Physics 208 Report 3

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I Statement of the problem

How many repeaters are needed to ensure signal transmission over TAT-13? For simplicity, one can assume that the pump power remains almost constant through the EDFA. Erbium atoms can be considered as a 3-level system with the following properties:

Quantity	Value	Quantity	Value
τ_{21}	11 (ms)	τ_{32}	$\sim$ µs
λ_{21}	1550 (nm)	λ_{13}	980 (nm)
σ_{21}	4×10^{-25} (m^2)	σ_{13}	6×10^{-25} (m^2)

II Solution

II.1 Model

To develop a model, we must first understand *why to use repeaters* in TAT-13. The signal experiences an attenuation due to molecular resonances and Rayleigh scattering. As the signal intensity drops and gets in the same order as the background noise, it will be impossible to recover the signal. Therefore, we have to keep the intensity high enough to avoid mixing with background noise. Naturally, signal amplification is a good solution and it is feasible to employ EDFAs.

Thus we have to model and combine two phenomena: signal attenuation and amplification using EDFA. The signal is transmitted at a wavelength regime far from molecular resonances. Therefore, we can model the attenuation with pure Rayleigh scattering phenomenon. The numerical value can be easily retrieved from the data shown in fig. 1. For EDFA, we will consider a 3-level quantum model and derive equations in section II.4.

To finalize, we want the same intensities at two ends of the TAT-13. Therefore, we will look for how many EDFAs we need to compensate for the effect of attenuation.

II.2 Variables

- N: number of repeaters (what we are looking for)
- g: gain
- l: length of short fiber
- τ: typical time of spontaneous emission
- $\Gamma = \frac{1}{\tau}$: spontaneous emission rate
- λ: wavelength
- σ: cross-section
- P_s: the power of the input signal
- P_p: the power of the pump laser
- ν_p: frequency of the pump laser
- I_p: intensity of the pump laser

II.3 Parameters of the system and Physical Constants

- $h = 6.62 \times 10^{-34}\,(\mathrm{J\,s})$: Planck's constant
- $c = 3 \times 10^{8}\,(\mathrm{m\,s^{-1}})$: the speed of light
- $L = 6321\,(\mathrm{km})$: distance between continents
- $n = 5 \times 10^{23}\,(\mathrm{m^{3}})$: density of Erbium atoms in the EDFAs
- $r = 2\,(\mu\mathrm{m})$: radius of the fiber

II.4 Derivation

As the signal propagates through the optical fiber bundle (which is a dielectric), its intensity will decrease due to absorption inside the fiber. The intensity of the signal then reads

$$I(x) = I_0 e^{-\mu x}$$

where I_0 is the initial intensity of the signal (which is equal to the initial power of the signal times the cross-section of the fiber) and μ ($[\mu] = \mathrm{m}^{-1}$) is the coefficient of absorption. We can determine this value from the graph in 1. As we can see, at the wavelength of the input signal ($\lambda = 1550\,(\mathrm{nm})$), the observed absorption is close to the theoretical value for Rayleigh scattering. We can then assume that all losses inside the fiber are due to scattering.

As mentioned in the II.1, the goal of these fibers is to have the same signal at the end of the fibers as it was at the beginning. It is for this reason, that the EDFAs (amplifiers) are placed along the fibers. The first question to be answered is how are these amplifiers distributed. If we inserted them at the end, the signal reaching the amplifiers would be too weak to be distinguishable from background noise, so the signal that would be

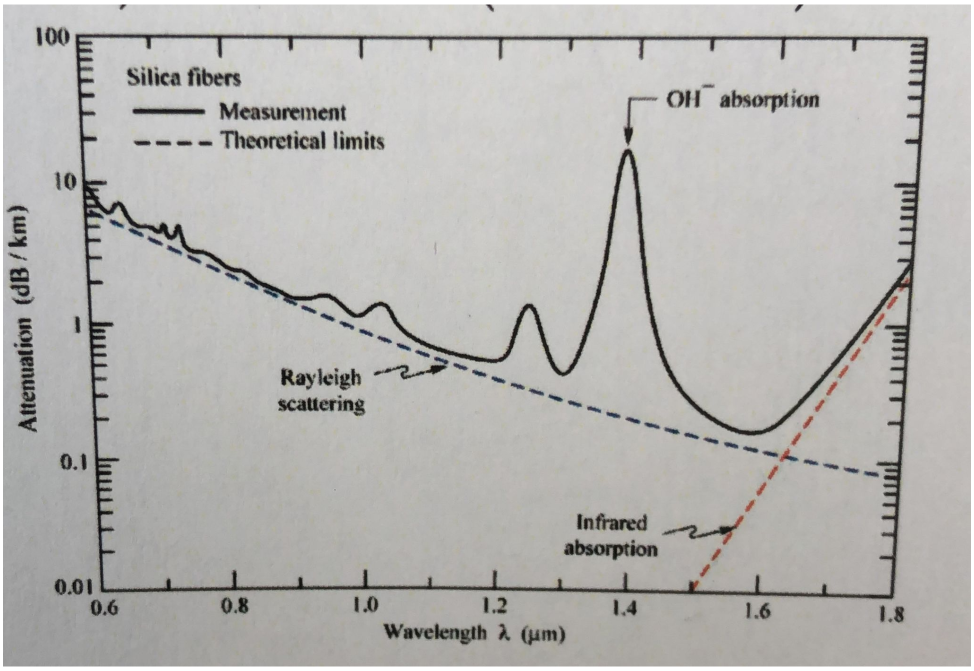

FIGURE 1: Graph provided with information of the problem

amplified would be different from the input one. Two other options are placing them at the beginning or distributing them along the fiber. To decide which one is most likely, we need to study the gain g, which is defined as

$$g = \frac{g_0}{1 + \frac{I_L}{I_{sat}}}$$

where I_L is the intensity of the incoming signal, and g_0 and I_{sat} are values depending on the system that we will explore later. Here, we were able to use this gain since $\tau_{32} \ll \tau_{21}$. It is clear that as the intensity of the input signal increases, the gain decreases. This means that if we were to put the amplifiers at the beginning, the gain from which one would decrease, meaning we would need more of them to reach an intensity high enough so that at the end of the fiber we would have the same intensity as at the beginning. We can therefore assume that they are uniformly distributed along the fiber.

The behavior of the signal intensity will therefore look like the graph on the right of 2 (we can notice that $L \gg l$ so we can consider the EDFAs as points so that the increase in instantaneous).

We will now study what happens to the intensity after each amplifier. We consider each EDFA as a three-level system, whose transitions between the first and the third level are governed by a pump ($\lambda_p = 980\,(\mathrm{nm})$). The intensity will then follow the following differential equation,

$$\frac{\mathrm{d}I}{\mathrm{d}x} = gI = \frac{g_0}{1 + \frac{I}{I_{sat}}} I$$

where g_0 is unsaturated gain and I_{sat} is saturation intensity.

g_0 is mathematically given as

$$g_0 = \frac{W_P - \Gamma_{12}}{W_p + \Gamma_{12}} n_{tot}\, \sigma_{12}$$

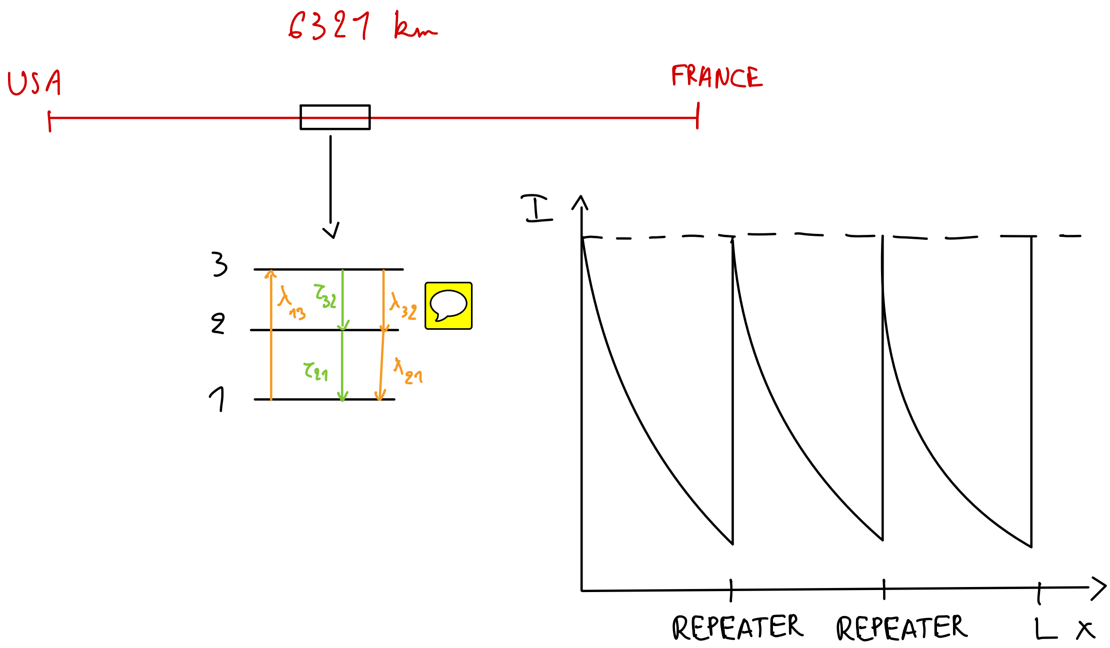

FIGURE 2: Sketch of problem and expected intensity evolution in the given distance

In particular, W_p is the pump rate and its value is $W_p = \frac{\sigma_{13} I_p}{h\nu_p} = \frac{\sigma_{13} P_p \lambda_p}{hc\pi r^2}$. Furthermore, $n_{tot} = n$ is the total concentration of Erbium atoms in the fiber and $\Gamma_{12} = \frac{1}{\tau_{12}}$ is the rate of spontaneous emission.

On the other hand, the saturation intensity is given as,

$$I_{sat} = \frac{h\nu(W_p + \Gamma_{12})}{2\sigma_{12}} = \frac{hc(W_p + \Gamma_{12})}{2\lambda_{12}\sigma_{12}}$$

We can now look for simplification in the differential equation. We can observe that $\frac{P}{\pi r^2}$ will always be smaller or equal to the input intensity, so if $I \ll I_{sat}$, then $g \approx g_0$. As we will see in II.5, this is the case, hence the intensity inside the amplifier will behave like

$$I(x) = I_{in} e^{g_0 x}$$

where I_{in} is the intensity at the beginning of the amplifier. We can then see that the intensity will be amplified by a factor of $e^{g_0 l}$. Since we place N of them, the effect of the amplifiers will be an increase in the amplitude of a factor of $e^{N g_0 l}$. All in all, we obtain that

$$I(x) = I_0 e^{N g_0 l - \mu x}$$

As said above, the goal is to have $I(L) = I_0$, hence we must have $e^{N g_0 l - \mu L} \iff N = \frac{\mu L}{g_0 l}$.

II.5 Numerical Results

We begin with determining μ. Using the data from fig. 1, we write,

$$G_{dB} = 10\log_{10}\frac{I_{in}}{I_{out}} \iff 0.2\,(\mathrm{dB}) = 10\log_{10} e^{\mu\cdot 1\,(\mathrm{km})}$$
$$\implies \mu = \frac{0.02}{\log_{10} e}\,(\mathrm{dB/km}) = 0.0460\,(\mathrm{dB/km})$$

After that, we calculate,

$$\begin{cases} I = \dfrac{P_s}{\pi r^2} = \dfrac{1\,(\mu\mathrm{W})}{\pi\cdot(2\,(\mu\mathrm{m}))^2} = 0.0796\times 10^6\,(\mathrm{W/m^2}) \\ I_p = \dfrac{P_p}{\pi r^2} = \dfrac{50\,(\mathrm{mW})}{\pi\cdot(2\,(\mu\mathrm{m}))^2} = 3.98\times 10^9\,(\mathrm{W/m^2}) \\ W_p = \dfrac{\sigma_{13} I_p}{hc}\lambda_{13} = \dfrac{6\times 10^{-25}\,(\mathrm{m^2})\cdot I_p}{hc} 980\,(\mathrm{nm}) = 1.18\times 10^4\,(\mathrm{s^{-1}}) \\ I_{sat} = \dfrac{hc(W_p + \frac{1}{\tau_{12}})}{2\sigma_{12}\lambda_{12}} = \dfrac{hc(W_p + (11\,(\mathrm{ms}))^{-1})}{2\cdot 1550\,(\mathrm{nm})\cdot 4\times 10^{-25}\,(\mathrm{m^2})} = 1.90\times 10^9\,(\mathrm{W/m^2}) \end{cases}$$

Here observe that $I/I_{sat} = 4.19\times 10^{-5} \ll 1$. Therefore, our assumption for derivation is valid and we conclude that,

$$g \approx g_0 = \frac{\tau_{12}W_p - 1}{\tau_{12}W_p + 1} n_{tot}\sigma_{21} = 0.197\,(\mathrm{dB/m})$$

Finally,

$$N = \left\lceil \frac{\mu L}{gl} \right\rceil = \left\lceil \frac{0.0460\cdot 6321}{0.197\cdot 10} \right\rceil = 148$$

Here, we used the ceiling function because we know that it is a better (or safer) option to have a highly-amplified signal compared to a signal lost in background noise due to not being sufficiently amplified. Assuming that EDFAs are equidistantly placed, we will place each amplifier with a step-size of $\Delta L = \frac{6321}{148} = 42.7\,(\mathrm{km})$. Notice that not only $L \gg l$, but also $\Delta L \gg l$. This strengthens our argumentation.

II.6 Conclusion

First of all, the value we found is feasible in industry standards: ~ 150 repeaters can be produced, employed and installed in the submarine cable network. Then, we will discuss our results comparing them to the literature value. We find in (Dawson & Trischitta, 1996) that the real number of repeaters used was 140 which is really close to the value that we find. This corresponds to repeater spacing of $\Delta L_{real} = 45\,(\mathrm{km})$. Therefore, we can calculate the relative error as,

$$\delta = \frac{45 - 42.7}{45} = 5\%$$

We conclude that we find the result 5% off the actual value, which is reasonable in terms of the approximations/assumptions we made.

Now, although our estimation is quite good, we should still try to understand why we obtained 8 more repeaters. Looking back at our reasoning, the only assumption that we can challenge is considering that the pump power remains constant through the EDFA. Moreover, we can see that having a greater gain g_0 leads to a smaller value for N, maybe then EDFAs have a slightly higher gain. Also, in (Dawson & Trischitta, 1996), we see that there have been developments over the years in EDFA technologies and they used different EDFA models (16 (dB) and 10 (dB) repeater for example) in different sections of the TAT-12/13 lines.

References

Dawson, S., & Trischitta, P. (1996). Tat-12/13 project overview. In *Iee colloquium on transoceanic cable communications - tat 12 and 13 herald a new era* (p. 2/1-2/5). doi: 10.1049/ic:19960443