April 2023

Around the World

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Problem Statement and Objective

The Trans-Atlantic Telephone 13 (TAT-13) is a 6321 km optical fiber cable connecting France to US. As a signal sent along the cable attenuates in intensity over distance, Erbium-doped fiber amplifiers (EDFA), each of length 10 m, are required as optical repeaters to compensate for this attenuation by amplifying the signal so that the intensity of the signal reaching France is the same as that emitted from the US. The objective of this report is to deduce the number of EDFA repeaters required to achieve this.

Model of EDFA

Each EDFA repeater is modelled as a 3-level system. A diagram of the 3 levels is shown below.

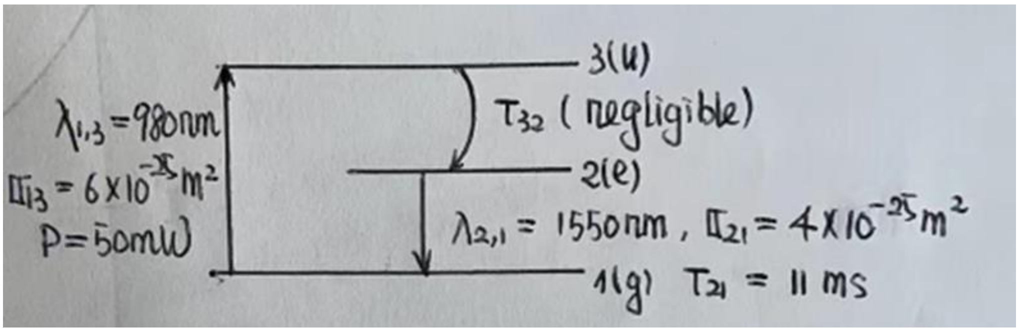

Figure 1: The diagram of the EDFA 3-level system

An external laser of wavelength λ_{13} pumps erbium atoms from level 1 to level 3. Due to adiabatic elimination assumption, these erbium atoms very quickly decay to level 2, causing a population inversion (number of erbium atoms in level 2 greater than that in level 1). The 1.55 μm photons of the signal passes through the EDFA cable and have the same wavelength as the photon emitted by a level 2 to level 1 transition (with wavelength λ_{21}), hence are amplified by this 3-level system.

By modelling the EDFA repeater as a 3-level system with adiabatic elimination assumption, the following variables are of relevance:

- τ_{21} spontaneous decay time from level 2 to level 1 ($\tau_{21} = 11\ ms$)

- λ_{21} wavelength emitted by an erbium atom transitioning from level 2 to 1 ($\lambda_{21} = 1.55\ \mu m$)
- σ_{21} cross section corresponding to level 2 to level 1 transition ($\sigma_{21} = 4 \times 10^{-25}\ m^2$)
- τ_{32} spontaneous decay time from level 3 to 1 (τ_{32} is not required by adiabatic elimination assumption)
- λ_{13} wavelength absorbed by an erbium atom transitioning from level 1 to 3 ($\lambda_{13} = 980\ nm$)
- σ_{13} cross section corresponding to level 1 to level 3 transition ($\sigma_{13} = 6 \times 10^{-25}\ m^2$)
- n_{tot} total density of erbium atoms in level 1 and level 2 combined ($n_{tot} = 5 \times 10^{23}\ m^{-3}$)

The following intermediate variables are to be determined:

- Γ_{21} rate of spontaneous decay from level 2 to 1
- W_{13} pump rate from level 1 to 3
- g gain of the signal after passing through the EDFA

Variables

Known variables from data

- Known variables directly related to the EDFA 3 level system have already been listed above
- c speed of light in vacuum ($c = 3 \times 10^8\ m\ s^{-1}$)
- h Planck's constant ($h = 6.63 \times 10^{-34}\ m^2\ kg\ s^{-1}$)
- L length of TAT-13 ($L = 6321\ km$)
- P_L power of signal emitting laser with λ_{21} wavelength ($P_L = 1\ \mu m$)
- P_{13} power of pump laser for EDFA, ($P_{13} = 50 \times 10^{-3}\ W$)
- l length of each EDFA cable ($l = 10\ m$)
- r radius of optical fiber for both EDFA and TAT-13 cable ($r = 2\ \mu m$)
- α attenuation of 1.55 μm signal per unit length ($\alpha = 0.2\ dB\ km^{-1}$, see appendix)

Intermediate variables

- S area cross section of TAT-13 cable ($S = \pi r^2 = 1.26 \times 10^{-11}\ m^2$)
- $\Gamma_{21} = \frac{1}{\tau_{21}} = 90.9\ s^{-1}$
- I_L intensity of laser emitter ($I_L = \frac{P_L}{S} = 7.96 \times 10^4\ W\ m^{-2}$)
- I_{13} intensity of pump laser ($I_{13} = \frac{P_{13}}{S} = 3.98 \times 10^9\ W\ m^{-2}$)
- $W_{13} = \frac{\sigma_{13} I_{13} \lambda_{13}}{hc} = 1.18 \times 10^4\ s^{-1}$
- I_{sat} saturation intensity of EDFA 3 level system ($I_{sat} = \frac{hc(W_{13}+\Gamma_{21})}{2\lambda_{21}\sigma_{21}} = 1.90 \times 10^9\ W\ m^{-2}$)
- $g = \frac{g_0}{1+\frac{I_L}{I_{sat}}} \approx g_0 = \frac{W_{13}-\Gamma_{21}}{W_{13}+\Gamma_{21}} n_{tot}\sigma_{21} = 0.197\ m^{-1}$ (since $I_{sat} \gg I_L$)
- I_0 intensity of the input signal
- I_1 the intensity of the signal at the output (assuming no repeaters in TAT-13)
- I_2 the intensity of the signal at the output (with N repeaters and no losse

Variable to be deduced

- N (number of repeaters required for TAT-13)

Solving the Problem

Attenuation

As the $1.55\ \mu m$ signal travels through TAT-13 of total length L, the attenuation in decibels per unit length is α, hence the total attenuation is $\alpha L\ (dB)$. Let us assume that there are no EDFA repeaters in TAT-13, then the intensity of the input signal and the intensity at the output are related through the loss formula $\alpha L = 10\log_{10}\left(\frac{I_0}{I_1}\right)$. We thus relate $\alpha L\ (dB)$ to the factor of attenuation in intensity by the formula:

$$Factor\ of\ attenuation\ in\ intensity = \frac{I_1}{I_0} = 10^{-\frac{\alpha L}{10}}$$

As a result by travelling through TAT-13, the intensity of the signal attenuates by a factor of $10^{-\frac{\alpha L}{10}}$.

Amplification

Now let us consider the EDFA repeaters assuming no attenuation. Since the signal passes through a total distance of Nl EDFA optical repeaters when travelling through TAT-13, by Beer Lambert's Law, the intensity of the signal increases by a factor of:

$$Factor\ of\ amplification\ in\ intensity = \frac{I_2}{I_0} = e^{gNl}$$

Amplification and Attenuation

In order for the signal to have same intensity at emission in US as at reception in France, N EDFA repeaters (each of length l) are required to amplify the signal so that the amplification of the signal compensates the attenuation of the signal.

Placing the repeaters at the end of the TAT-13 would result in the amplification of a weak signal, containing a considerable amount of noise. Placing the repeaters at the beginning would provide a weaker amplification as the gain would be smaller (a strong signal is amplified less). It is thus desired to place the EDFA repeaters in a dispersed manner along the TAT-13.

A labelled diagram illustrating the intensity pattern due to attenuation and amplification is shown below, in this case assuming the N EDFA repeaters are equally spaced along the TAT-13 cable and that the distance l of each amplification is much smaller than the distance d of attenuation between each EDFA repeater.

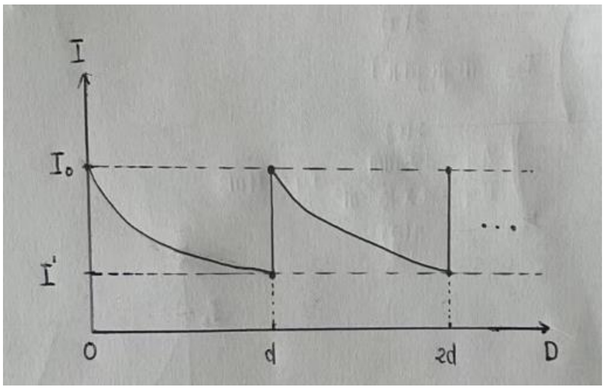

Figure 2: The intensity evolution along the length of the cable (We note that the length of a single EDFA is so small compared to the total distance of propagation that we omit it on the graph.)

We want the intensity at emission in US to be equal to that at reception in France, hence the following relation must hold:

$$Factor\ of\ attenuation\ in\ intensity \times Factor\ of\ amplification\ in\ intensity = 1$$

As a result, we have

$$10^{-\frac{\alpha L}{10}} \times e^{gNl} = 1$$

Solving this equation to find the number of EDFA repeaters required N and using the values of α, L, g, l listed in the section above, we deduce:

$$N = \frac{\alpha L}{10gl} \ln(10) \approx 148$$

Conclusion

A number of 148 EDFA repeaters is a reasonable amount which can be physically added to an optical fiber bundle. It would correspond to a mean spacing of approximately 43km between repeaters. However, one would expect each segment of the network to be designed independently, so the spacing between repeaters can be adjusted to optimize the signal-to-noise ratio in any given segment. The TPC-5CN (Trans-Pacific Cable 5 Cable Network), started its operation in 1996, and was the first submarine optical fiber network which employed EDFA. Its repeater spacing ranges across sectors from 33km to 85km[1], which further consolidates our estimate. The design and configuration of such a cable system can however vary depending on several factors, such as the distance between endpoints, the data rate of the signal, and the type of fiber used.

References

*[1] Barnett, W. Christopher; Takahira, Hitoshi; Baroni, James C.; Ogi, Yoshihiro (February 1996). "The TPC-5 Cable Network". IEEE Communications. **34** (2): 36–40.*

Appendix

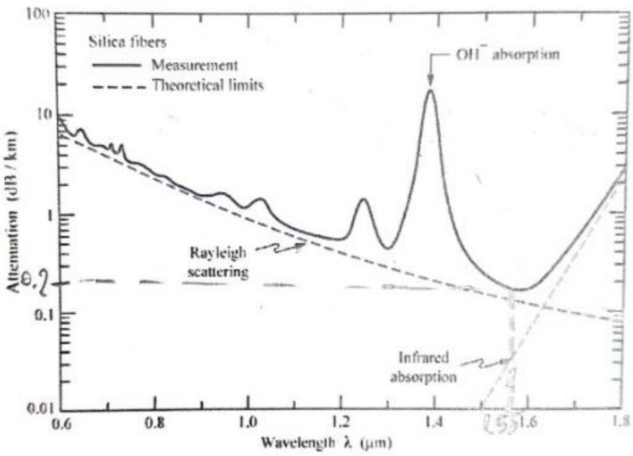

Figure 3: Attenuation of signal due to propagation induced losses as a function of the signal wavelength. An attenuation of $\alpha = 0.2\ dB\ km^{-1}$ corresponds to a $1.55\mu m$ signal.