Evaluation PHY555 Energy & Environment

Friday 16st December 2022

Part II, 5 points, recommended time 30 min

Note 1: Use pink paper sheets for your answers.

Exercise: Sky Radiator – Solutions

By carefully tuning the optical properties of a material, it is possible to take advantage of radiative heat transfer in order to provide a cooling power with a passive system. The main idea is to take advantage of a spectral window where the atmosphere is almost transparent to radiatively expel heat towards space.

1. Demonstrate that, when a hot and a cold system are in contact, heat flows spontaneously from the hot to the cold system. What is required in a usual cooler to take heat away from the cold source towards the hot source?

   Consider an isolated system composed of a hot body at T_H and a cold body at T_C. If an amount $\mathrm{d}Q$ of heat flows from the cold to the hot source, the energy variation of the sub systems is $-\mathrm{d}Q$ and $+\mathrm{d}Q$ respectively according to the first law.

   To estimate the entropy change, we use the second law and we consider a virtual transformation where the systems is not isolated anymore, but each sub system is transformed, independently from the other, in a reversible way, from the initial state to the same final state as the real transformation.

   For the cold system, this is achieved by draining $\mathrm{d}Q$ using a thermostat at temperature $\mathrm{d}Q$, with an entropy exchange of $-\mathrm{d}Q/T_C$. For the hot system, we inject heat using a thermostat at temperature T_H, increasing the entropy by $+\mathrm{d}Q/T_H$. No entropy is created since the transformation is reversible.

   Entropy is a variable of state, so the total change in the virtual path is the same as in the real path : $\mathrm{d}S=\mathrm{d}Q(1/T_H-1/T_C)<0$; which is impossible for an isolated system.

   To get heat to flow from a cold source towards a hot source, work must be provided, so that the total heat delivered to the hot source in larger than that taken from the cold source, and the entropy flows can be (at least) equal.

2. Estimate from basic principles the solar irradiance (in W/m²) received by an absorber under direct sunlight on Earth.

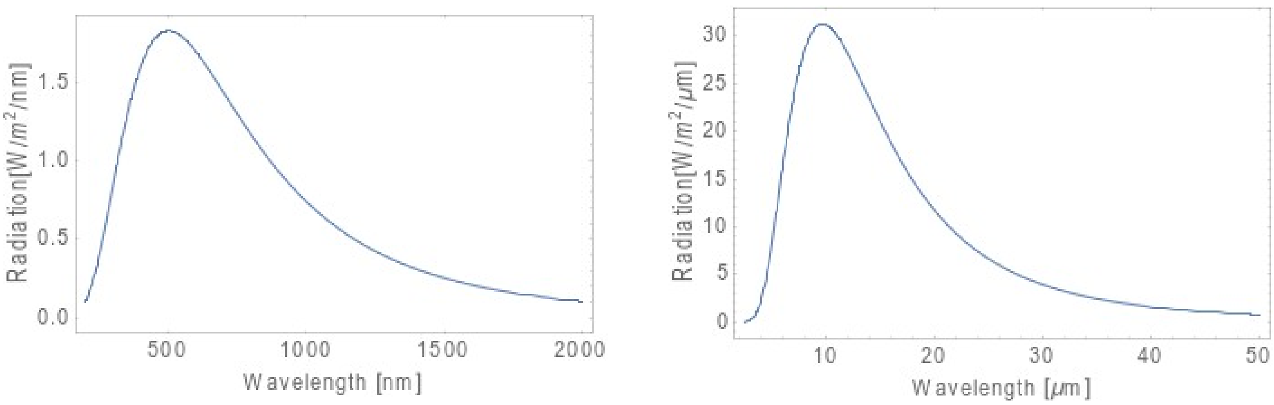

Figure 1: Blackbody radiation at the sun's temperature (left) and at ambient temperature (right).

One obtains

$$\frac{\Omega}{\pi}\sigma T_{sun}^4 \simeq \frac{R_{sun}^2}{D_{Earth-sun}^2}\sigma T_{sun}^4 = 1380\,\mathrm{W/m^2} \quad \textit{(Eq. 1)}$$

3. Consider the absorber as gray-body (constant absorptivity ≤ 1). Estimate the steady state temperature of the absorber considering only the solar radiation as a power input.

   The sun power is:

   $$P_{sun} = \alpha\frac{\Omega}{\pi}\sigma T_{sun}^4 \quad \textit{(Eq. 2)}$$

   The Krichhoff's law indicates that emissivity = absorptivity, so

   $$P_{rad} = \alpha\sigma T_{abs}^4 \quad \textit{(Eq. 3)}$$

   Once the steady state is reached:

   $$T_{abs} = \left(\frac{\Omega}{\pi}\right)^{1/4} T_{sun} = 395\,\mathrm{K} \quad \textit{(Eq. 4)}$$

4. According to the previous situation, what is the temperature of the absorber in the dark (for instance, at night)? Comment this result.

   No sun so $P_{sun} = 0$ *, so* $T_{abs} = 0\,\mathrm{K}$*, obviously missing something.*

5. We now also consider that radiation emitted by the atmosphere, which we consider as a black body at ambient temperature. Estimate the atmospheric irradiance received by the absorber. What is now the temperature of the absorber in the dark?

   The absorber perceives the atmosphere radiation coming from all directions, and absorbs it with the same absorptivity α

   $$P_{atm} = \alpha\sigma T_{atm}^4 \quad \textit{(Eq. 5)}$$

   In the dark, $P_{abs} = P_{atm} \quad \Rightarrow \quad T_{abs} = T_{atm}$*, which makes sense.*

6. Consider that the absorber has a given absorptivity in the 0.2 – 2 μm spectral region, and a (possibly) different absorptivity in the 2 – 20 μm. Show that in this simple model, the absorber cannot be colder than ambient temperature, regardless of the solar irradiance.

   For simplicity, we consider that the solar radiation is entirely in the first spectral interval, while the atmospheric radiation is entirely in the second interval. The radiation of the absorber is a priori closer to ambient temperatures than to $6000\,\mathrm{K}$*, and will be treated also as being entirely emitted in the second interval*

   $$P_{abs} = P_{atm} + P_{sun} \quad \Rightarrow \quad \alpha_{IR} T_{abs}^4 = \alpha_{IR} T_{atm}^4 + \alpha_{vis}\frac{\Omega}{\pi} T_{sun}^4 \quad \textit{(Eq. 6)}$$

   Which is clearly larger than T_{atm} *; can be made close to* T_{atm} *with a weak absorption in the visible range (a mirror is not hot under sunlight) or much hotter with a strong absorption in the visible range.*

7. The atmosphere is actually not a black body, but shows transparency windows (ie spectral intervals with almost no absorptivity), notably in the 8-13 μm region. It is possible to take advantage of this feature to achieve a passive chiller – a system which spontaneously reaches temperatures 5K lower than ambient temperature, even under sunlight (Raman et al., Nature volume 515, pages 540–544 (2014)). Explain qualitatively how it is possible, and how this result can be compatible with question 1 and 4.

   In the transparency window, the atmosphere does not absorb radiations ; so it doesn't emit radiations either.

   Consider a system can with a low absorptivity (and emissivity) before 8 μm and after 13 μm, and a strong emissivity in between. The system gets little radiation neither from the sun, nor from the atmosphere, but emits a lot of radiation so dissipates a lot of power.

   The "cold" source in this case is the outer space, at $3\,\mathrm{K}$*! If no heat transfer were occurring, the system would be at equilibrium with the cosmological background.*