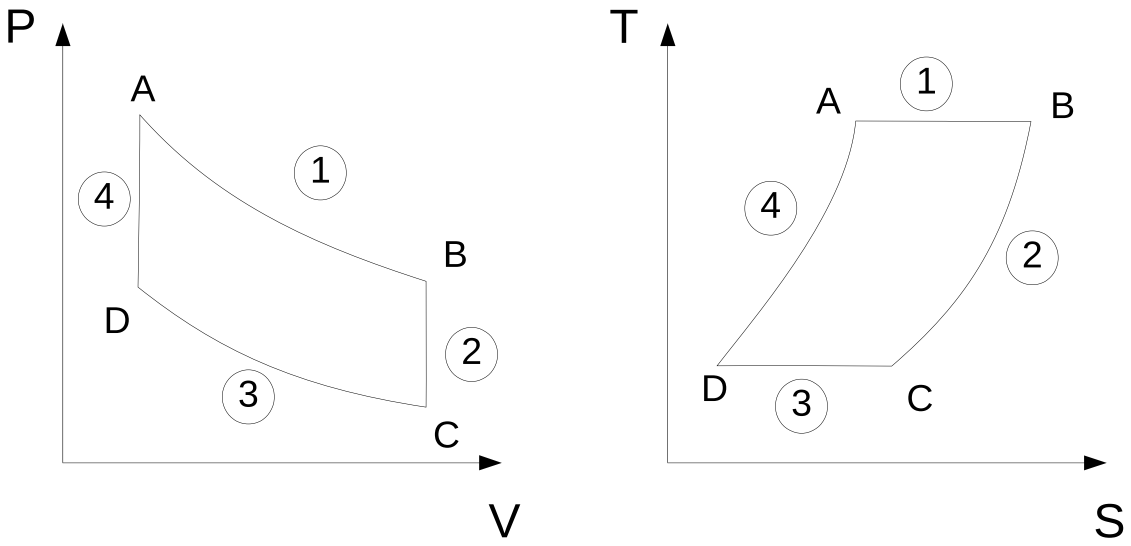
Exercise: Stirling Engine - Solutions

1. *Cycle drawings*

   - A-B: Isothermal expansion phase: $T = C_{str}$ so $P = nR\frac{T}{V} \propto 1/V$. Curves are hyperbola in P-V diagram. V increases as P decreases. In T-S diagram, horizontal line (T constant).
     Since $\Delta S = nC_v \ln\left(\frac{V}{V_0}\right)^{\gamma-1} = nR\ln\left(\frac{V}{V_0}\right)$, S increases with V.
   - B-C: Isochoric heat-removal: vertical, down-going line (constant volume) in P-V
     Since $\Delta S = nC_v \ln\left(\frac{T}{T_0}\right)$, down-going exponential in T-S diagram
   - C-D: Isothermal compression: hyperbola in P-V, decreasing V, increasing P.
   - D-A: Isochoric heating: vertical, up-going line in P-V.
     Since $\Delta S = nC_v \ln\left(\frac{T}{T_0}\right)$, up-going exponential in T-S diagram



2. *Compression and expansion works*

For both phases:

$$\mathrm{d}W = -P\,\mathrm{d}V = -nRT_H\frac{\mathrm{d}V}{V} \quad \Rightarrow \quad W_1 = -nRT_H \ln\left(\frac{V_B}{V_A}\right) < 0, \quad W_3 = nRT_L \ln\left(\frac{V_B}{V_A}\right) 1 0$$

Since $\Delta T = 0$, we have $\Delta U = 0$ and thus $W = -Q$.

3. *Isochoric phases*

Since the volume is constant, $\mathrm{d}W = -p\,\mathrm{d}V = 0 \quad \Rightarrow \quad W = 0$. Thus $Q = \Delta U = nC_v \Delta T$. In particular $Q_4 = -Q_2$ (the regenerator gives back the energy it absorbed).

4. *Yield*

The cycle work is therefore

$$W = W_1 + W_3 = -nR(T_H - T_L)\ln\left(\frac{V_B}{V_A}\right)$$

The yield is simply the same as for the Carnot cycle:

$$\eta = -W/Q_1 = -W/W_1 = 1 - \frac{T_L}{T_H}$$

Note that the incoming heat is Q_1. The heat exchages

5. Advantages and drawbacks

Closed cycles → No consumables, work in any environment.

No pistons, no admission, … very few mechanical parts, very little maintenance

Can work with any heat source

But very slow due to isothermal transformations, not very powerful, sealing, cost.