Evaluation PHY555 Energy & Environment

Friday 16st December 2022

Part I, 5 points, recommended time 30 min

Q1: **Orders of Magnitude / Storage** – You have in your possession a button cell (small, coin-sized battery) with the following characteristics: 200 mAh at a nominal voltage of 1.5 V, mass of 2.3 g, diameter of 2 cm. If you design a perfect system where all the energy of the battery is used to spin it like a flywheel, at what angular velocity would the battery spin?

The energy stored in the battery is

$E_e = U \times I \times t = 1.5\,\mathrm{V} \times 200 \times 10^{-3}\,\mathrm{A} \times 3600\,\mathrm{s} = 1080\,\mathrm{J} = 0.3\,\mathrm{Wh}$.

It should correspond to the kinetic energy of the flywheel: $E_{\mathrm{kin}} = \frac{1}{2} J \Omega^2$.

For an homogeneous cylinder of height h *and density* ρ*:*

$$\left.\begin{aligned} M &= \rho h \int_0^R 2\pi r\,\mathrm{d}r = \frac{2\pi}{2}\rho h R^2 \\ J &= \rho h \int_0^R 2\pi r^3\,\mathrm{d}r = \frac{2\pi}{4}\rho h R^4 \end{aligned}\right\} \Rightarrow J = \frac{1}{2} M R^2 \quad \Rightarrow E_{kin} = \frac{1}{4} M R^2 \Omega^2$$

Equality between electrical and kinetic energy gives:

$$\Omega = \sqrt{\frac{4 E_{\mathrm{kin}}}{M R^2}} = \sqrt{\frac{4 \times 1080\,\mathrm{J}}{2.3\ 10^{-3}\,\mathrm{kg} \times 10^{-4}\,\mathrm{m}^2}} = 137\ 10^3\,\mathrm{rad/s}, \qquad \nu = \frac{\Omega}{2\pi} = 22\,\mathrm{kHz}$$

Q2: **Global Warming –** If atmospheric CO_2 increases to 450 ppm, using the data of PC2, over what bandwidth (centered around 670 cm^{-1}) would atmospheric optical absorption due to CO_2 be greater than 50%?

From PC2: $A_\lambda = 1 - \exp\left(-c_{CO_2} n_0 L \sigma(\lambda)\right) \geq 0.5 \quad \Leftrightarrow \quad c_{CO_2} n_0 L \sigma(\lambda) \geq \ln(2)$

Using $c_{CO_2} = 450 \times 10^{-6}$, $n_0 = 2{,}5 \times 10^{25}\,\mathrm{m}^{-3}$ *and* $L = 8 \times 10^3\,\mathrm{m}$ *the condition becomes:*

$$\sigma(\lambda) \geq \frac{\ln(2)}{c_{CO_2} n_0 L} = 7.7 \times 10^{-27}\,\mathrm{m}^2 = 0.7 \times 10^{-22}\,\mathrm{cm}^2$$

From graph of PC2, $\Delta\nu = (770 - 580) = 190\,\mathrm{cm}^{-1}$

One can also use the result from PC2:

$$\frac{\Delta\lambda}{\lambda_0^2} \approx \frac{2}{r} \ln\left(\sigma_0 c_{CO_2} n_0 L\right) \quad \text{with} \quad \lambda_0 = 15\,\mu\mathrm{m},\ \sigma_0 = 10^{-23}\,\mathrm{m}^2 \text{ and } r = 0.09\,\mathrm{cm}$$

From which one obtains $\Delta\lambda = 3.4\,\mu\mathrm{m}$

Q3: **CCS** – Congratulations, you have invented a machine that can perform Direct Air Capture of CO_2 from the atmosphere (420 ppm). The machine has a 2nd law efficiency of 50% (it only requires twice the power to operate than the thermodynamic minimum). For the power source driving the machine, what must be the carbon intensity (kg_{CO2}/kWh) to break even in terms of atmospheric CO_2?

The minimal theoretical work needed to extract one mole of CO_2 *from atmosphere is*

$$W_{\mathrm{min}}=RT(1-\ln(x))=8.31\times300\times(1-\ln(420\times10^{-6}))=21.9\,\mathrm{kJ/mol_{CO_2}}$$

Working with an efficiency $\eta=0.50$*, the machine work is:*

$$W_{\mathrm{machine}}=W_{\mathrm{min}}/\eta=43.7\,\mathrm{kJ/mol}=12.14\,\mathrm{Wh/mol_{CO_2}}.$$

Taking into account the modal mass $M_{CO_2}=44\,\mathrm{g/mol}$ *one obtains a massive work:*

$$W_{\mathrm{machine}}=0.276\,\mathrm{kWh/kg_{CO_2}}$$

The machine should produce less CO_2*; that is a carbon intensity:*

$$m_{CO_2}\leq\frac{M_{CO_2}}{W_{\mathrm{machine}}}=\frac{0.044\,\mathrm{kg/mol}}{12.14\,\mathrm{Wh/mol}}=3.6\,\mathrm{kg_{CO_2}/kWh}$$

Q4: **Thermal Machines** – You seek to build an electrical power generation station. You have access to a hot source at 350°C and a cold sink at 10°C. At what efficiency should the station be run to extract the maximum available electrical power?

The Curzon Ahlborn efficiency gives the maximum power with an efficiency:

$$\eta=1-\sqrt{\frac{T_L}{T_H}}=1-\sqrt{\frac{273.15+10}{273.15+350}}=32\,\%$$

This should not be confused with the Carnot efficiency, which gives the maximum theoretical efficiency but usually with a null power:

$$\eta_{\mathrm{Carnot}}=1-\frac{T_L}{T_H}=1-\frac{273.15+10}{273.15+350}=55\,\%$$

Q5: **Nuclear** – CANDU reactors use uranium with a very small fraction of ^{235}U. Why is it more critical for them than for other technologies to use heavy water as a moderator, rather than graphite or H_2O?

To avoid their loss by capture to the much more prevalent ^{238}U*, emitted neutrons must be slowed down by the moderator to energies where the* ^{235}U *fission cross section is much higher. However, this must be done with the lowest possibility of capture loss to the moderator, to maintain criticality.*

Heavy water has the lowest net capture cross section after the necessary number of slowing collisions. Regular water has a higher capture cross section.

Q6: **Insulation** – A fashionable front door to a house is made of wood, 5 cm thick, and has dimensions of 80 cm x 220 cm. It has a small decorative window made of safety glass that is 10 cm x 30 cm and 8 mm thick. Using the information of PC6, how much more heat is lost through the window versus the rest of the door?

The thermal resistances of wood and glass are respectively, taking into account the boundary layer:

$$R_{wood} = \frac{0.05}{0.13} + 0.17 = 0.55\,\mathrm{K\,W^{-1}\,m^2}$$
$$R_{glass} = \frac{0.008}{1} + 0.17 = 0.178\,\mathrm{K\,W^{-1}\,m^2}$$

The heat fluxes are therefore, taking into account the respective area of wood and windows:

$$j_{wood} = \frac{S_{\mathrm{wood}}}{R_{\mathrm{wood}}} = \frac{0.8 \times 2.2\,\mathrm{m^2}}{0.55\,\mathrm{K\,W^{-1}\,m^2}} = 3.2\,\mathrm{W/K}$$
$$j_{glass} = \frac{S_{\mathrm{glass}}}{R_{\mathrm{glass}}} = \frac{0.1 \times 0.3\,\mathrm{m^2}}{0.178\,\mathrm{K\,W^{-1}\,m^2}} = 0.17\,\mathrm{W/K}$$

The heat losses through the window correspond to $0.17/3.2 = \mathbf{5.3\,\%}$ *of the total heat losses although it corresponds to only* $(0.1 \times 0.3)/(0.8 \times 2.2) = \mathbf{1.7\,\%}$ *of the surface.*

Note: Without taking into account the boundary layers, one would obtain:

$$\begin{array}{l} R_{wood} = \frac{0.05}{0.13} = 0.385\,\mathrm{K\,W^{-1}\,m^2} \\ R_{glass} = \frac{0.008}{1} = 0.008\,\mathrm{K\,W^{-1}\,m^2} \end{array} \Rightarrow \begin{cases} j_{wood} = \frac{S_{\mathrm{wood}}}{R_{\mathrm{wood}}} = \frac{0.8 \times 2.2\,\mathrm{m^2}}{0.385\,\mathrm{K\,W^{-1}\,m^2}} = 4.6\,\mathrm{W/K} \\ j_{glass} = \frac{S_{\mathrm{glass}}}{R_{\mathrm{glass}}} = \frac{0.1 \times 0.3\,\mathrm{m^2}}{0.008\,\mathrm{K\,W^{-1}\,m^2}} = 3.75\,\mathrm{W/K} \end{cases}$$

so ignoring the boundary layers gives the wrong result that 45% of the heat would go through the small window.

Q7: **Wind -** In a well-designed wind turbine blade, the "chord" of the blade is thinner at the tips. Why is it so?

Answer, tick the right box(es):

☐ all parts of the blade face directly into the apparent wind

☐ the ratio between the lift and drag coefficient remains constant over the length of the blade

☒ the tangential force on the blade remains constant over its length

☐ to minimize the total mass of the blade

☐ it looks cool

Further comments:

In the design, the ratio between the lift and drag coefficient remains constant over the length of the blade, but this is just a consequence of the twisting angle, not of the chord length. The chord length is adjusted to produce a uniform thrust on the rotor, which also produces a uniform tangential force.

Q8: **Solar** – A modern, commercial electric car consumes about 0.2 kWh/km. If the roof of such a car were to be covered in solar cells, under optimum conditions (sun directly overhead, using values calculated in PC 8), how fast can a solar-cell powered car travel (if the batteries were disconnected)?

The solar flux, at the top of the atmosphere and under normal incidence (Solar *constant) is* $F_s = 1368\ \mathrm{W/m^2}$*, out of which ~30% is absorbed in the atmosphere. In the absence of clouds and under optimum conditions, the flux reaching the ground is* $F_T \sim 1000\ \mathrm{W/m^2}$*. Assuming* $\eta \sim 20\,\%$ *efficiency and a surface of* $S \sim 2\,\mathrm{m^2}$*, the solar power received by the car is:*

$$P_{\mathrm{solar}} = F_T \times \eta \times S \approx 400\ \mathrm{W}$$

Assuming 100% efficiency of the electrical motors the corresponding velocity is:

$$v = \frac{P_{\mathrm{solar}}}{0.2\ \mathrm{kWh/km}} = 2\ \mathrm{km/h}$$