AROUND THE WORLD

PHY208 Atoms and Lasers

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1 Question

How many repeaters are needed to ensure the signal transmission over TAT-13?

2 Variables

2.1 Known data and constants

From the problem statement we can extract the following data:

- L - the total distance of the segment TAT-13 given as 6321 km
- P_L - power of the laser given as $\sim 1\ \mu$W
- λ_{21} - laser's wavelength given as 1550 nm
- τ_{21} - average decay time from second state to ground state given as 11 ms
- σ_{21} - cross section for the laser given as 4×10^{-25} m^2
- l - length of one optical repeater given as 10 m
- r - radius of an optical repeater given as 2 μm
- n - density of Erbium atoms in EDFA given as $\sim 5 \times 10^{23}$ m$^{-}3$
- τ_{32} - average decay time from third state to second state assumed very fast
- λ_{13} - laser's wavelength given as 980 nm
- σ_{13} - cross section for the laser pump given as 6×10^{-25} m^2
- P_p - power of the pump laser given as 50 mW

The constants used in this problem are:

- c - speed of light
- h - Planck constant taken to be 6.626×10^{-34} m^{2}kg

2.2 Intermediate variables

- x - distance along the fiber
- α - absorption coefficient
- g - gain due to the amplifying medium
- g_0 - unsaturated gain
- I_L - intensity of the laser beam (signal)
- $I_{in} = I_L(0)$ - intensity of the laser beam when it enters the amplifying medium
- $I_o = I_L(d)$ - initial intensity of the laser beam (also intensity after exiting the amplifying medium)
- I_p - intensity of the pump
- I_{sat} - saturation intensity
- $I_{removed}$ - intensity removed from the system after passing through the amplifying medium (only on the drawing)

2.3 Sought after variable

The variable we are looking for is:

- N - number of repeaters needed to ensure the signal transmission over TAT-13

3 Model

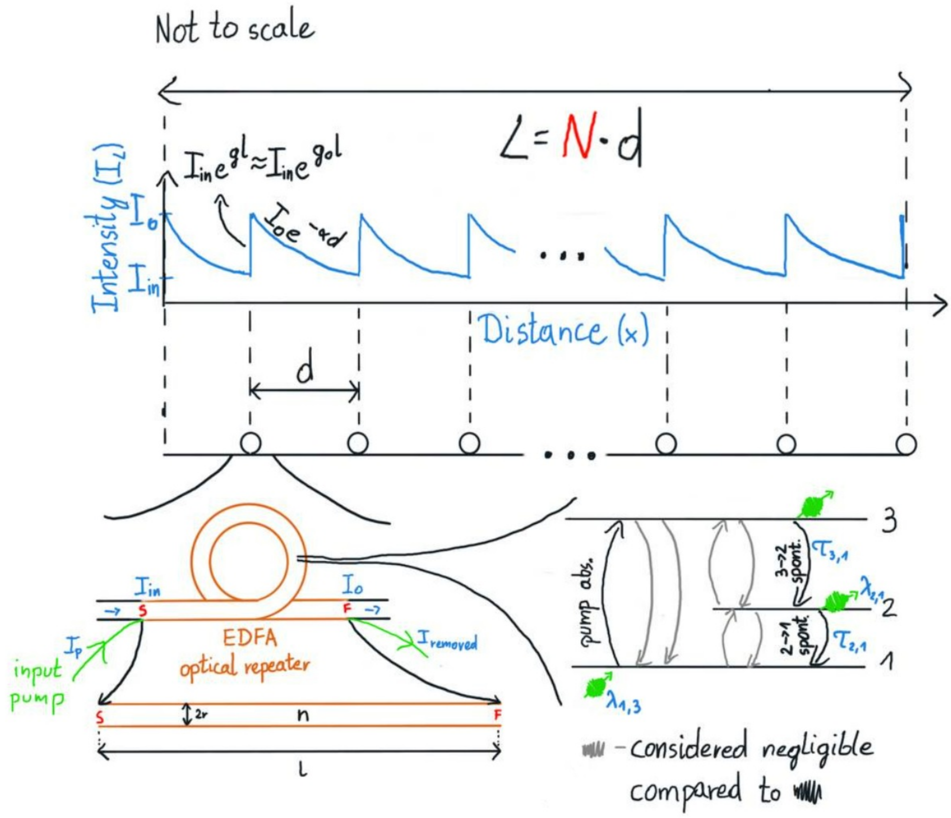

Figure 1: Sketch of the problem

We use the semi-quantum model of light-matter interaction to model the amplifying Erbium atoms as 3-level systems since to get a population inversion we need a 3-level system, and population inversion is needed for the amplification of light.

In order to efficiently amplify the given signal, one must choose a good placement of the amplifiers in the cable. In practice, it would seem easier to make changes to the already placed cable near the shores of the ocean rather than in the middle. However, amplifying the signal at its beginning and/or its end is extremely inefficient. Indeed, if the signal was sufficiently amplified at the start to travel through the whole ocean with no amplification (except perhaps near its end), it would most likely be saturated, hence, there would be losses in gain. On the other hand, if we choose to amplify the signal in the end, but not in the middle, the signal will reach the other shore attenuated enough to be of the same order of magnitude as noise (which is added by the amplifier, for example, pump noise due to pump power fluctuations and become indistinguishable. Therefore, it is fair to say that placing the amplifiers equidistant to each other is an optimal setup.

Furthermore, the signal will be amplified sufficiently enough if the gain in the Erbium-doped fiber fully compensates the losses (see figure 1 for a sketch of the physical situation):

$$e^{gNl}e^{-\alpha L} = 1 \tag{1}$$

The equation yields:

$$N = \frac{\alpha L}{gl} \tag{2}$$

We can obtain α from the graph (figure 2) in the problem which plots attenuation as a function of wavelength.

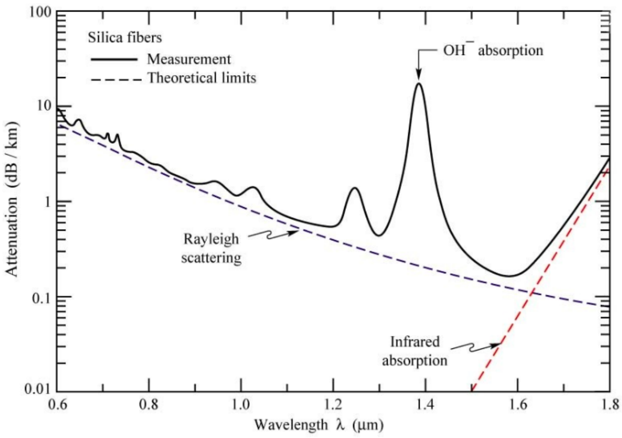

Figure 2: Attenuation vs Wavelength (figure taken from ref. [1])

In this graph the attenuation is given in dB/Km. The real definition of attenuation is $\frac{I_{out}}{I_{in}}$. The total attenuation across a distance d (in units of Km) can be given as $-Gd$ where G is the attenuation coefficient which is depicted on the y-axis of the graph. We have that

$$-Gd = 10\log_{10}\frac{I_{out}}{I_{in}}$$

$$I_{out} = I_{in}10^{-\frac{Gd}{10}} = I_{in}e^{-\frac{G\ln 10}{10}d} \tag{3}$$

Also due to Beer-Lambert's law we have that the intensity after traveling a distance d through a medium (fibre optic cable) will be given by:

$$I_{out} = I_{in}e^{-\alpha d} \tag{4}$$

Equating 3 and 4 and then substituting the experimentally measured value of $G = 0.2$ corresponding to the wavelength of 1.55 μm (from the graph) we find that

$$\alpha = \frac{G}{10}\ln 10 = 0.046 \text{ Km}^{-1}$$

We used the experimental value, not the theoretical one since the experimental value is the real value that takes into account both scattering and absorption effects and other unknown effects that theory might not account for. The fact that the attenuation is minimum for this wavelength is not a coincidence. The wavelength is chosen in a way to minimize attenuation.

We will now analyze what happens inside a repeater node. The nodes consist of fibers that contain Erbium atoms. Being three-level systems, Erbium atoms can amplify the input signal. If we have population inversion, the gain is:

$$g = \frac{g_0}{1 + \frac{I_L}{I_{sat}}} \tag{5}$$

It depends on the distance x along the fiber from its input and we can find the dependence using the following differential equation (from the definition of gain):

$$\frac{dI_L}{dx} = \frac{g_0}{1 + \frac{I_L}{I_{sat}}} I_L \tag{6}$$

This would be difficult to solve analytically but we can make the following simplification: if the laser intensity is much smaller than saturation intensity I_{sat} then we can ignore the term $\frac{I_L}{I_{sat}}$ (this is equivalent to saying that since we are well below saturation, increasing the intensity does not affect our gain negatively). We can verify this assumption by computing:

$$\frac{I_{in}}{I_{sat}} = 4 \cdot 10^{-5} \tag{7}$$

which is indeed very small. Then we can easily integrate equation 6 to obtain the following amplification factor:

$$I_L(d) = I_L(0)e^{g_0 l} \tag{8}$$

Thus the final expression for the number of repeater nodes is:

$$N = \frac{\alpha L}{n\sigma_{21} l} \frac{\frac{\sigma_{13}P_p}{\pi r^2 hc} + \frac{1}{\tau_{21}}}{\frac{\sigma_{13}P_p}{\pi r^2 hc} - \frac{1}{\tau_{21}}} \tag{9}$$

After performing the substitution we get the numerical value of $N = 148$. The deviation from the real value of 140 amplifiers[2] used is less than 6%. However, we have made a number of simplifying assumptions, including: we assumed the pump power is constant, we ignored losses within the amplifier, and we neglected the effects of saturation on gain.

4 Reference

1. Schubert, E. (2006). Light-Emitting Diodes (2nd ed.). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511790546

2. P. Trischitta, M. Colas, M. Green, G. Wuzniak and J. Arena, "The TAT-12/13 Cable Network," in IEEE Communications Magazine, vol. 34, no. 2, pp. 24-28, Feb. 1996, doi: 10.1109/35.481240.