Evaluation PHY555 Energy & Environment

Friday 21st December 2018

Part II, Lecture notes allowed, duration 2h

Note 1: The exercise and the three parts of the problem are all **independent.**

Note 2: Quality of the redaction of justification of calculation will be taken into account in the evaluation. Answers can be written in **French** or in **English.**

Note 3: The astrophysical and physical constants needed in the problem are grouped in a table at the end of the subject

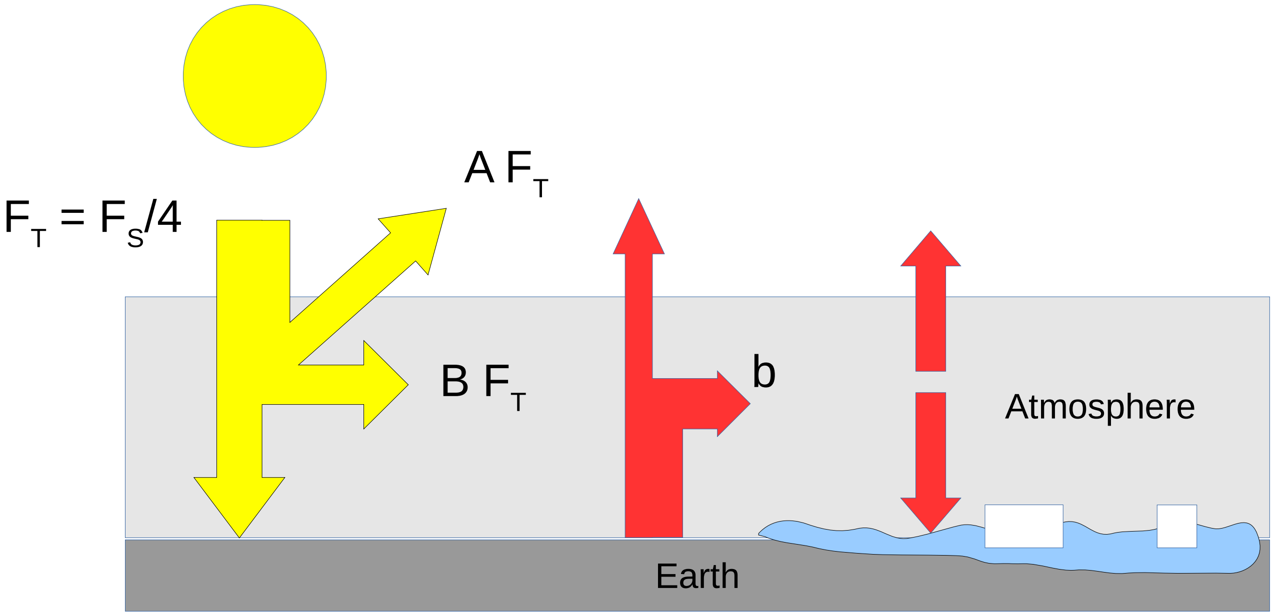
Note 4: There is no need to copy the question text. Just identify them by their numbers.

Note 5: **Lecture notes and small class documents (notes and slides)** are allowed as well as **French-English dictionary**. No other document is allowed.

Note 6: Documents mentioned above might be consulted in a **digital form** on a tablet/computer, but the device (and other devices with connection capabilities, such as phones, calculator, ...) must be put in **flight mode**. **No Wifi connection** is allowed.

Exercise: Greenhouse Effect – Retroaction of Albedo (5 points)

The Earth is modelled as a black body of uniform temperature T_T, covered by an atmosphere with a uniform temperature T_A, illuminated by the incoming average solar radiation $F_T = F_S / 4$ (where $F_S = 1360\,\mathrm{W/m^2}$, corresponding to the solar flux under normal incidence, is called the "*solar constant*"). A fraction $B \sim 20\,\%$ of the incoming solar radiation is absorbed in the atmosphere, and a fraction A, called "*albedo*", is reflected to space. The rest is absorbed by the earth, which remits, as a black body, an infrared radiation. A fraction $b = 91\%$ of the infrared radiation from the Earth is absorbed by the atmosphere. The atmosphere behaves as a grey body, emitting a radiation per unit surface $\epsilon \sigma T_A^4$ in both direction (up and down), where the emissivity ϵ is equal to the absorbency b:



1 Establish the thermal balance for the Earth and for the atmosphere, under assumption of thermal equilibrium.

2 Show that the Earth equilibrium temperature can be written as:

$$T_T = \left[\frac{F_S}{\sigma (4 - 2 \times b)} \left(1 - A - \frac{B}{2} \right) \right]^{1/4}$$

3 The albedo is made of different contributions: reflection by high atmosphere – including clouds – and supposed to be constant ($A_A \sim 24\,\%$), and ground reflection, affected by atmospheric absorption in the down-going and up-going direction. Assuming a surface reflectivity α_T, and an atmospheric transparency t_A in each direction, show that the earth effective albedo (total fraction of incoming visible radiation sent back to space) can be expressed as:

$$A_E = \alpha_T t_A^2 (1 - A_A) + A_A$$

Show as well that the effective atmospheric absorption (Fraction of solar flux absorbed in the atmosphere, denoted B above) can be expressed as:

$$B_E=(1-t_1)(1-A_A)(1+\alpha_T t_A)$$

4 Compute the values of A_E , B_E and current earth temperature for $\alpha_T=15\%$, and $t_A=76\%$.

5 Now consider the case of an icy planet: the earth is fully covered by Ice ($\alpha_T \sim 80\%$), Compute the new albedo value and the new temperature, assuming no change in other parameters. Is the albedo feedback amplifying or stabilizing temperature variations? Explain why.

Problem: Hydrogen Vehicle (15 points)

The Toyota Mirai (Fig. 1) is one of the very first hydrogen fuel personal vehicle available on the market, at a price of ~ 78 900 €. More of 3000 vehicles have been sold in California since 2015 (despite limited hydrogen infrastructures), and Toyota expects to sell more than 30 000 by 2020. In this problem, we will study several aspects of hydrogen fuel car. In the first part, we investigate the needed power, and look into conventional gasoline motors. The second par is devoted to Fuel Cell technology, and the third one investigates some aspects of the production, storage and distribution of hydrogen.

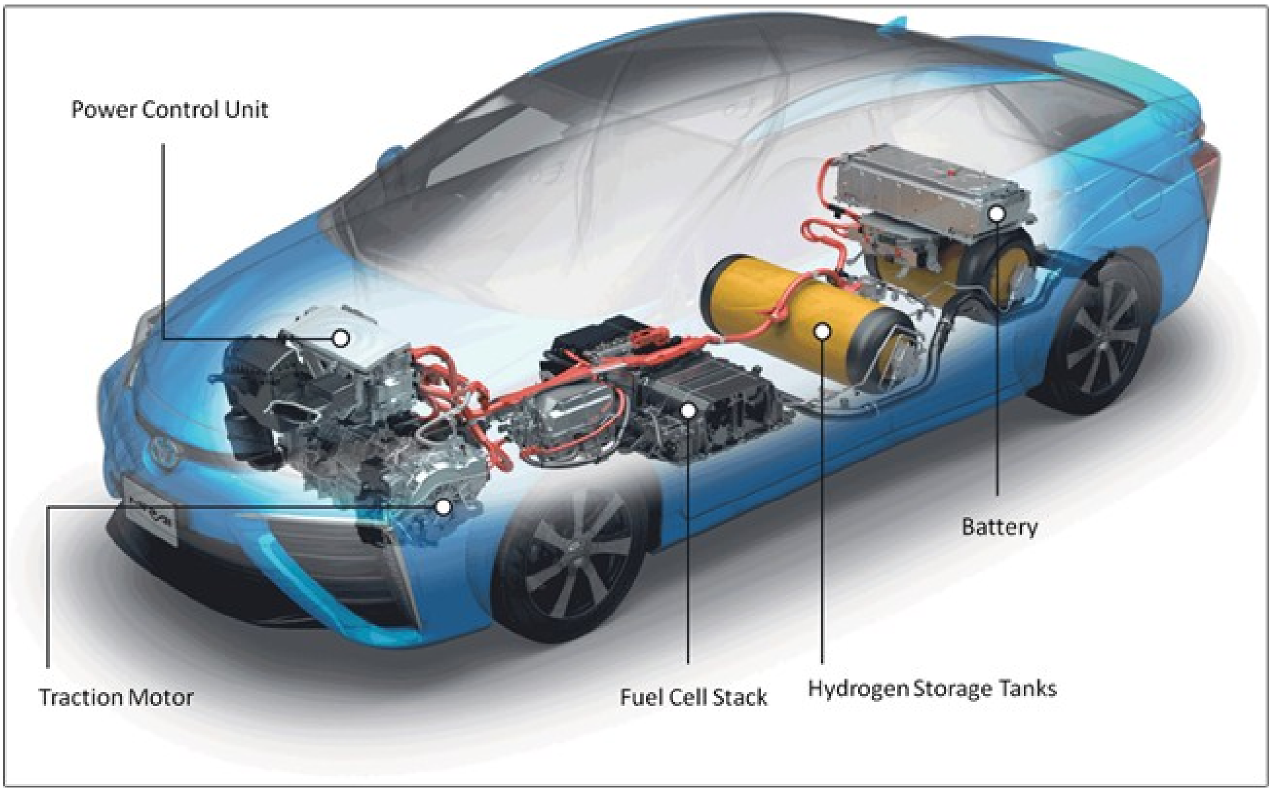


Figure 1: Toyota Mirai

Part 1. Conventional Gasoline Car (5 points)

1. We remind that the *drag force* exerted on a moving object can be expressed as:

$$F_D = \frac{1}{2}\rho S C_x v^2$$

where C_x is the air penetration coefficient, ρ the fluid density, S the surface and v the speed. Computes the power needed to sustain a constant speed of $100\,\mathrm{km/h}$, considering only the resistance of air.

2. Rolling resistance can be expressed by a constant force, with expression:

$$F_R = C_{rr}\, m\, g$$

where C_{rr} is the rolling resistance coefficient, usually of the order of $C_{rr} \approx 0.01$. Estimate the power needed to counteract rolling resistance at speed $100\,\text{km/h}$. At which speed are the rolling resistance and air resistance equivalent?

3. Overall mechanical yields (transmission, gear box, cooling…) are of the order of 50%. Estimate the needed motor power.
4. The spark ignition piston engine follow the Otto cycle, with 4 different transformations:
    - Adiabatic compression of fuel-air mixture (A → B)
    - Isochorous combustion (spark ignition) (B → C)
    - Adiabatic expansion in cylinder (C → D)
    - Isochorous cooling (D → A)

   Draw the cycle in the P-V and T-S diagrams.

   Show that the maximum efficiency can be expressed as: $\eta_{th}=1-\alpha^{1-\gamma}$

   where $\alpha=V_A/V_B$ is the compression ratio. For realistic compression ratio are $\alpha\approx 10$, estimate the thermodynamical yield.
5. Estimate the fuel consumption of the car at constant speed (in litter og gasoline per 100 km)

Part 2. Hydrogen fuel car (5 points)

1. Fuel cells are a promising alternative energy conversion technology. Most fuel cells use hydrogen as a fuel which reacts with oxygen to produce electricity. Different types of fuel cells are distinguished by the nature of the electrolyte. For example, a *proton exchange membrane fuel cell* (PEMFC) contains a polymer electrolyte that transports protons as shown in Figure 2.

   The PEMFC reactions are:

    - Anode: $H_2 \Rightarrow 2H^+ + 2e^-$
    - Cathode: $\frac{1}{2}O_2 + 2H^+ + 2e^- \Rightarrow H_2O$
    - Overall: $H_2 + \frac{1}{2}O_2 \Rightarrow H_2O$

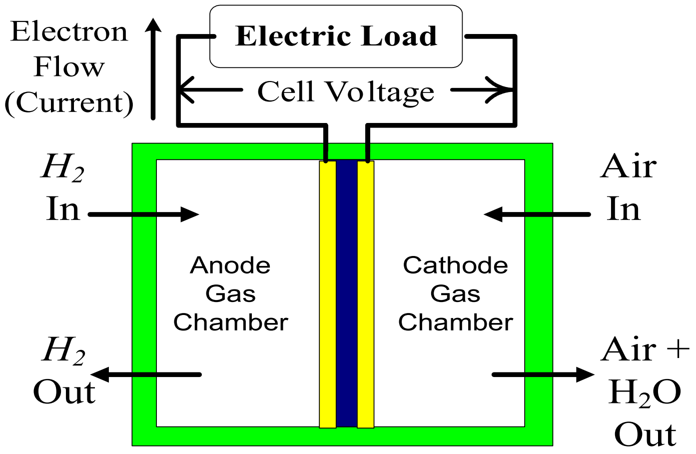


Figure 2: Principle of a proton membrane fuel cell

   At low electrical intensities, the voltage difference between the cathode and the anode is $U_{\text{cell}}\approx 1\,\text{V}$. Estimate the yield of the fuel cell in this low intensity regime.

2. The cell stack (Fig. 3) is made from $n_{\text{cells}}=370$ cells of $d_{\text{cell}}=1.34\,\text{mm}$ thickness each, for a total volume $V_{\text{stack}}=37\,l$. Estimate the current density I_S on each cell (electrical current per unit area of the cell) for the power computed at question 1.3 and for the maximum power of the stack assuming a voltage difference of $U_{\text{cell}}\approx 1\,\text{V}$.
3. The polarization curve (Figure 4) gives insight into both chemical and electrical events occurring within the active region of the fuel cell. The voltage drop, also called *overvoltage*, increases with current density and can be attributed to three different physical mechanisms:

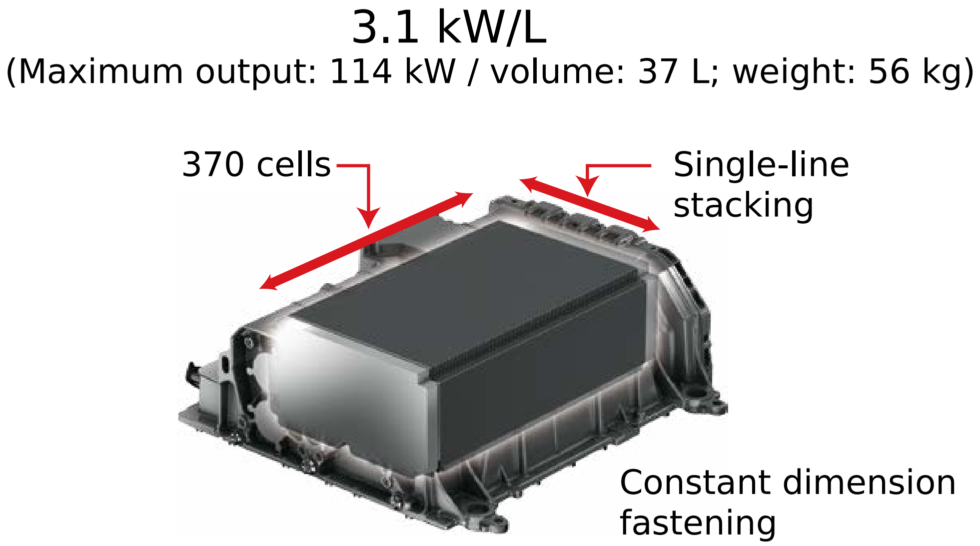


Figure 3: Fuel stack "Toyota FCStack"

- kinetic losses (for current densities less than $50\,\mathrm{mA/cm^2}$),
- ohmic losses (for current densities in the range of $50-900\,\mathrm{mA/cm^2}$),
- and mass transport losses (for current densities above $900\,\mathrm{mA/cm^2}$).

It is noted that all of the above losses result in fuel being converted into heat instead of electricity. The curve can be approximated by (dashed red line on fig. 4):

$$U_{cell} \approx 0.9\,\mathrm{V} - 0.4\,\mathrm{V} \times \left(\frac{I_s}{1000\,\mathrm{mA/cm^2}} \right)$$

Draw the stack power and yield as function of current density.

4. What intensity would be needed to operate the car at $100\,\mathrm{km/h}$ corresponding to a motor power of $P_M \approx 30\,\mathrm{kW}$? What would be the yield of the stack? Estimate the hydrogen consumption for a distance of $100\,\mathrm{km}$.

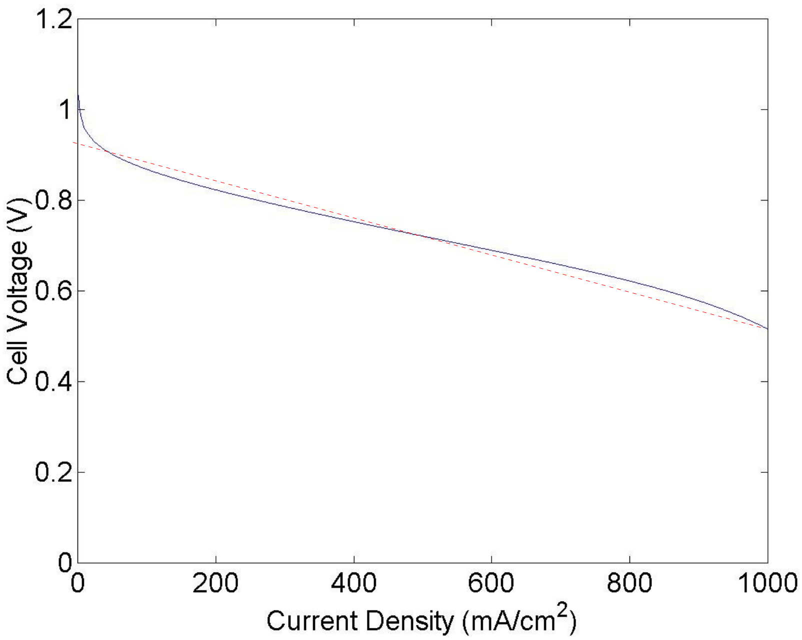


Figure 4: Polarization Plot

5. Hydrogen is stored at 700 bars in a tank of $122\mathrm{l}$. Compute the hydrogen density at this pressure, and estimate the autonomy of the car.

Part 3. Hydrogen production and transportation (5 points)

1. Production: several techniques are available on the market to produce hydrogen. The most widely used are gasification of biomass/fossil fuel and electrolysis of water:
   1. **gasification of fossil fuel or methane**:
      - For this process high temperature (700–1100 °C) steam (H_2O) reacts with gas or coal in an endothermic reaction to yield syngas (*Steam Reforming*):

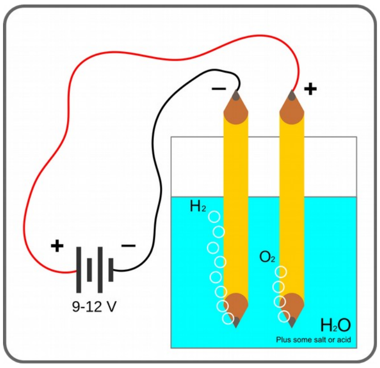
        $$C_nH_m + nH_2O \Rightarrow nCO + (n+m/2)H_2$$

        in particular, $CH_4 + H_2O \Rightarrow CO + 3H_2$ and $C + H_2O \Rightarrow CO + H_2$
      - in a second stage, the additional hydrogen is generated through the lower-temperature, exothermic, *water gas shift reaction*, performed at about 360 °C:

        $$CO + H_2O \Rightarrow CO_2 + H_2$$
   2. **partial combustion of fossil fuel**:
      - The first stage if this process is a partial combustion in a fuel-oxygen mixture to generate syngas:

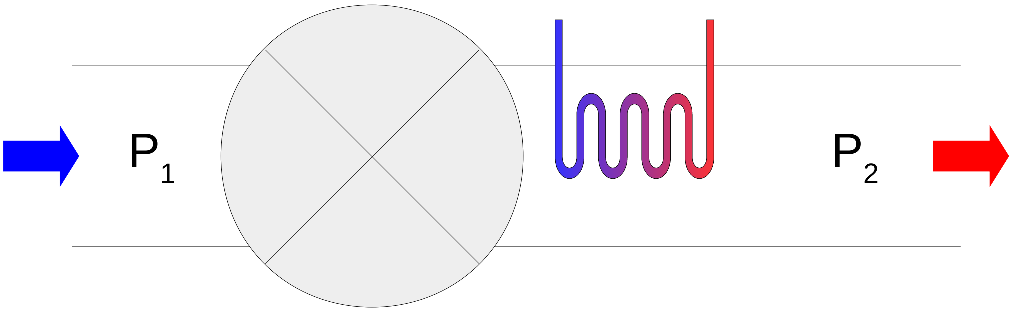
        $$C_nH_m + n/2O_2 \Rightarrow nCO + m/2H_2$$

        (this can also be applied to coal: $C + 1/2O_2 \Rightarrow CO$)
      - The second stage is identical to the gasification technique (*water gas shift reaction)*
   3. **electrolysis of water**: electricity is used to split water into hydrogen and oxygen. The reaction has a standard potential of −1.23 V, meaning it ideally requires a potential difference of 1.23 volts to split water (see schematics on the right). Hydrogen will appear at the cathode (where electrons enter the water), and oxygen will appear at the anode.



Compare the benefits and drawbacks of the three techniques in terms of efficiency, implications and environmental aspects.

2. Compute the energetic cost of compression to 700 bars (per mole) under the two following assumption:



- Adiabatic compression (compute also the gas temperature at the end of compression)

- Isothermal compression (i.e. the compression heat is recovered, e.g. in a co-generation unit)

Note: the compression is done in a turbo-compressor, with constant flow, like shown in the picture above.

3. Liquefaction: Estimate the energetic cost (per mol) of liquefaction at $T_{\text{vap}} = 20.271\,\text{K}$. One can consider that the gaz is first cooled down to T_{vap} using a reversible Carnot machine operating continuously between the gaz temperature and the atmospheric one ($T_0 = 298\,\text{K}$), followed by liquefaction at $T_{\text{vap}} = 20.271\,\text{K}$ (still using a Carnot machine). Current industrial processes reach an energetic cost of liquefaction of $\sim 12\,\text{kWh/kg}$. What is their efficiency?
4. Transportation: Two transportations techniques are currently on the market: Liquid hydrogen tanker (Fig. 5 - tank of 25 000 l) and compress hydrogen trailer (Fig. 6 25 000 l at 250 bars). The fuel consumption of semi-trailer is typically 40 l (diesel or gasoline) per 100 km. To what fraction of the energy stored in transported hydrogen does a 600 km delivery distance correspond to (do not forget the round trip: the trailer comes back empty after fuel delivery)?
5. Personal conclusion

Figure 5: Liquid hydrogen tanker of 25 000 l

Figure 6: Composite tube trailer of 25 000 l of gaseous hydrogen at 250 bar.

Numerical Values & constants

Energy Data

Ton Oil Equivalent	1 t.o.e. = $41\ 855 \times 10^6$ J
Gasoline Fuel Lower Heating Value	$FLH = 35.4 \times 10^6$ J/l
Gasoline Density	$\rho = 0.75$ kg/l
Solar Constant	$F_S = 1360$ W/m^2

Constants

Stefan constant	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Gravity at Earth	$g = 9.81$ m s^{-2}
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Absolute Zero	$T_0 = -273{,}15$ °C
Planck constant	$h = 6{,}626\ 0\ 755 \times 10^{-34}$ J.s
Perfect gas constant	$R = 8{,}314$ J mol^{-1} K^{-1}
Electronvolt	1eV = 1.6×10^{-19} J
Air density	$\rho_{air} = 1.2$ kg/m^3

Hydrogen Thermodynamics

Boiling point	$T_{liq} = 20.271$ K
Heat of vaporization	$L_v = 0.904$ kJ/mol
Density (Liquid)	$\rho_{liq} = 0.0708$ kg/l
Adiabatic Index	$\gamma = 7/5$
Lower heating value	FLH = 242 kJ/mol
Calorific Constant	$C_p = 28.53$ J/K/mol

Toyota Mirai Specifications

Fuel	Hydrogen
Max Speed	178 km/h
0 – 100 km/h	9.6 s
Motor Power	113 kW
Hydrogen Cell Power	114 kW
Hydrogen Tank Capacity	5 kg
Type of Batteries	Ni-Mh
Battery Capacity	1.6 kWh
Autonomy	> 500 km
Average Consumption	0.76 kg/100 km
Length	4.890 m
Width	1.815 m
Height	1.535 m
Weight	1850 kg
C_x (air penetration coefficient)	0.29