Evaluation PHY555 Energy & Environment
Friday 15th December 2017

Part II, Lecture notes allowed, duration 2h

Note 1: The exercise and the three parts of the problem are all **independent.**
Note 2: Quality of the redaction of justification of calculation will be taken into account in the evaluation. Answers can be written in **French** or in **English.**
Note 3: The astrophysical and physical constants needed in the problem are grouped in a table at the end of the subject
Note 4: There is no need to copy the question text. Just identify them by their numbers.
Note 5: **Lecture notes and small class documents (notes and slides)** are allowed as well as **French-English dictionary**. No other document is allowed.
Note 6: Documents mentioned above might be consulted in a **digital form** on a tablet/computer, but the device (and other devices with connection capabilities, such as phones, calculator, ...) must be put in **flight mode**. **No Wifi connection** is allowed.

Exercise: Stirling Engine (5 points)

From Wikipedia: A Stirling engine is a heat engine that operates by cyclic compression and expansion of air or other gas (the working fluid) at different temperatures. More specifically, the Stirling engine is a closed-cycle regenerative heat engine with a permanently gaseous working fluid. Closed-cycle, in this context, means a thermodynamic system in which the working fluid is permanently contained within the system, and regenerative describes the use of a specific type of internal heat exchanger and thermal store, known as the regenerator. The inclusion of a regenerator differentiates the Stirling engine from other closed cycle hot air engines.

Stirling engines are used in particular in the context of thermal solar energy, in association with parabolic dishes (Fig. 1).

In the so-called "Alpha-type Stirling engine", there are two cylinders connected by a flywheel. The expansion cylinder (red in table 1) is maintained at a high temperature (T_H) while the compression cylinder (blue) is cooled (T_L). The passage between the two cylinders contains the regenerator. The idealised Stirling cycle consists of four thermodynamic processes acting on the working fluid, as described in the table 1:

- Isothermal expansion. The expansion-space and associated heat exchanger are maintained at a constant high temperature T_H , and the gas undergoes near-isothermal expansion absorbing heat from the hot source.
- Constant-volume (or isochoric) heat-removal. The gas is passed through the regenerator, where it cools, transferring heat to the regenerator for use in the next cycle.
- Isothermal compression. The compression space and associated heat exchanger are maintained at a constant low temperature T_L so the gas undergoes near-isothermal compression rejecting heat to the cold sink
- Constant-volume (or isochoric) heat-addition. The gas passes back through the regenerator where it recovers much of the heat transferred in process 2, heating up on its way to the expansion space.

1 Draw the P-V and T-S diagrams of the cycle, indicating the position of each phase, and give the equations of the curves corresponding to each transformation.

2 Compute the compression and expansion works., and the heat exchange during the two isothermal phases.

3 Compute the work and heat exchange in the isochoric phases.

4 Compute the yield of the motor as function of the temperatures T_H and T_L of the hot and cold sources.

5 What are the advantages and drawbacks of such motors?

Figure 1 : Point focus parabolic mirror with Stirling engine at its centre and its solar tracker at Plataforma Solar de Almería (PSA) in Spain

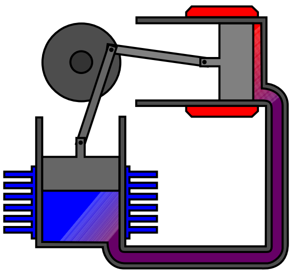
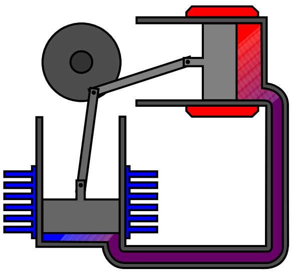
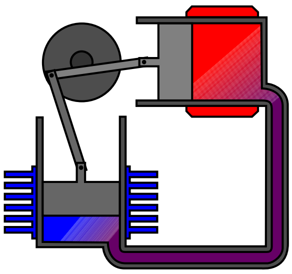
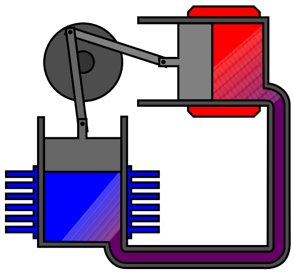
Phase 1: Isothermal expansion	Phase 2: Isochoric cooling	Phase 3: Isothermal compression	Phase 4: isochoric heating
			
Most of the working gas is in the hot cylinder and has more contact with the hot cylinder's walls. This results in overall heating of the gas. Its pressure increases and the gas expands. Because the hot cylinder is at its maximum volume and the cold cylinder is at the top of its stroke (minimum volume), the volume of the system is increased by expansion into the cold cylinder.	The system is at its maximum volume and the gas has more contact with the cold cylinder. This cools the gas, lowering its pressure. Because of flywheel momentum or other piston pairs on the same shaft, the hot cylinder begins an upstroke reducing the volume of the system.	Almost all the gas is now in the cold cylinder and cooling continues. This continues to reduce the pressure of the gas and cause contraction. Because the hot cylinder is at minimum volume and the cold cylinder is at its maximum volume, the volume of the system is further reduced by compression of the cold cylinder inwards.	The system is at its minimum volume and the gas has greater contact with the hot cylinder. The volume of the system increases by expansion of the hot cylinder.

Table 1: Phases of Stirling engine.

Problem: Tidal Energy (15 points)

In this problem, we will study the tidal forces induced on the Earth by the gravitational attraction of the Moon and of the Sun. These forces are responsible for a deformation of the ocean free surface which, due to the rotation of the Earth, generates tides. We will first derive the tidal amplitude and an estimation of the total energy in the tides. Then, in a second part, we will derive an approximate value of the water currents induced by the succession of high and low tides. In a third and last part, a practical example of a tidal power plant will be studied

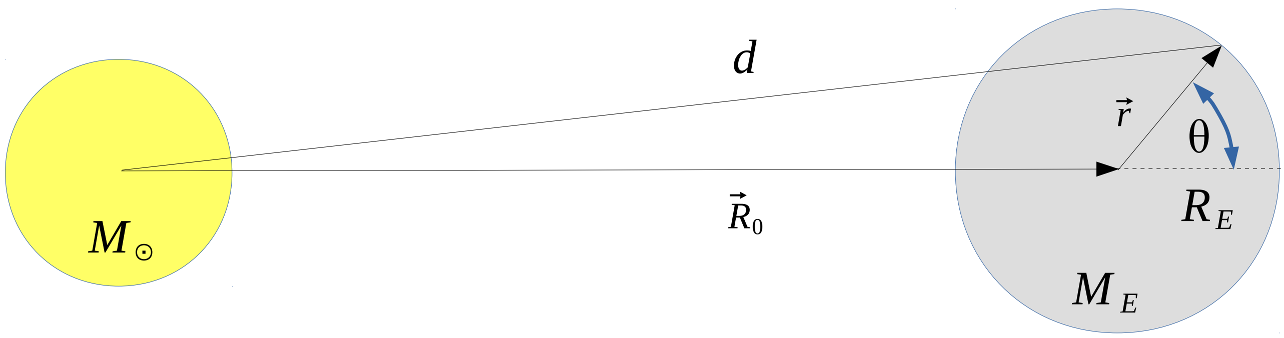
Part 1. Tidal Forces (6 points)

Unless otherwise specified, the following simplifications will be made:

- The Earth is supposed to be fully covered by oceans. Effect of continents (reflection of waves, …) will be ignored.
- The *obliquity* of the Earth is neglected: the rotation axis of the Earth is supposed to be orthogonal to the ecliptic plane (i.e. the plane of the orbit). Similarly, the Moon is supposed to orbit in the equatorial plane of the Earth.
- All orbits are assumed to be *circular*, i.e. the *eccentricity* of the orbits is ignored.

1 Gravitational Potential

Let's consider a system of two astrophysical bodies: Earth + (Moon or Sun) of respective masses M_E and ($M_\odot$ or M_M) (Sketch 1). We will place ourself in the "*geocentric*" frame: frame centred on the Earth, with fixed orientation axis with respect to stars.

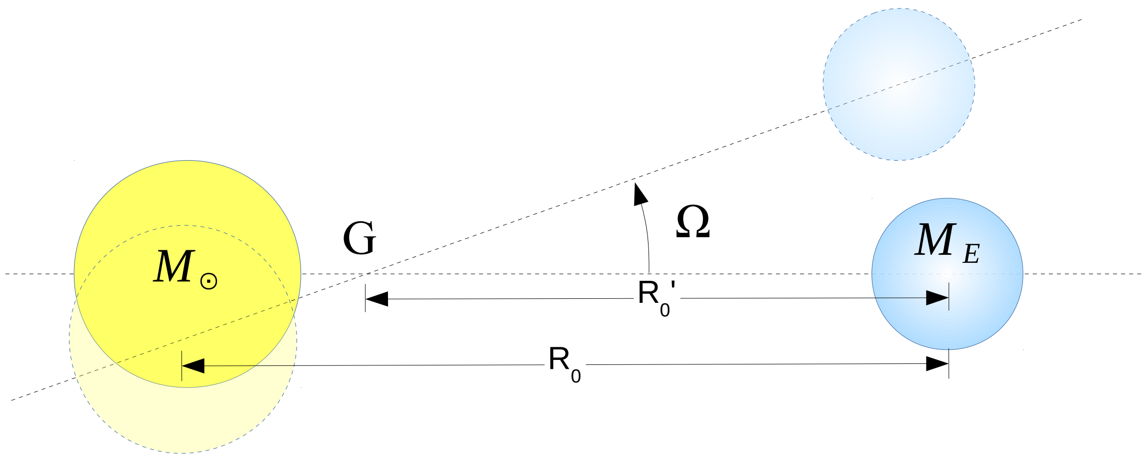


Sketch 1: Geometry used for tidal calculations

- Express the distance d as function of R_0 , $\cos\theta$ and $\epsilon = R_E / R_0$.
- Show that the potential energy of a mass m located at the surface of the Earth, due to gravitational field of the second body, can be approximated by:

$$E_p^{(g)} \approx -\frac{G m M_\odot}{R_0}\left(1 - \frac{\epsilon^2}{2} - \epsilon\cos\theta + \frac{3}{2}\epsilon^2\cos^2\theta\right)$$

2 Potential of inertial forces



Sketch 2: Geometry of geocentric frame

The geocentric frame is not Galilean, but in accelerated translation with respect to the center of mass G of the two bodies (See sketch 2).

- Show that the angular velocity is expressed by:

$$\Omega^2 = \frac{GM}{R_0^3} \quad \text{with} \quad M = M_\odot + M_E$$

- Express the inertial forces, and show that they can be expressed by a potential energy of the form:

$$E_p^{(i)} = -\frac{Gm M_\odot}{R_0} \epsilon \cos\theta$$

3 The rotation of the Earth around its center introduces a deformation of the surface of the Earth (ellipsoidal shape) which can be ignored in the context of tidal, as it does not vary with time. The effective interaction potential is then the combined effect of solar attraction and inertial forces:

$$E_p = E_p^{(g)} + E_p^{(i)}$$

Compute the average value at this potential energy at the surface of the Earth.

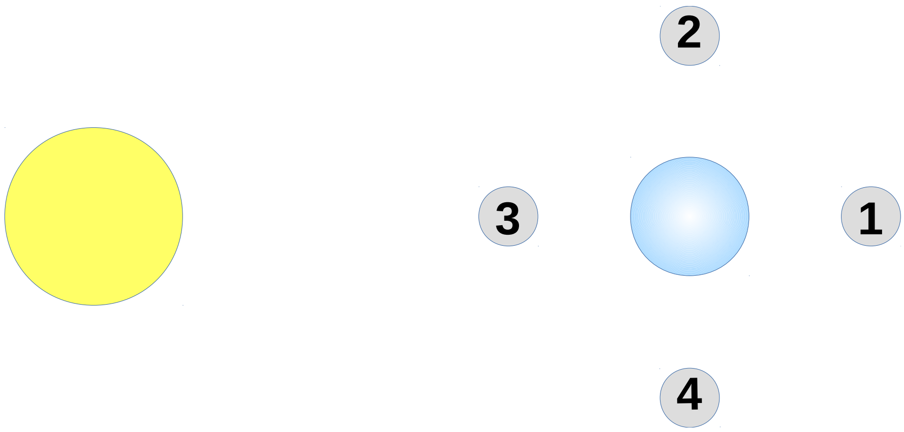
4 We now include the effect of the Earth gravitational field: in reaction to the tidal forces, the surface of the ocean will adjust itself to a constant potential energy. We note $h \ll R_E$ the local change of altitude of the surface.

- Show that the equation describing the surface of the ocean (with respect to the equilibrium shape in the absence of tidal) is given by:

$$\frac{h}{R_E} = \frac{M_\odot}{M_E} \times \left(\frac{R_E}{R_0}\right)^3 \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

- What is the tide amplitude? Compute the values for the Sun and for the Moon

5 High and low tide amplitudes



Sketch 3: Relative positions of Moon & Sun with respect to the Earth

The moon orbits the Earth in 27,25 days. How will the gravitational effect of the moon and the sun interfere? For each of the positions in the sketch 3, indicate whether it would correspond to a high or to a low tide amplitude. Estimate the amplitude in each case.

6 Compute the water volume above the low tide.

7 Compute the corresponding potential energy (total available energy).

8 What is neglected in the current theory, and how would it affect the tidal?

Part 2. Tidal Currents (3 points)

We assume that the tidal amplitude follows a wave propagation equation:

$$h(x,t) = h_0 \cos(kx - \omega t)$$

with a period $T = 2\pi/\omega = 12\,\text{hours}$. For gravity waves in shallow water (which is a valid approximation here, as the wavelength is several thousand kilometres), the wave speed is found to be:

$$c = \frac{\omega}{k} = \sqrt{g H_0}$$

where H_0 is the ocean depth.

1 Compute the propagation speed of a tidal wave on the Earth for an average ocean depth of 3.7 km. How does it compare with the surface rotation speed of the Earth? Conclusion?

2 Figure 2 shows the amplitude and delays of the tide in the English Channel (average depth $63\,\text{m}$). Describe briefly how the shape of the coast can alter the propagation of the tidal wave.

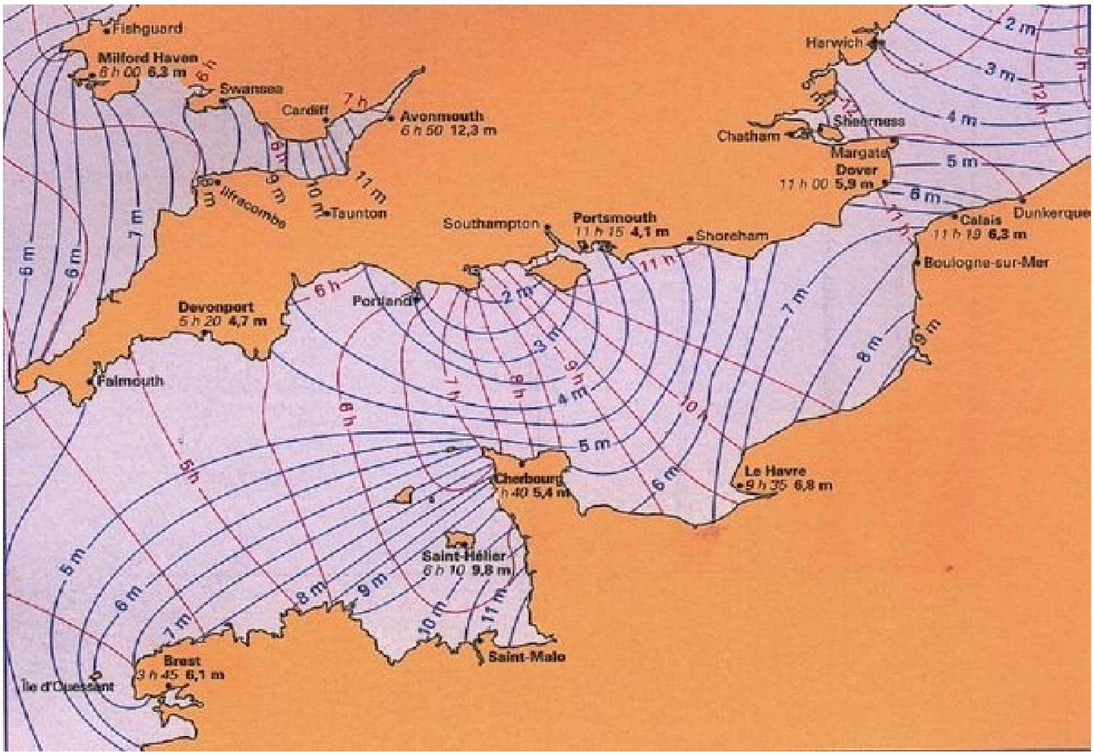


Figure 2: Tidal amplitude (blue lines) and delay (red lines) in the English Channel

3 Using the equation of continuity on a small layer of fluid, show that the velocity of tidal current can be expressed as

$$v_x = h_0 \sqrt{\frac{g}{H_0}} \cos(kx - \omega t)$$

What velocities are obtained for a tide of $\sim 5\,\text{m}$ in a thickness of $\sim 30\,\text{m}$?

4 One technique to gather tidal energy is the use of submarine turbines. What power can be obtained with a turbine of $10\,\text{m}$ diameter in the above current?

Part 3. Tidal power plan (6 points)

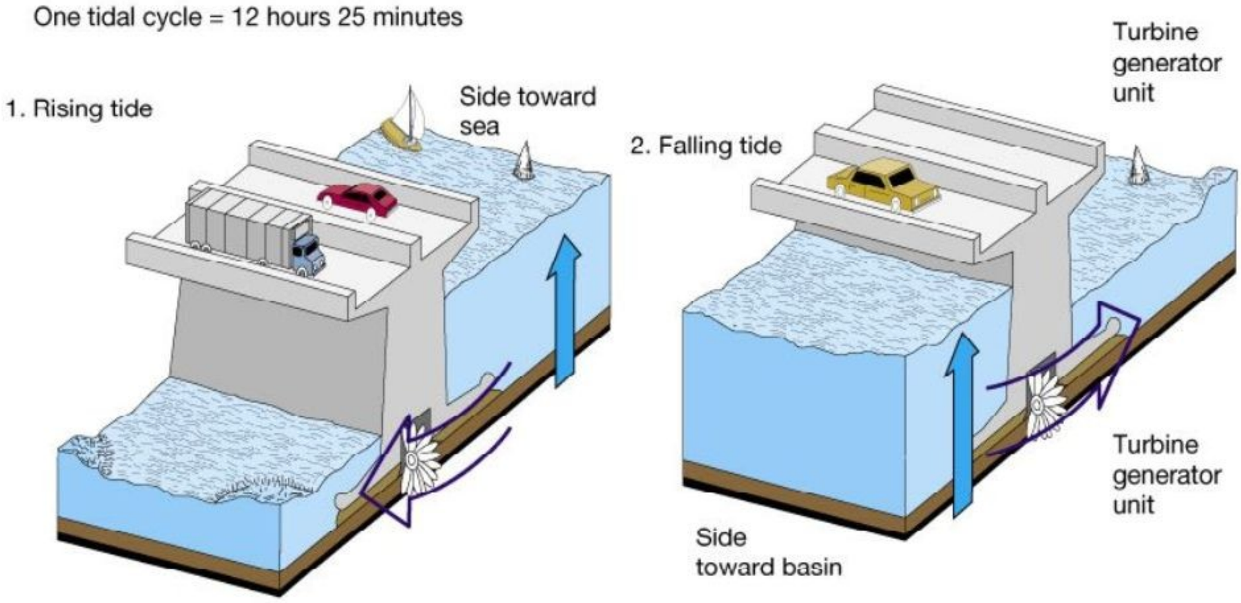


Figure 3: Operation mode of a tidal plant

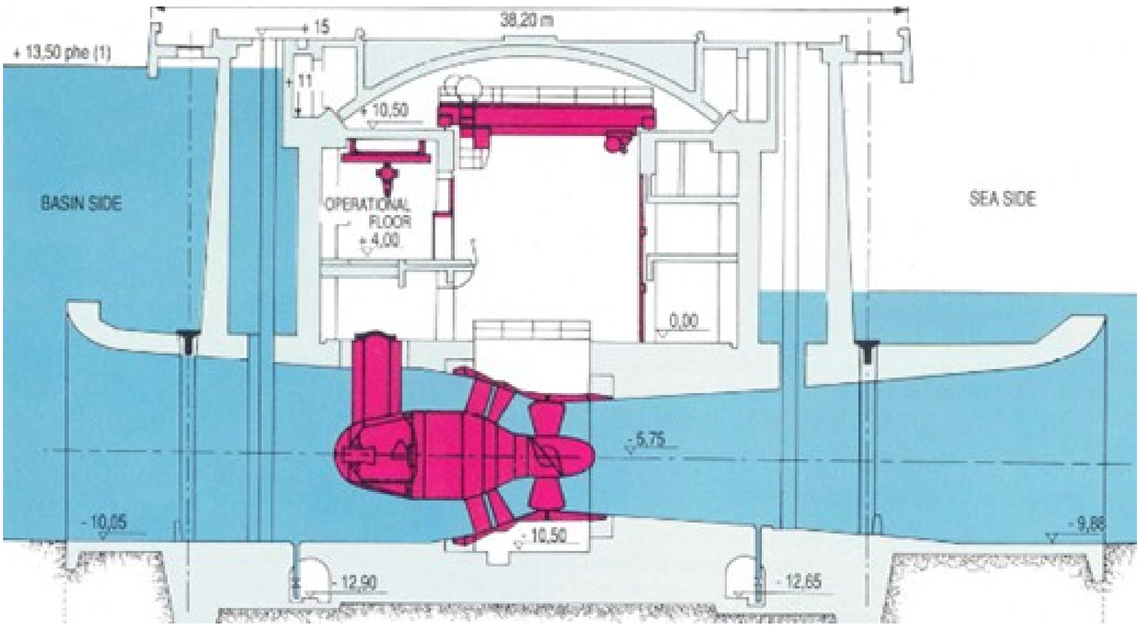


Figure 4: Cut of the "La Rance" dam

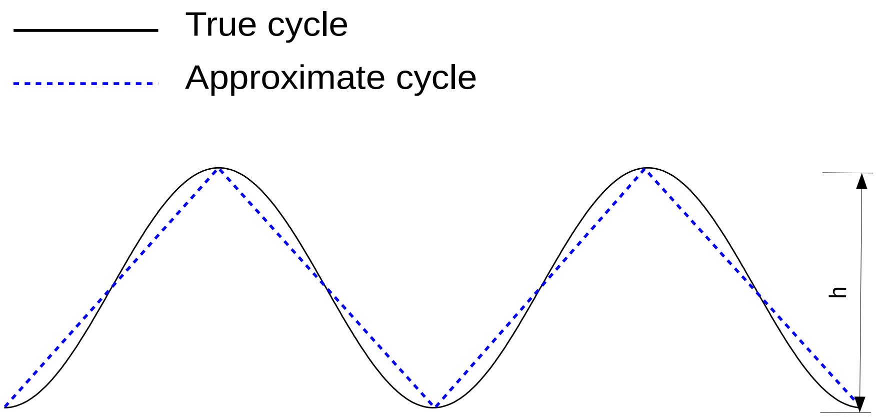
Figure 3 shows the operation modes of a *reversible* tidal power plant (i.e. in which power can be generated both during the high and low tides). The first plant of that type ever built is the “La Rance” tidal plant, located next to Saint Malo, France. It uses reversible turbines (Figure 4), whose blades can be oriented according to the flow direction. The turbines can also be used in pumping mode, to increase the water level in the basin during the high tides. The parameters of the plant are the following:

- Dam size of $332\,\mathrm{m}$ length by $33\,\mathrm{m}$ width
- Storage basin of $22\,\mathrm{km}^2$
- 24 turbines of $5.34\,\mathrm{m}$ diameter, nominal power $10\,\mathrm{MW}$, rotating at 93 rotations per minute. The turbines are reversible (bi-directional flow) , and can also work in pumping mode (to increase the level of water in the basin)
- nominal water flow $260\,\mathrm{m}^3/\mathrm{s}$ for each turbine
- tide level from $5\,\mathrm{m}$ (low amplitude tides) up to $12\,\mathrm{m}$ (high amplitude tides), for an average tide amplitude of $8\,\mathrm{m}$. This almost record breaking tide level is due to a resonance phenomenon in a closed bay.
- Installed nominal Power : 240 MW
- Annual Net Production : 544 GWh (pumping energy deduced)
- Annual Energy Consumption in Pumping Mode: 64,5 GWh

1 Compute the total capacity of the basin, the stored potential energy (for the average tide of $8\,\mathrm{m}$) and the specific energy.

2 At the nominal flow speed, what is the time needed to fill/empty completely the basin?

3 We assume two high tides per day. To simplify the description of the cycle, the tidal cycle is approximated by a triangular shape of height $h\approx 8\,\mathrm{m}$ (constant level rise and drop speeds as shown on sketch 4).



Sketch 4: Approximation of tidal cycle (sea level)

In order for the turbine to operate, a minimum altitude difference $b\approx 2\,\mathrm{m}$ between the basin and the sea is requested.

The different phase of operation in "*double cycle mode*", starting from the low tide, are:

- Waiting (sea level rises, basin is closed)
- Turbining (sea level rises, water enters the basin, turbines produce power)
- Filling (sea level starts to drop, water still enters the basin, but altitude difference is not high enough to enable the turbines to operate)
- Waiting (sea level drops, basin is closed)
- Turbining (sea level drops, water leaves the basin, turbines produce power)
- Emptying (water leaves the basin, altitude difference not high enough for the turbines)

Draw on the same plot the cycle of the water level in the basin an in the sea, and identify each phases

4 Assuming a tidal cycle duration of 12 hours (ignoring the period of rotation of the moon around the earth), compute, as function of b, the duration of each phase. Do the numerical calculation for $b=2\,\mathrm{m}$.

5 Assuming a turbine efficiency $\eta_t \approx 90\%$, compute the water flow and power in turbining mode.

6 Show that the yield over a full cycle can be expressed as:

$$\eta=\frac{E}{E_p}=\eta_t\times 2\times\frac{b}{h}\times\left(1-\frac{b}{h}\right)\times\left(1-\frac{3}{2}\frac{b}{h}\right)$$

Which value of b maximizes the yield (numerically)? What is the approximate maximal yield?

Constants

Astrophysical Data

Sun :

Average Diameter : 1 392 000 km

Mass ($M_{\odot}$): $1,9891\times10^{30}$ kg

Surface Temperature ($T_{\odot}$) : 5800 K

Moon :

Mass (M_L): $7,349\times10^{22}$ kg

Semi major axis (R_0) :384 403 km

Earth :

Equatorial Radius (R_E) : 6 378,1 km

Mass (M_E) : $5,9736\times10^{24}$ kg

Average density : $5,515\times10^{3}$ kg/m^3

Axis inclination : 23,4392°

Semi major axis (R_0) : 149 597 887,5 km

Energy data

1 toe = $41\ 855\times10^{6}$ J

1eV = 1.6×10^{-19} J

Stefan constant :
$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}

Universal constant of Gravitation :
G = 6.6742×10^{-11} m^3 kg^{-1} s^{-2}

Acceleration of Gravity at Earth:

g = 9.81 m s^{-2}

Avogadro's number :
$N_A = 6.02\times10^{23}$ mol^{-1}

Absolute Zero :
T_0 = -273,15 °C

Planck constant :
$h = 6,626\ 0\ 755\times10^{-34}$ J.s

Perfect gas constant :
R = 8,314 J mol^{-1} K^{-1}