PHY208 Problem 5

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1 Problem Statement

A Doppler LIDAR can be used to estimate the velocity of the wind as function of altitude. In this assignment, we will investigate this technique and the concepts behind it to estimate the wind speed at an altitude of 7.5km along the line of sight of a laser.

2 Model

We work with a laser of wavelength $\lambda_0 = 2\mu m$ or equivalently of angular frequency $\omega_0 = \frac{2\pi c}{\lambda_0}$, where c is the speed of light, incident on a beam splitter creating two identical beams b_1 and b_2. b_1 is going to be aimed towards the sky and by putting a gate in its path we can send pulses by quickly opening and closing it. Once those laser pulses hit an atom, some of it will be back scattered directly back to our apparatus and will interfere with b_2. This step is crucial as we expect the frequency of the back scattered light to be too high to be measured by our apparatus. Therefore, by interfering a high frequency signal with a lower frequency that we can measure (i.e ω_0 from b_2) we will be able to measure that high frequency. We assume that the angle θ at which b_1 is sent (with respect to the horizontal) is very small so that the back scattered light will interfere coherently with b_2 such that to find the intensity we simply need to add the electric fields together. This is all nice but how will we measure the speed of the wind? The answer to this lies in the concept of Doppler shift: the atoms in the atmosphere are not fixed in position; they are moving around (for simplicity, we assume that the winds movement is along the line of sight). Then, as the particle will be moving towards or away from the laser pulse, the frequency perceived by the atom will be different than the original frequency ω_0. Furthermore, as we are talking about a reflection, we should expect two doppler shifts. Indeed, not only will the frequency change due to the pulse going towards the moving atom but when back scattering, the atom will still be in movement and therefore lead to another doppler shift. Therefore, what we are interested in is ω_2, i.e the frequency of the laser pulse that will interfere with our beam b_2. Finally, we can use the change in frequency to determine the speed of the wind.

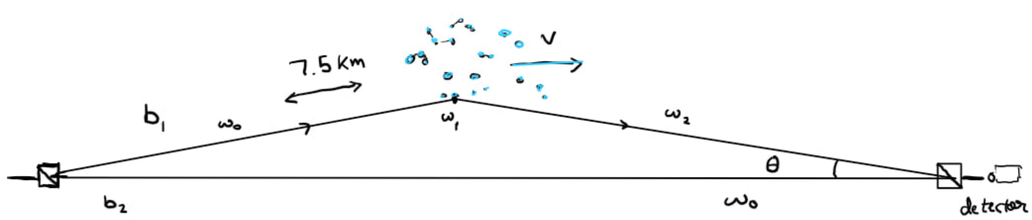

Figure 1: Depiction of b_1 and b_2 paths to the detector.

3 Calculation

We have a coherent superposition of the back scattered light with frequency ω_2 and source laser with frequency ω_0, meaning that we sum the electric fields:

$$E = Ae^{i\omega_0 t} + Be^{i\omega_2 t} = Ae^{i\omega_0 t}(1 + be^{i\Delta\omega t})$$

We can make an approximation that b=B/A is really small, because the amplitude of the back scattered wave is small compared to the laser one. Not all the light gets scattered, and the one that does, it gets scattered along all directions radially, and we only measure the one along the line of sight. Finally, we calculate the intensity as modulus squared of the E field:

$$I = A^2(1 + b^2 + 2bcos(\Delta\omega t))$$

We see that the intensity has an oscillating factor that depends on $\Delta\omega$. On the intensity graph, we can see that I oscillates with a frequency that changes in time. This is because $\Delta\omega$ is time-dependent. We will see that it depends on the wind velocity v, which varies along the distance in the air, and the time we are talking about is actually the time for light to make a two-way trip back to the source, meaning that this time is related to the distance(height). The intensity graph is thus, just a result of our measurement, it depends on how the wind velocity layers are structured. In our case, for 7.5km distance, this time is very close to 50 microseconds.

The light hits air particles along its way and gets scattered back, not only at the 7.5km distance (that is why we see the change in frequency of the intensity immediately, way before t=50 microseconds). So, we read the value of $\Delta\omega$ from the middle graph that is given to us (for the time around 50 microseconds).

Using the formula for the Doppler effect, while being careful that we count in the double Doppler effect and the fact that the roles of transmitter-receiver change when we do the second effect, we obtain the relation:

$$\omega_2 = \omega_0 \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}$$

The speed of wind is clearly non-relativistic so the term v/c is much less than 1. Applying the Taylor expansion of 1/(1-x) and neglecting the higher order terms gives us:

$$\Delta\omega = \omega_0 \frac{2v}{c}$$

Using this relation and $\Delta\omega$ obtained from the graph, we can calculate the wind velocity v.

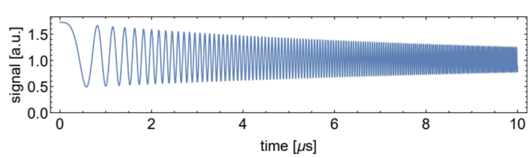

Figure 2: Recorded intensity of the combined back-scattered and continuous signal.

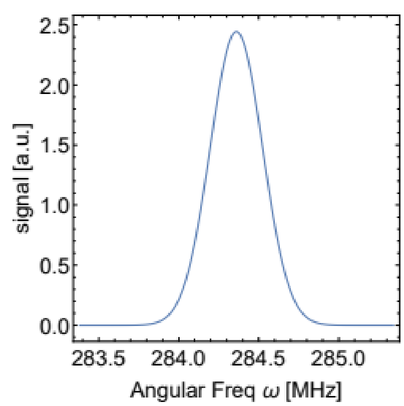

Figure 3: Spectrum of the signal at 50 μs

4 Numerical application

Assuming that the light travels with velocity $c = 3 \times 10^8 \frac{m}{s}$ (since $n_{air} = 1.0003$), we get that for the light to travel 15km (two ways - back and forth), it needs the time $t = \frac{15km}{3\times10^8 \frac{m}{s}} = 50\mu s$. Furthermore, knowing that the wavelength of the laser pulse is equal to $2\mu m$, we get $\omega_0 = 9.42 \times 10^{14} Hz$. Thanks to the theoretically derived formula above, we can compute the velocity of the wind at the height 7.5km. Indeed using the a spectral analysis of the signal after $50\mu s$, we obtain:

$$\Delta\omega = 284.3Mhz$$

Plugging it into equation:

$$v = \frac{c\Delta\omega}{2\omega_0} = 45.27\frac{m}{s} = 162.972\frac{km}{h}$$

We can compare this value to the one we would acquire using the same method, but 2.5km above the ground. Using first graph from the spectral analysis, we obtain:

$$v = 14.57\frac{m}{s}$$

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We have that the result from altitude of 2.5km is close the the average wind velocity taken from the paper:

$https://www.researchgate.net/figure/Average-wind-speeds-in-m-s-vs-altitude-in-km-On-an-average-wind-speeds-are-minimum-in_fig1_3 28173177$

Which further validates the experimental procedue partaken in this part.

5 Conclusions

The model used to compute the velocity of the wind along the line of sight has few limitations, which are due to the simplifications partaken in order to achieve this model. Firstly, we have the fact that the scattered light incoming to the detector is not reflected at the same height. This creates a distribution of scattered light along the differing altitude. The second approximation is the assumption that the air flow at the given height is laminar (hence constant velocity for the light scattered at the given height, which is a result after applying Bernoulli's principle). This needs not to be the case in general. Despite those shortcomings, theoretical predictions agree with experimental data. As time increases, $\Delta\omega$ rises, as the scattered light scatters through air at a higher altitude, hence from air with higher velocity. Furthermore, we see that as light scatters from higher amplitude, the intensity of scattered light decreases (as the way it needs to take increases) - from some point on the graph of intensity of signal vs time stabilizes - term b in equation from section 3 tends to 0.