École Polytechnique
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PC4 — Energy Storage

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1 – Energy transfer pumping station

The pumped storage is a storage technique of electrical energy. It is based on the principle of pumping water to store it in accumulation basins in altitude when energy demand is low – pumping mode — to churn the water later to produce electricity when demand is high.

The hydroelectric installation of Verney-Grand'Maison, the largest in France, was commissioned in 1987. It has two factories, one outdoor (drop height $922\,\mathrm{m}$, flow $76\,\mathrm{m^3/s}$) and one underground (drop height: $955\ \mathrm{m}$, flow speed: $140\,\mathrm{m^3/s}$). Only the last one can operate in pumping mode (see figure 1). Rated power in electricity production mode is $1800\ \mathrm{MW}$ for the two plants together. The Grand'Maison dam (upper basin) covers an area of $2{,}2\,\mathrm{km^2}$ and may contain 137 million $\mathrm{m^3}$ of water.

1. What are the specific energy (in $\mathrm{kWh/kg}$) and the total capacity of the system? How long can the station operate without water income?
2. What is the overall turbine yield?
3. Power consumption while pumper is $1275\ \mathrm{MW}$ with a flow rate of $135\,\mathrm{m^3/s}$.What is the pumping yield?
4. Part of the power losses comes from friction of water in pipes (pressure drop). Show that the pressure drop can be expressed by an equivalent reduction on the drop height.

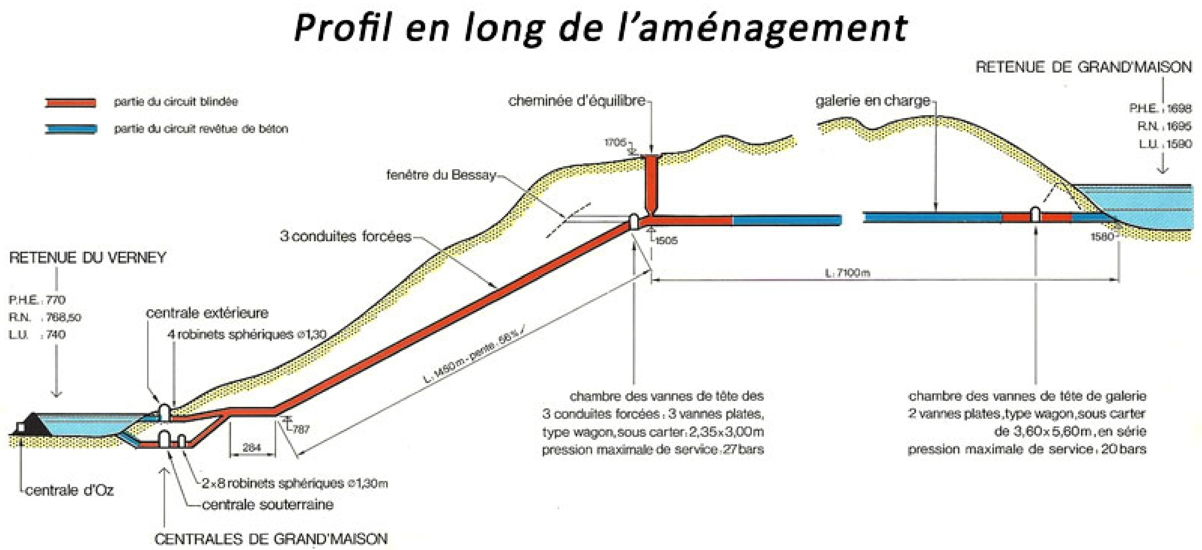


Figure 1 : Profile of the Grand'Maison power station (Savoie, France)

The pressure drop in the flow of an incompressible fluid density ρ with an average speed V in a circular pipe of length L and diameter D is given by the Darcy-Weisbach equation:

$$\Delta p = f \frac{L}{D} \frac{\rho v^2}{2}$$

Here, f is the Darcy friction coefficient which depends on the flow regime (laminar or turbulent) and the roughness of the line (see Fig. 2). In the Grand'Maison station there 3 parallel pipes of diameter $D=3\ \text{m}$ and length $L=1800\ \text{m}$ each.

5. Determine the flow regime in the pipes from the Reynolds number $\text{Re}=\rho v D/\eta$ .The dynamic viscosity η of the water varies between $0.00157\ \text{Pa.s}$ at 4 ºC to $0.00131\ \text{Pa.s}$ at 10 ºC. The transition from laminar to turbulent takes place for Reynold numbers close to 3000.
6. (*) Prove that, in the case of laminar flow, the Darcy friction coefficient is equal to $64/\text{Re}$.
7. Estimate the losses in pumping yield induced by the friction in the pipes.
8. The same technique can also be used in a tidal power station. In the factory of la Rance (France), water is pumped at high tide (and the level in the basin is raised approximately by $1\ \text{m}$ above the level of high tide). At low tide the water is churned. What is the point of the process? What is the return to a tidal range of $5\ \text{m}$?

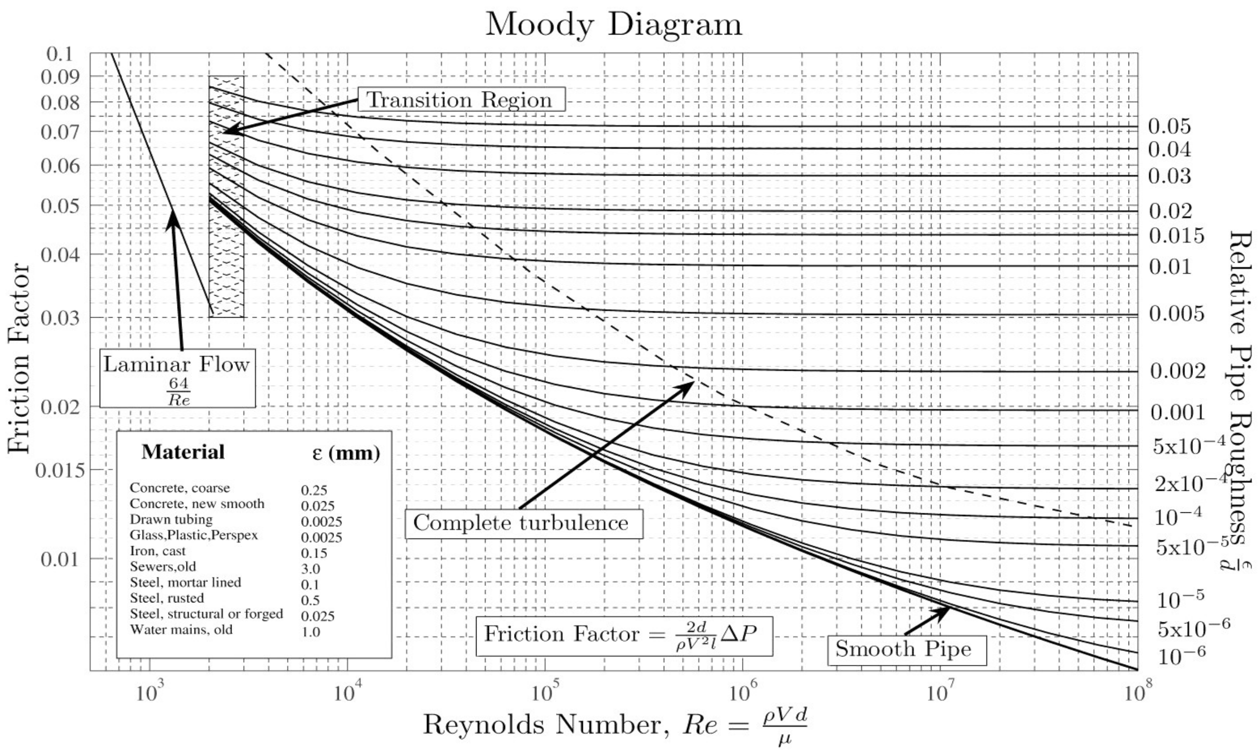
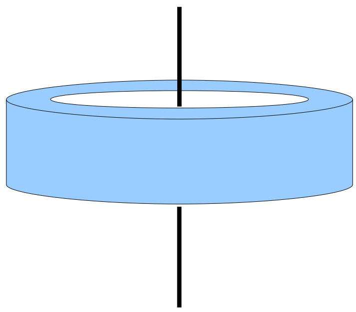


Figure 2 : Friction coefficient f depending on the Reynolds number (Moody diagram)

2 – Designing a flywheel

Flywheels are a relatively convenient way to store energy transiently. They are used as regulators in many mechanical systems, in order to make the system operate more consistently, by opposing itself to jerks due to the motor driving the device or to the receiver consuming the energy transmitted. They are also used in cars and lorries. In the 1950s, a bus was operated by means of a flywheel: The “Gyrobus” is powered by an electric motor, whose power is supplied by a large wheel launched at high speed. The engine powering the energy storage wheel is reversible. Once the steel wheel of $1{,}6\,\mathrm{m}$ diameter and $1500\,\mathrm{kg}$, is launched at a rotation speed of $3000\,\mathrm{tr/min}$, the motor becomes a generator and feeds electric motors for propulsion of the vehicle.

Charging the flywheel was done during ascents and descents of passengers, by means of a boom installed on the vehicle. It took 30 seconds to 3 minutes, depending on the voltage applied at the motor terminals to charge the flywheel. Fully charged, a gyrobus was completely autonomous over a distance of about $6\,\mathrm{km}$ at a speed of about $50-60\,\mathrm{km/h}$.



1. Let's Consider a flywheel of radius R and mass M, rotating at the angular velocity Ω. Compute the disk rotational kinetic energy, for a full disk and for a thin ring. The power consumption of a bus is around $150\,\mathrm{kW}$. What was the autonomy of the Gyrobus?
2. Determine the tangential stress σ_t in the rotating ring (stress, homogeneous to a pressure, is the binding force divided by the contact surface). For molten steel, the ultimate tensile stress is about $200\times10^{6}\,\mathrm{N\,m^{-2}}$ (see Table 1). The most resistant alloy steel (nickel-chrome vanadium) a tensile strength of $1800\times10^{6}\,\mathrm{N\,m^{-2}}$.Determine the maximum rotational speed of the flywheel. What relation connects the maximum available energy to the flywheel mass for each of the materials given in Table 1? Conclusions?
3. Balance: the flywheel is inclined by an angle θ relative to its rotation axis. What actions are exerted on the rotation axis? We consider a steel axis, of $2\,\mathrm{cm}$ in diameter. What is, as function of the power of the device and the diameter of the axis, the limit tilt angle?
4. What are the side effects of the use of an inertia disc in a vehicle? It is possible to counterbalance them?

Material	Elastic strength (MPa)	Density (g/cm^3)
Concrete	10	2,30
Aluminium	600	2,70
alloy Steel	1800	7,86
Titanium	1300	4,51
Fibreglass	3400	2,60
carbon fibre	4300	1,75
Kevlar	3620	1,44
carbon Nano-tubes	62 000	0,037 – 1,34

Table 1 : Tensile strength of conventional materials. Wikipedia

3 – Comparison with other energy carriers

Largest values of specific energy are obtained with (electro-)chemical energy carriers: $43\,\mathrm{MJ/kg}$ for gasoline, $120\,\mathrm{MJ/kg}$ for di-hydrogen. This makes them particularly interesting for the sector of transportation. In addition, the conversion of dihydrogen in a fuel cell has a significantly higher yield (~ 50%) than an internal combustion engine (33%).

1. Estimate the needed mass of H_2 to cover $400\,\mathrm{km}$. What volume does it represent? What can we conclude from it?
2. Estimate the specific energy for a lead battery (atomic mass 207u). The specific energy given by the manufacturers are in the range $30-40\,\mathrm{Wh/kg}$. Where does the difference come from? By what means can we increase specific energy?