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PHY 555 — Energy and Environment

PC6 — Solar Power Plant – Solution Strategies

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1 – Photovoltaics

1. The cell receives black-body radiation corresponding to the integrated value of Planck's law. The prefactor of two is due to the integral for a flat surface, and notice that the source is at $T_{ambient}$.

$$\phi_{\text{amb}}(\hbar\omega) = \int_0^1 \text{d}(\cos\theta) \int_0^{2\pi} \text{d}\phi \frac{\dot{N}_{BB}(\hbar\omega,\Omega)}{S_{\text{emit}}} = 2 \frac{1}{4\pi^2\hbar^3 c^2} \times \frac{\hbar^2\omega^2}{\exp(\frac{\hbar\omega}{k_B T_{\text{amb}}})-1} \text{ (Eq. 1.1)}$$

The cell absorbs a fraction $\alpha(\hbar\omega)$ of this, which only has non-zero values above the bandgap. The number of thermal electrons generated per second is thus:

$$G_0 = \int_{\hbar\omega \geq E_g} \text{d}(\hbar\omega)\alpha(\hbar\omega)\phi_{\text{amb}}(\hbar\omega) = \int_{\hbar\omega \geq E_g} \text{d}(\hbar\omega) \frac{1}{4\pi^2\hbar^3 c^2} \times \frac{\hbar^2\omega^2 \times \alpha(\hbar\omega)}{\exp(\frac{\hbar\omega}{k_B T_{\text{amb}}})-1} \text{ (Eq. 1.2)}$$

Conservation of the number of electrons in the medium (steady state) imposes that the same amount of electrons have to recombine per unit time:

$$R_0 = G_0 \text{ (Eq. 1.3)}$$

2. For $E_g = 1\text{eV}$ one has, for a black body ($\alpha(\hbar\omega) \equiv 1$):

$$G = \int_{\hbar\omega \geq E_g} \text{d}(\hbar\omega)\phi_{\text{sun}}(\hbar\omega) = \int_{\hbar\omega \geq E_g} \text{d}(\hbar\omega) \frac{\Omega_{\text{sun}}}{\pi} \times \frac{1}{4\pi^2\hbar^3 c^2} \times \frac{\hbar^2\omega^2}{\exp(\frac{\hbar\omega}{k_B T_{\text{sun}}})-1} \text{ (Eq. 1.4)}$$

$$G \approx 3.46 \times 10^{21} \text{electrons/s/m}^2$$

$$R_0 = \int_{\hbar\omega \geq E_g} \text{d}(\hbar\omega)\phi_{\text{amb}}(\hbar\omega) = \int_{\hbar\omega \geq E_g} \text{d}(\hbar\omega) \frac{1}{4\pi^2\hbar^3 c^2} \times \frac{\hbar^2\omega^2}{\exp(\frac{\hbar\omega}{k_B T_{\text{amb}}})-1} \text{ (Eq. 1.5)}$$

$$R_0 \approx 1.07 \times 10^{7} \text{electrons/s/m}^2$$

Note that the term Ω_{sun}/π in the expression of G comes from the optical étendue. For small angles, the étudue is $\Omega \times \cos\theta$ and we assume here a normal incidence.

3. The output power P can be written as: $P = E_g \times G$

It tends to 0 when $E_g \to 0$, and it also tends to 0 when $E_g \to \infty$ as $G \to 0$ in this case. This can be easily understood: if the gap is very small, one collects many electrons but only recover a tiny fraction of the incident photon energy. It the gap is large one only collects the small fraction of photons which have an energy larger that the gap energy and therefore miss a large fraction of the photons. There must be an optimal in the middle. Numerically, Trivich-Finn limit is at 44%. This corresponds to the total energy provided by the electrons, not just the work, a faction of this energy is released as heat in the semi-conductor.

4. The conservation of charge implies $G = R + I/q$ where R is the recombination rate under sunlight.

Assuming that all electrons have the same recombination probability ($n/R = const.$) one can write:

$$\frac{n}{n_0} = \frac{R}{R_0} = \frac{G - I/q}{R_0} \text{ (Eq. 1.7)}$$

(a) Knowing that the work done by an electron passing through a voltage difference is $w=qV$, we can use the equation from the question ($w = k_B T_{\text{cell}} \ln(\frac{n}{n_0})$ *)* and Eqn 1.7 to link the electron density n to the operating point voltage:

$$\frac{n}{n_0} = \exp(\frac{qV}{k_B T_{\text{cell}}}) \text{ (Eq. 1.10)}$$

Knowing that $P = V \times I$ one obtains:

$$P = qV \times (G - R_0 \exp(\frac{qV}{k_B T_{\text{cell}}})) \text{ (Eq. 1.11)}$$

One can then note two "zero-power values": $V = 0$, and $I = 0$, and their corresponding values of $I_{V=0}$ and $V_{I=0}$.

The optimal operation point is between these two extreme values.

(b) Differentiate equation 1.11 against the gap energy and set to 0 to get maximum.

One can then differentiate equations 1.4 and 1.5 to get $\partial G / \partial E_g$ and $\partial R_0 / \partial E_g$. Be careful about the relevant temperatures (sun, ambient, cell)

Many pre-factors disappear, and:

$$\frac{\Omega_{\text{sun}}}{\pi} \frac{1}{\exp(\frac{E_g}{k_B T_{\text{sun}}}) - 1} = \frac{1}{\exp(\frac{E_g}{k_B T_{\text{amb}}}) - 1} \times \exp(\frac{qV}{k_B T_{\text{cell}}})$$

Assume that (1) the cell is at thermal equilibrium with the environment ($T_{\text{cell}} = T_{\text{amb}}$) and (2) that the gap energy is sufficiently large compared to $k_B T_{\text{sun}}$ to simplify this equation. One eventually obtains:

$$\frac{E_g}{k_B} \times (\frac{1}{T_{\text{amb}}} - \frac{1}{T_{\text{sun}}}) = (\frac{qV}{k_B T_{\text{amb}}}) - \ln(\frac{\Omega_{\text{sun}}}{\pi}) \text{ (Eq. 1.16)}$$

This relation can be inverted to get the desired formula.

(c) We can easily calculate numerically that the optimal gap is around $E_g \approx 1.1eV$. Estimate the maximum conversion efficiency for a simple solar cell.

The maximum power is (Eq. 1.11):

$$P_{\text{opt}} = qV_{\text{opt}} \times (G - R_0 \exp(\frac{qV_{\text{opt}}}{k_B T_{\text{amb}}})) \approx 0.3 P_{\text{sun}} \text{ (Eq. 1.18)}$$

(d) The best efficiency ever reached for a silicon solar cell is about 27% under sunlight. Comment this value. Despite many practical challenges (reflection, absorption, actual collection mechanism) very close to theoretical limit.

2 – Solar Thermal

1. The absorbed and radiated flux are, taking into account absorbency α and emissivity ϵ (we keep the possibility that α and ϵ are different as they are dealing with very different spectral regions)

$$P_{\text{sun}} = \alpha S_{\text{abs}} \times \phi_{\text{sun}} \approx \alpha S_{\text{abs}} \times \frac{\Omega_{\text{sun}}}{\pi} \sigma T_{\text{sun}}^4 \quad P_{\text{rad}} = \epsilon S_{\text{abs}} \sigma T_{\text{abs}}^4 \text{ (Eq. 2.1)}$$

The maximum temperature is obtained when both fluxes are equal (nothing transmitted). This gives:

$$\epsilon T_{\max}^4 = \alpha \frac{\Omega_{\text{sun}}}{\pi} T_{\text{sun}}^4 \text{ (Eq. 2.2)}$$

2. The heat flow (energy/time = power) provided to the engine is the difference between the absorbed and radiated power:

$$\dot{Q} = P_{\text{sun}} - P_{\text{rad}} = \alpha S_{\text{abs}} \times \frac{\Omega_{\text{sun}}}{\pi} \sigma T_{\text{sun}}^4 - \epsilon S_{\text{abs}} \sigma T_{\text{abs}}^4 = S_{\text{abs}} \times \frac{\Omega_{\text{sun}}}{\pi} \sigma T_{\text{sun}}^4 (\alpha - \epsilon \frac{\pi}{\Omega_{\text{sun}}} \frac{T_{\text{abs}}^4}{T_{\text{sun}}^4}) \text{ (Eq. 2.3)}$$

A heat engine can transform this heat flow into work with an efficiency bounded by the Carnot efficiency. The overall efficiency will be the product of two efficiencies (absorber and machine). Notice that the first expression is the ratio of heat flow to engine and power incident from sun (without considering absorbency):

$$\eta = \frac{-W}{Q} \leq (\alpha - \epsilon \frac{\pi}{\Omega_{\text{sun}}} \frac{T_{\text{abs}}^4}{T_{\text{sun}}^4}) \times (1 - \frac{T_{\text{amb}}}{T_{\text{abs}}}) = \alpha (1 - \frac{T_{\text{abs}}^4}{T_{\text{max}}^4}) \times (1 - \frac{T_{\text{amb}}}{T_{\text{abs}}}) \text{ (Eq. 2.5)}$$

The only free parameter in this equation is the temperature of the absorber. Numerical resolution gives a maximum yield of about 6% for black bodies.

3. The reflector forms the image of the sun in its focal plane. The absorber should be at least as large as the geometrical image. The image size is determined by the focal length of the concentrator mirror and the apparent diameter of the sun.
4. For a perfect reflector, the incident energy flow on the collector and the absorber are identical. The solar concentration ratio C_S is given by the ratio of the areas:
   - Collector area: $D \times L$ (rectangle of width D)
   - Absorber area: $= d_A \times L$ (rectangle of width d_A)

In Gaussian optics the image is assumed to be flat and aberrations are ignored. The width of the absorber must be greater than the width of the image:

$$C_S = \frac{DL}{d_A L} < \frac{D/f}{\sin\theta_S} = \frac{a}{\sin\theta_S} \approx \frac{a}{\theta_S} \text{ (Eq. 2.6)}$$

The concentration ratio seems to grow without any limit with the aperture a. But from a given opening, the image size will not be determined only by the focal length but by the geometric aberrations that not included in the paraxial optics. The absorber maximum temperature is now given by:

$$\epsilon T_{\text{max}}^4 = \alpha C_S \frac{\Omega_{\text{sun}}}{\pi} T_{\text{sun}}^4 \text{ (Eq. 2.7)}$$

For very large values of a, the absorber temperature can exceed the sun temperature: the sun could heat the absorber to a temperature higher than its own, which clearly violates the second principle of thermodynamics

5. Shape of parabolic reflector will follow $y = x^2/f$. For an incident ray (blue) at an angle $\theta_S/2$ to strike the absorber, the radius r_A of the latter must exceed $FX \times sin(\theta_S/2)$ where FX is the distance from the focus to the point of impact on the mirror,

$$|FX| = \sqrt{x^2 + (y-f)^2} = \sqrt{x^2 + \left(\frac{x^2}{4f} - f\right)^2} = f \times \left[\frac{1}{4}\left(\frac{x}{f}\right)^2 + 1\right] \text{ (Eq. 2.8)}$$

For a marginal ray, i.e. $x = D/2$ under the incident angle $\theta_S/2$, this gives:

$$r_A > \sin(\theta_S/2) \times f \times (\frac{1}{4}(\frac{D}{2f})^2 + 1)$$

$$\Rightarrow C_S = \frac{D}{2\pi r_A} < \frac{1}{\theta_S} \times \frac{16a}{\pi(a^2+16)}$$

$$C_S(a) < \frac{1}{\theta_S} \frac{16a}{\pi(a^2+16)} \text{ (Eq. 2.9)}$$

6. The derivative of $C_S(a)$ with respect to a gives a maximum for $a = 4$, i.e. $D = 4f$ with

$$C_S^{\text{max}} = \frac{2}{\pi\theta_S} \approx 70 \text{ (Eq. 2.10)}$$

7. *For a 2D concentrator (parabolic dish), what is the maximum concentration ratio?*

For a paraboloid (Dish-Stirling) concentrator, one obtains similarly:

$$C_S(a) < \left(\frac{1}{\theta_S}\frac{4a}{(a^2+16)}\right)^2 \text{ for a maximum at } a = 4 \; : C_S^{\max} = \frac{1}{4\theta_S^2} \approx 2922 \text{ (Eq. 2.11)}$$

But for paraboloids of revolution, one generally cannot obtain better than $a = 1$ (see Fig. 1) which gives a maximum value of C_S around 650.

8. Things to consider: non-normal incidence of rays on absorber, surfact defects, alignment inaccuracy, imperfect reflectivity, dust,

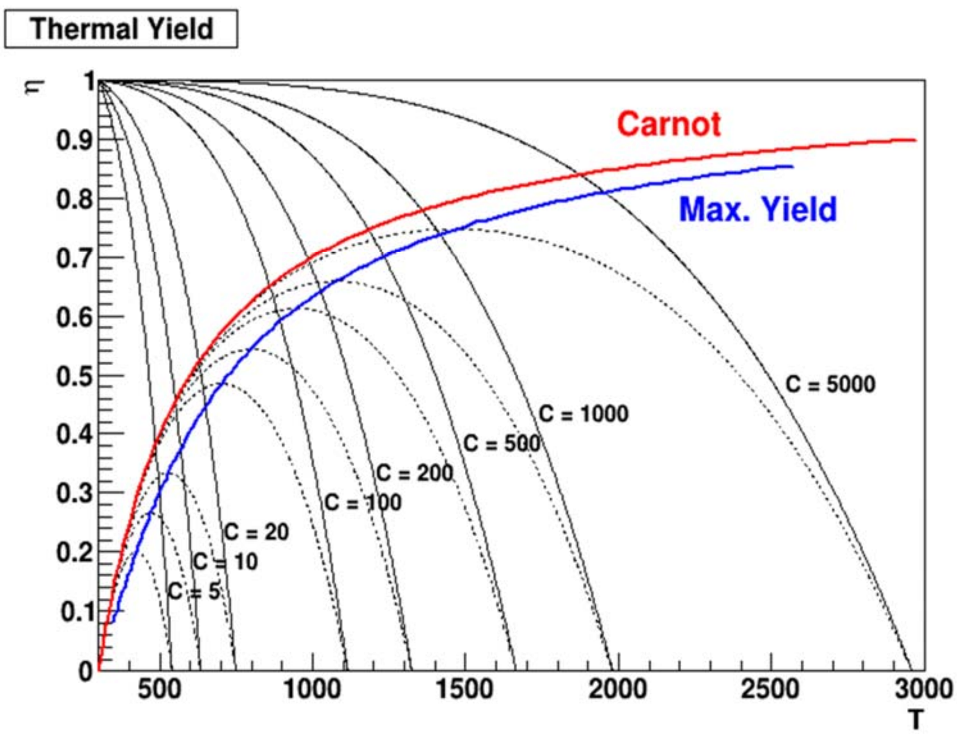
9. The expression of the overall yield is unchanged:

$$\eta \leq \alpha\left(1 - \frac{T_{\text{abs}}^4}{T_{\max}^4}\right) \times \left(1 - \frac{T_{\text{amb}}}{T_{\text{abs}}}\right) \text{ (Eq. 2.12)}$$

with the maximum temperature being now:

$$\epsilon T_{\max}^4 = \alpha C_S \frac{\Omega_{\text{sun}}}{\pi} T_{\text{sun}}^4 \text{ (Eq. 2.13)}$$

The overall shape is displayed below for various concentration ratio:



10. *For an average solar flux $F_S = 250Wm^{-2}$, a solar concentration ratio of $C_S = 600$, and a temperature of the cold reservoir $T_0 = 200°C$, verify that the overall performance is greatest when the temperature of the absorber is $T = 605°C$. What then is the value of overall yield?*

The extremality condition of the net yield η with respect to T leads to a fifth degree equation:

$$0 = \frac{d\eta}{dT} = -\frac{\alpha T^2}{T_{\max}^3}\left[4\left(\frac{T}{T_{\max}}\right)^5 - 3\left(\frac{T_0}{T_{\max}}\right)\left(\frac{T}{T_{\max}}\right)^4 - \frac{T_0}{T_{\max}}\right]$$

For the parameter values given here we obtain $T_{\max} = 1275\text{K}$, so

$\frac{T}{T_{\max}} = \frac{878\text{K}}{1275\text{K}} = 0{,}688$ and $\frac{T_0}{T_{\max}} = \frac{473\text{K}}{1275\text{K}} = 0{,}371$. It is easily verified that

$| 4 \times (0{,}688)^5 - 3 \times 0{,}371 \times (0{,}688)^4 - 0.371 | < 0.004$, and thus $\left|\frac{d\eta}{dT}\right| < 1{,}4 \cdot 10^{-6}\text{K}^{-1}$ with $\eta \approx$ 36%

11. *Comment the choice of the cold reservoir temperature (200°C). Does it sound realistic? Can the yield be increased by changing this setting? Is it sounds technically feasible?*

The temperature of the cold reservoir appears high in comparison to that of ambient air. One can consider how cooling is implemented (fins, fans, liquids, etc) and relative costs.