PHY208 TD1 - Force due to laser

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1 Introduction

In this report we will compute the force applied by a laser pointer on the screen when it is used to comment a lecture slide.

2 Solving the Problem

We start by considering as shown on Figure 1 one photon moving along the x-axis and incident normally on a screen located at x_s. We denote $\vec{p_0} = p_0 \vec{u_x}$ its momentum and using de Broglie's relation:

$$\vec{p_0} = \hbar k_0 \vec{u_x} = \frac{2\pi\hbar}{\lambda_0} \vec{u_x} \tag{1}$$

where λ_0 is its wavelength. From this, we can get the energy of the photon:

$$E_0 = \vec{p_0} \cdot \vec{c} = \frac{2\pi\hbar}{\lambda_0} c \tag{2}$$

where $\vec{c} = c\vec{u_x}$ is the speed of light in vacuum.

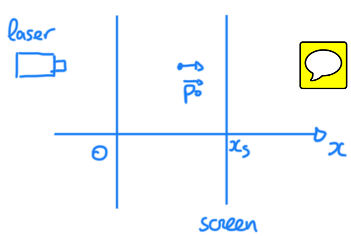

Figure 1: Single photon

We now concider a cylinder of radius R, length dt (in unit time) and with n photons inside incident normally on the screen as shown in figure 2. The initial time t_i corresponds to when the face of the cylinder at $t + dt$ is at x_s so no photon has been reflected off the screen yet and the final time t_f corresponds to when the face at t is at position x_s in other words when all the photons have been reflected off the screen (note that $t_f - t_i = dt$). The total power inside this cylinder is given by $P = \frac{E}{dt}$ where E is the total energy or equivalently $E = nE_0$ so

$$P = \frac{nE_0}{dt} \tag{3}$$

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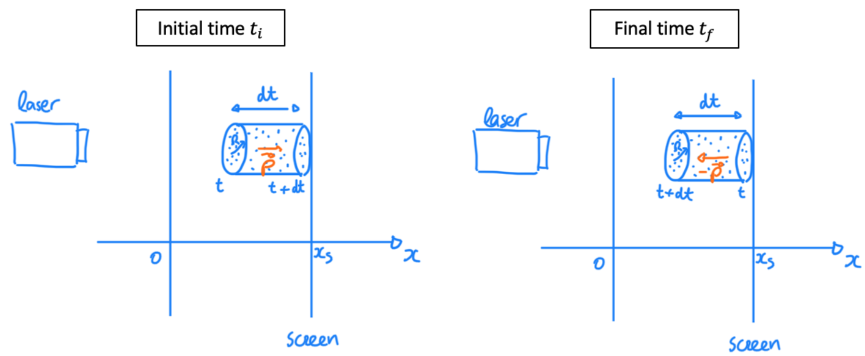

Figure 2: Situation considered

In order to get the force, we will use the following formula:

$$\vec{F} = \frac{\Delta\vec{p}}{dt} = \frac{\vec{p_{t_f}} - \vec{p_{t_i}}}{dt} \tag{4}$$

where $\vec{p}$ denotes the total momentum of the photons inside the cylinder at time t (i.e $\vec{p} = \sum_{n=1}^{n} \vec{p_{n,t}}$). Then, at time t_i all the photons inside the cylinder have momentum:

$$\vec{p_{n,t_i}} = p_0 \vec{u_x}$$

So,

$$\vec{p_{t_i}} = \sum_{n=1}^{n} \vec{p_{n,t_i}} = np_0 \vec{u_x} \tag{5}$$

And at time t_f, all the photons have been reflected hence:

$$\vec{p_{n,t_f}} = -p_0 \vec{u_x}$$

So,

$$\vec{p_{t_f}} = \sum_{n=1}^{n} \vec{p_{n,t_f}} = -np_0 \vec{u_x} \tag{6}$$

Therefore, combing equations 4, 5, 6, we obtain:

$$\vec{F} = \frac{\vec{p_{t_f}} - \vec{p_{t_i}}}{dt} = \frac{-np_0 - (np_0)}{dt} \vec{u_x} = -p_0 \frac{2n}{dt} \vec{u_x} \tag{7}$$

We need to find an expression for the number of photons n inside the cylinder using equation 3, $P = \frac{nE_0}{dt} \iff n = \frac{Pdt}{E_0}$ where $E_0 = \frac{2\pi\hbar}{\lambda_0}c$ from equation 2 so

$$n = \frac{\lambda_0}{2\pi\hbar c} P dt \tag{8}$$

Combining equations 7 with 1 and 8, we obtain:

$$\vec{F} = -2p_0 n \frac{1}{dt}\vec{u_x} = -2(\frac{2\pi\hbar}{\lambda_0})(\frac{\lambda_0}{2\pi\hbar c}Pdt)(\frac{1}{dt})\vec{u_x} = -\frac{2P}{c}\vec{u_x}$$

Note that the force that we are calculating here is the force that the screen exerts on the photons to make them change direction. Thus, by Newton's 3rd law, the force on the screen due to a laser is equal to:

$$\vec{F} = \frac{2P}{c}u_x \iff \|\vec{F}\| = \frac{2P}{c}$$

3 Numerical value and extenstion

3.1 Numerical Value

From one we found in class, we notice that a typical power of a laser pointer is around 1 mW. Then, we obtain

$$||F|| = \frac{(2)(1 * 10^{-3})}{3 * 10^8}$$

$$||F|| = 6.67 * 10^{-12}$$

To confirm that this is a valid value for the force of a laser pointer, we approximate the mass of a dust particle. For dust made of silica, we have

$$\rho = 3300 kgm^{-3}$$

Then, we have for $r = 2 * 10^{-5}m$

$$m = \rho V = 1.3 * 10^{-10} kg$$

In harsh sunlight, we observe that dust particles in the air do not seem to gain any acceleration from the light incident on them. Assuming about 1% the power from the beam is transferred to the particle (due to a tiny surface area), and that dust is a perfectly reflective material, we obtain

$$|a| = \frac{|F|/100}{m} = 5.13 * 10^{-3} ms^{-2}$$

While this is a very small value, it is still more than we observe in the environment, particularly due to our assumptions of normal incidence and the photon model of light.

3.2 Extension

We attempt to extend the model by allowing an arbitrary angle of incidence as shown on Figure 3.

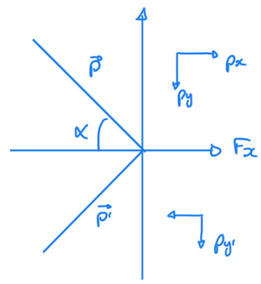

Figure 3: Arbitrary angle of incidence

Following a similar process to normal incidence, we obtain

$$F_x = \frac{\Delta p_x}{dt} = \frac{(-np_0 cos\alpha) - (np_0 cos\alpha)}{dt} = \frac{2Pcos\alpha}{c}$$

$$F_x = \frac{\Delta p_x}{dt} = \frac{0}{dt} = 0$$

with P power of the laser.

For light incident on a screen with 100% absorption, we get

$$|F| = F_x = \frac{\Delta p_x}{dt} = \frac{(0 - (np_0 cos\alpha)}{dt} = \frac{Pcos\alpha}{c}$$

It is worth noting that for any screen, the percentage reflected will be between 0% and 100%, leading to a coefficient that we denote ϵ with $1 < \epsilon < 2$ and

$$|F| = \epsilon\frac{Pcos\alpha}{c}$$

We now attempt to reach the same solution using the wave model of light. allowing scattering with equal probability. Then, we have

$$I = | < \vec{\Pi} > | = \frac{Power}{UnitArea} = \frac{\frac{\Delta(\vec{F}\cdot\vec{x})}{\Delta t}}{A} = \frac{|F|c}{A}$$

With A the area the fields are incident on, I the intensity and Π the Poynting vector of the incident radiation. Then, multiplying by A on both sides

$$I * A = P = |F|c$$

and the force due to the incident light is

$$|F| = \frac{P}{c}$$

Note that the result is the same as the photon model for a material that fully absorbs the light. Now, for a material that reflects all light, we add the force due to the emission of the reflected light. For a screen with specular reflection, we get that the force of the emission is the same as the force of the absorption, leading to a net force

$$|F|_{net} = 2|F|_{inc} = \frac{2P}{c}$$

same as the photon model.

For diffusive reflection, which is more accurate in the case of the screen, we have

$$F_{ref} = \frac{P}{c} \;\; = \frac{A}{c}\frac{1}{2\pi}\int_0^{\pi/2} Icos\theta d\Omega$$

With $d\Omega = 2\pi sin\theta d\vartheta$ the infinitesimal solid angle with respect to the point of emission and A the area of incidence of the light

Then, we have

$$F_{ref} = \frac{P}{c} \;\; = \frac{1}{c}\frac{1}{2\pi}\int_0^{\pi/2} IAcos\theta 2\pi sin\theta d\theta = \frac{IA}{2c}\int_0^{\pi/2} sin2\theta d\theta$$

and

$$F_{ref} = -\frac{1}{2}\frac{P}{c}$$

and we have net force

$$F_{net} = \frac{3P}{2c}$$

We again realize that the total force can be expressed with

$$|F| = \epsilon\frac{P}{c}$$

with $1 < \epsilon < 2$ depending on the assumptions of the model.