PHY 555 – Energy and Environment

PC9 – Energy Storage – Solutions

1 – Energy transfer pumping station

1. What are the specific energy (in $\mathrm{kWh/kg}$) and the total capacity of the system? How long can the station operate without water income?

potential energy:	$E=m\,g\,h$
specific energy:	$E/m=g\,h=9.81\,\mathrm{m\,s^{-2}}\times 955\,\mathrm{m}\sim 9.4\,\mathrm{kJ/kg}=2.6\,\mathrm{Wh/kg}=2.6\,\mathrm{kWh/t}$
	$E/V=2.6\,\mathrm{kWh/m^3}$
total capacity:	$E=2.6\,\mathrm{kWh/m^3}\times 137\,106\,\mathrm{m^3}=356\,\mathrm{GWh}$
autonomy:	$t=356\,\mathrm{GWh}/1.8\,\mathrm{GW}\sim 8\,\mathrm{days}$

2. What is the overall turbine yield?

design:

$$P_{th}=\rho\,g\,(h_1 Q_1+h_2 Q_2)=1000\ \mathrm{kg\,m^{-3}}\times 9.81\,\mathrm{m\,s^{-2}}(922\times 76+955\times 140)\,\mathrm{m^4\,s^{-1}}\sim 2000\,\mathrm{MW}$$

yield:

$$P_{nom}/P_{th}=1800\,\mathrm{MW}/2000\ \mathrm{MW}=90\,\%$$

3. Power consumption while pumper is $1275\ \mathrm{MW}$ with a flow rate of $135\,\mathrm{m^3/s}$ .What is the pumping yield?

In pumping mode, we have to consider the difference between the free surfaces, because the pumps (underground) benefit from the pressure of the amount of water above them. This difference varies while pumping as the lower dam empties and the upper one fills up.

The theoretical output power is therefore:

$$Pt_h=\rho\,g\,(h_2 Q_2)=1000\,\mathrm{kg\,m^{-3}}\times 9.81\,\mathrm{m\,s^{-2}}\times 889\,\mathrm{m}\times 135\,\mathrm{m^3\,s^{-1}}=1177\,\mathrm{MW}$$

4. Part of the power losses comes from friction of water in pipes (pressure drop). Show that the pressure drop can be expressed by an equivalent reduction on the drop height. (This is termed "head loss", in the parlance of our times.)

The hydrostatic pressure is given by $p=\rho\,g\,h$. Hydrostatic pressure at the bottom of the pipe is about $9.4\,\mathrm{MPa}\,(94\,\mathrm{bar})$.
A pressure drop Δp due to friction is equivalent to a loss of drop height $\Delta h=\Delta p/\rho\,g$

5. Determine the flow regime in the pipes from the Reynolds number

average water velocity: $Q=n\times v\times S=n\times v\times \pi/(4\,D^2)$

$$\Rightarrow\quad v=Q/(n\times S)=4\,Q/(\pi\,n\times D^2)=4\times 135\,\mathrm{m^3/s}/(3\,\pi\times 9\,\mathrm{m^2})\sim 6.4\,\mathrm{m/s}$$

Reynolds number: $\mathrm{Re}=\rho\,v\,D/\eta=\dfrac{4\rho}{\pi\eta}\dfrac{Q}{nD}\approx 10^6\dfrac{Q[\mathrm{m^3/s}]}{n\,D[\mathrm{m}]}=\dfrac{10^6\times 135}{3\times 3}=1.5\times 10^7$

7. *Estimate the losses in pumping yield induced by the friction in the pipes.*

From the Darcy-Weisbach equation one obtains:

$$\Delta h = f\frac{L}{D}\frac{v^2}{2g} = \frac{f}{2g}\frac{L}{D}\left(\frac{Q/n}{\pi/4D^2}\right)^2 = \frac{8f}{\pi^2 g}L\left(\frac{Q}{n}\right)^2 D^{-5}$$

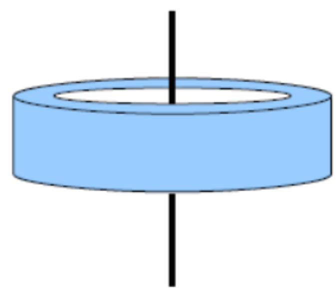
Figure 2 shows that the Darcy friction coefficient does not vary much with R_e in turbulent regime, The pressure losses therefore depends a lot in the pipe diameter, and there will be more interest in increasing the diameter of the pipes instead of their number.

8. The same technique can also be used in a tidal power station. In the factory of la Rance (France), water is pumped at high tide (and the level in the basin is raised approximately by 1 m above the level of high tide). At low tide the water is churned. What is the point of the process? What is the return to a tidal range of 5 m?

The technique benefits from the difference between high tide and low tide. For a 5m tide, if one pumps water at an average altitude of 1m above the sea level, and that the same water is churned at low tide, the turbine yield will be significantly greater than 1: 0,9×6 –1/0,9=4,3 (90 % of the energy corresponding to 6m tidal range is recovers, while only 1/0.9 was spent while pumping, corresponding to only one meter of elevation).

2 – Designing a flywheel

1. *Let's Consider a flywheel of radius* $\mathbf{R}$ *and mass* $\mathbf{M}$, *rotating at the angular velocity* Ω. *Compute the disk rotational kinetic energy, for a full disk and for a thin ring. The power consumption of a bus is around* $150\,\mathrm{kW}$. *What was the autonomy of the Gyrobus?*



The kinetic energy of rotation is expressed as:

$$E_c = \frac{1}{2}J_z\Omega^2$$

The moment of inertia along the z axis is:

$$J_z = \int r^2 \mathrm{d}m = h\int \rho r^2 r\,\mathrm{d}r\,\mathrm{d}\theta = 2\pi\rho h\frac{R^4}{4} = M\frac{R^2}{2}$$

thus:

$$E_c = \frac{MR^2\Omega^2}{4}$$

At constant weight, a hollow (ring) wheel is more efficient than a full disk, because the mass in the center contributes little to energy kinetic. For such a ring :

$$E_c = \frac{MR^2\Omega^2}{2}$$

For the Gyrobus, the rotational angular velocity is: $\Omega = 2\pi\times 3000/60 = 100\pi\,\mathrm{rad\,s^{-1}}$.We find, for a radius of $R = 0{,}8\,\mathrm{m}$,and a mass $M = 1500\,\mathrm{kg}$ a kinetic energy $E_c = 47\,\mathrm{MJ}$ .For a power $P = 150\,\mathrm{kW}$, the autonomy is $t = E_c/P \sim 5\,\mathrm{mn}$ .At lower power, autonomy was increased and could reach about $10\,\mathrm{km}$ at $50\,\mathrm{km/h}$ .The tangential speed of the flywheel is

$R\Omega = 0{,}8\times 2\pi\times 3000/60 = 250\,\mathrm{m\,s^{-1}} = 900\,\mathrm{km\,h^{-1}}$.

2. *Determine the tangential stress σ_t in the rotating ring (stress, homogeneous to a pressure, is the binding force divided by the contact surface). For molten steel, the ultimate tensile stress is about $200\times10^6\,\mathrm{N\,m^{-2}}$ (see Table 1). The most resistant alloy steel (nickel-chrome vanadium) a tensile strength of $1800\times10^6\,\mathrm{N\,m^{-2}}$. Determine the maximum rotational speed of the flywheel. What relation connects the maximum available energy to the flywheel mass for each of the materials given in Table 1? Conclusions?*

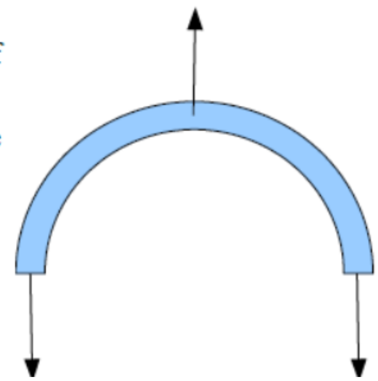
The simplest is to consider a half ring, and to compute the balance of actions. On the ring, the following actions are undertaken:

- Tangential binding forces, on both sides. Depending on the section:

  $$\vec{F}_l=-2\times S\times\sigma_t\vec{u}_x$$

- Centrifugal force:

  $$\vec{F}_c=\int_0^{\pi}\rho S r\,\mathrm{d}\theta\sin\theta\, r\,\omega^2=2\rho S r^2\omega$$



(Action = force. Sum of forces is set to zero to calculate maximum tangential stress.)

Ultimately, the maximum energy can be written as:

$$E\propto\frac{\sigma_t}{\rho}M$$

3. *Balance: the flywheel is inclined by an angle θ relative to its rotation axis. What actions are exerted on the rotation axis? We consider a steel axis, of $2\,\mathrm{cm}$ in diameter. What is, as function of the power of the device and the diameter of the axis, the limit tilt angle?*

(action = force)

At the limit of a thin ring,

$$J_{x'}=h\int\rho(r^2\sin^2\theta)r\,\mathrm{d}r\,\mathrm{d}\theta=\frac{1}{2}J_{z'}$$

In the tilted frame the inertia tensor is diagonal and thus:

$$\vec{\sigma}=J_{z'}\cos\theta\,\Omega\,\vec{u}_{z'}+J_{x'}\sin\theta\,\Omega\,\vec{u}_{x'}=M R^2\Omega\left(\cos\theta\,\vec{u}_{z'}+\frac{1}{2}\sin\theta\,\vec{u}_{x'}\right)$$

in the end, after re-projecting on Laboratory axes:

$$\vec{\sigma}=M R^2\Omega\left(\left(\cos^2\theta+\frac{1}{2}\sin^2\theta\right)\vec{u}_z-\frac{1}{2}\sin\theta\cos\theta\,\vec{u}_x\right)$$

The vertical component does not change over time. In contrast, By construction, $\frac{\mathrm{d}\vec{u}_x}{\mathrm{d}t}=-\Omega\vec{u}_y$ and therefore:

$$\vec{\Gamma}=\frac{\mathrm{d}\vec{\sigma}}{\mathrm{d}t}=\frac{1}{2}M R^2\Omega^2\sin\theta\cos\theta\,\vec{u}_y$$

This torque is exerted on the axle (shaft) of the flywheel. For a cylindrical shaft of diameter D (and a second moment of section I_{Gz}) , one finds that the maximum stress is

$$\sigma_{max} = \frac{M\,D}{2\,I_G z} = \frac{32\,M}{\pi\,D^3}$$

We can then write the maximum angle of misalignment based on the diameter of the axis, of the stored kinetic energy E_c and the yield strength of the material R_e :

$$\Gamma = \frac{1}{2} M\,R^2 \Omega^2 \sin\theta \cos\theta \approx E_c \theta_{max} = \pi\,D^3 \frac{R_e}{32} \quad \Rightarrow \quad \theta_{max} = \frac{\pi\,D^3 R_e}{32\,E_c}$$

3 – Comparison with other energy carriers

1. *Estimate the needed mass of H_2 to cover* 400 km . *What volume does it represent? What can we conclude from it?*

Energy consumption of a gasoline car: $6\,l/100\,km \times 0.7\,kg/l \times 43\,MJ/kg \times 0.33 \approx 60\,MJ/100\,km$, which isequivalent to 1 kg of H_2 with a yield of 50 %. So 1 kg of H_2 per 100 km.

Volume of tank and then compression from PV=nRT.

2. *Estimate the specific energy for a lead battery (atomic mass 207u). The specific energy given by the manufacturers are in the range* $30-40\,Wh/kg$. *Where does the difference come from? By what means can we increase specific energy?*

Most electrochemical couples exude an energy of about 1 eV per electron. This is particularly true for the couple Pb/PbO_2 .

$$1\,eV/m\,(\text{lead atom}) = 1.6\times10^{-19}\,J \times 6{,}00\times10^{23}\,mol^{-1}/207\,g/mol = 464\,kJ/kg \sim 130\,Wh/kg$$

Evolution evident from next generation of batteries (lithium, etc).