty of the wind speed should also be proportionally correlated to that of the particular angular frequency with the same coefficient, which generates a value of 0.032 m/s. This is less than 1% of the obtained result for the wind speed at 7.5km, which indicates that this technology gives a rather accurate measurement.

Another noteworthy point is that we have a solid reason to superpose the scattered signal with the original one. The Doppler effect is so weak that the scattered signal has a very similar frequency as the original one since the wind travels generally at a velocity far smaller than the light speed. Therefore, the scattered signal has generally a frequency of magnitude $10^{14}s^{-1}$, which is impossible for any machine available now to measure. However, as we can see in the previous part, the superposition generates a signal that oscillates at two different angular frequencies, with one being the average of the sum of the two frequencies, which is still very large and the other being the average of their difference, which is much smaller. The large frequency will function as the chaos of the signal and we end up only with the smaller frequency by smoothing the obtained pattern, enabling the machine to perform the spectral analysis and provide the pattern we need.

Bibliography

[1] Wikipedia. *Lidar*. 2023. URL: https://en.wikipedia.org/wiki/Lidar (visited on 14th May 2023).