Corrections

Exercise: Greenhouse Effect – Retroaction of Albedo

1 *Establish the thermal balance for the Earth and for the Atmosphere, under assumption of thermal equilibrium.*

The earth receives solar radiation and atmospheric radiation re-emitted to the Earth:

$$F_T = \underbrace{\frac{F_S}{4}(1-A-B)}_{\text{solar flux}} + \underbrace{b\,\sigma T_A^4}_{\text{atmosphere}} - \underbrace{\sigma T_T^4}_{\text{radiated}} = 0 \;\Rightarrow\; T_T^4 = b\,T_A^4 + \frac{F_S}{4\,\sigma}(1-A-B) \qquad (1)$$

the atmosphere absorbs a fraction b of the land surface radiation, a fraction B of the incident solar radiation and re-emits heat radiation in both directions:

$$F_A = \underbrace{b\,\sigma T_T^4}_{\text{land}} + \underbrace{\frac{F_S}{4}B}_{\text{solar flux}} - \underbrace{2\,b\,\sigma T_A^4}_{\text{radiated}} = 0 \qquad (2)$$

2 *Show that the Earth equilibrium temperature can be written as….*

Combining the two equations (1) and (2), one obtains:

$$2\,b\,T_A^4 = b\,T_T^4 + \frac{F_S}{4\,\sigma} \times B$$

Inserting this in eq. (1) gives immediately the Earth Equilibrium temperature:

$$T_T = \sqrt[4]{\frac{F_S}{\sigma(4-2\times b)}\left(1-A-\frac{B}{2}\right)} \qquad (3)$$

3 *The albedo is made of different contributions: reflection by high atmosphere – including clouds – and supposed to be constant ($A_A \sim 24\,\%$), and ground reflection, affected by atmospheric absorption in the down-going and up-going direction. Assuming a surface reflectivity α_T , and an atmospheric transparency t_A in each direction, show that the earth effective albedo (total fraction of incoming visible radiation sent back to space)…..*

We have the following fluxes:

- At the top of the atmosphere: F_T
- Reflected by the high atmosphere to space: $A_A F_T$
- Down-going flux reaching the ground: $t_A(1-A_A)F_T$
- Down-going flux absorbed in the atmosphere: $(1-t_A)(1-A_A)F_T$
- Up-going flux reflected by the ground surface: $\alpha_T t_A(1-A_A)F_T$
- Up-going flux transmitted to space: $\alpha_T t_A^2(1-A_A)F_T$
- Up-going flux absorbed in the atmosphere: $\alpha_T t_A(1-t_A)(1-A_A)F_T$

So the total flux sent back to space is:

$$A_E F_T = A_A F_T + \alpha_T t_A^2 (1 - A_A) F_T \quad \Rightarrow \quad A_E = A_A + \alpha_T t_A^2 (1 - A_A)$$

And the total flux absorbed in the atmosphere is

$$B_E F_T = (1 - t_A)(1 - A_A) F_T + \alpha_T t_A (1 - t_A)(1 - A_A) F_T \quad \Rightarrow \quad B_E = (1 - A_A)(1 - t_A)(1 + \alpha_T t_A)$$

4 *Compute the values of* A_E *,* B_E *and current earth temperature for* $\alpha_T = 15\%$ *, and* $t_A = 75\%$ *.*

One obtains

- $A_E = 0.30$
- $B_E = 0.20$
- $T_T = 12\,°\mathrm{C}$ (close to current values)

5 *Now consider the case of an icy planet: the earth is fully covered by Ice (* $\alpha_T \sim 80\%$ *), Compute the new albedo value and the new temperature.*

One obtains:

- $A_E = 0.59$
- $B_E = 0.29$
- $T_T = -41\,°\mathrm{C}$

Despite the fact that the atmospheric absorption is actually increasing, the dominant effect is the increase of albedo, leading to a huge drop in temperature: the land coverage is actually an essential element determining the earth temperature.

Is the albedo feedback amplifying or stabilizing temperature variations? Explain.

Albedo feedback is amplifying the variations of the climate system: decrease in temperature lead to a larger fraction of earth covered by ice, leading to a further decrease in temperature and so on. Similarly, global warming result in ice melt down, resulting in white area being replaced by darker ones, and therefore decreased albedo, further increasing the temperature.

Problem: Hydrogen Vehicle

Part 1. Conventional Gasoline Car (5 points)

1. *Computes the power needed to sustain a constant speed…*

   The front surface is $S=1.815\times1.535=2.78\,\mathrm{m}^2$. The needed mechanical power reads:

   $$P_D=F_D\times v=\frac{1}{2}\rho S C_x v^3=\frac{1}{2}\times1.2\times2.78\times0.29\times\left(\frac{100}{3.6}\right)^3=10.4\,\mathrm{kW}$$

2. *Estimate the power needed to counteract rolling resistance.*

   The rolling resistance power is:

   $$P_R=F_R\times v=C_{rr}\,m\,g\,v=0.01\times1850\times9.81\times\left(\frac{100}{3.6}\right)\approx5\,\mathrm{kW}$$

   At which speed are the rolling resistance and air resistance equivalent?

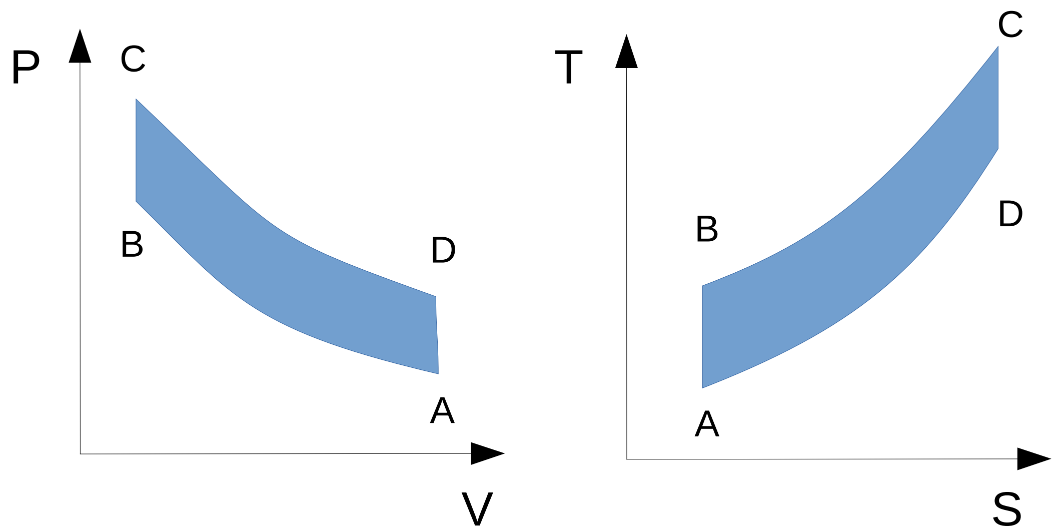
   The two forces are equivalent for

   $$v=\sqrt{\frac{2C_{rr}\,m\,g}{\rho S C_x}}\approx71\,\mathrm{km/h}$$

3. *Estimate the needed motor power.*

   The needed motor power is $P_M=\dfrac{P_D+P_R}{\eta_{\mathrm{transmission}}}\approx30.1\,\mathrm{kW}$

4. *Draw the cycle in the P-V and T-S diagrams.*



Show that the maximum efficiency can be expressed as: $\eta_{th}=1-\alpha^{1-\gamma}$

where $\alpha=V_A/V_B$ *is the compression ratio. Realistic compression ratio are* $\alpha\approx10$.

Heat Q_{in} is produced during the combustion phase (B-C) and released (Q_{out}) during the exhaust phase (D-A). The yield reads:

$$\eta_{\text{th}} = -\frac{W}{Q_{\text{in}}} = 1 + \frac{Q_{\text{out}}}{Q_{\text{in}}}$$

The combustion and exhaust phases being isochorous,

$$\mathrm{d}W = -p\,\mathrm{d}V = 0 \quad \Rightarrow \quad Q = \Delta U = C_v \Delta T$$

Thus $\eta_{\text{th}} = 1 + \dfrac{T_A - T_D}{T_C - T_B}$.

The phases C-D and A-B being isentropic (reversible adiabatic):

$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$ and $T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$.

Moreover $V_D = V_A$ and $V_C = V_B$.

Finally

$$\eta_{\text{th}} = 1 + \frac{T_A - T_C \left(\dfrac{V_C}{V_D} \right)^{\gamma-1}}{T_C - T_A \left(\dfrac{V_A}{V_B} \right)^{\gamma-1}} = 1 + \frac{T_A - T_C \alpha^{1-\gamma}}{T_C - T_A \alpha^{\gamma-1}} = 1 + \alpha^{1-\gamma} \frac{T_A \alpha^{\gamma-1} - T_C}{T_C - T_A \alpha^{\gamma-1}} = 1 - \alpha^{1-\gamma}$$

For $\alpha = 10$, the theoretical yield of the motor would be $\eta_{\text{th}} \approx 60\,\%$.

5. *Estimate the fuel consumption of the car at constant speed (in litter / 100 km)*

The total needed motor power is of the order of $30\,\mathrm{kW}$. Fuel instantaneous consumption is therefore

$$C = \frac{P_M}{\mathrm{FLH}\, \eta_{\text{th}}} = \frac{30.1 \times 10^3}{35.4 \times 10^6 \times 0.6} \approx 1.45 \times 10^{-3}\, \mathrm{l/s}$$

Corresponding to an average consumption of $C \times 3600 = 5.2\,\mathrm{l}/100\,\mathrm{km}$. This of course does not take into account changes of speed (energy lost in cycle), which would further increase this value.

Part 2. Hydrogen fuel car (5 points)

1. *At low electrical intensities, the voltage difference between the cathode and the anode is* $U_{\text{cell}} \approx 1\,\mathrm{V}$ *. Estimate the yield of the fuel cell.*

The power of the cell can be written as $P = U_{\text{cell}} \times I$. For n moles of exchanged hydrogen, $2 \times N_A \times n$ electrons are exchanged, corresponding to a total energy $E = 2 \times n \times N_A \times e \times U_{\text{cell}}$ where $e = 1.6 \times 10^{-19}\,\mathrm{C}$ is the electric charge of the electron. The produced energy per mole is therefore $E = 2 \times N_A \times e \times U_{\text{cell}} \approx 192.6\,\mathrm{KJ/mol}$. The yield of a single cell is therefore the recovered energy divided by the lower heating value:

$$\eta_{\text{cell}} = E / P_{\text{ci}} \approx \frac{192.6}{242} = 79\,\%$$

2. *Estimate the current density on each cell (electrical current per unit area of the cell) for the power computed at question 1.3 and for the maximum power of the stack.*

Each cell has a surface of $S_{\text{cell}} = V_{\text{stack}} / d_{\text{cell}} \approx 740\,\mathrm{cm}^2$. The power per cell is $P_{\text{cell}} = P_M / n_{\text{cells}} \approx 80\,\mathrm{W}$, corresponding to a current density

$I_S = P_{\text{cell}} / U_{\text{cell}} \times S_{\text{cell}} \approx 100\,\text{mA/cm}^2$.

The max power of the stack is $114\,\text{kW}$, corresponding to a current density $I/S \approx 400\,\text{mA/cm}^2$.

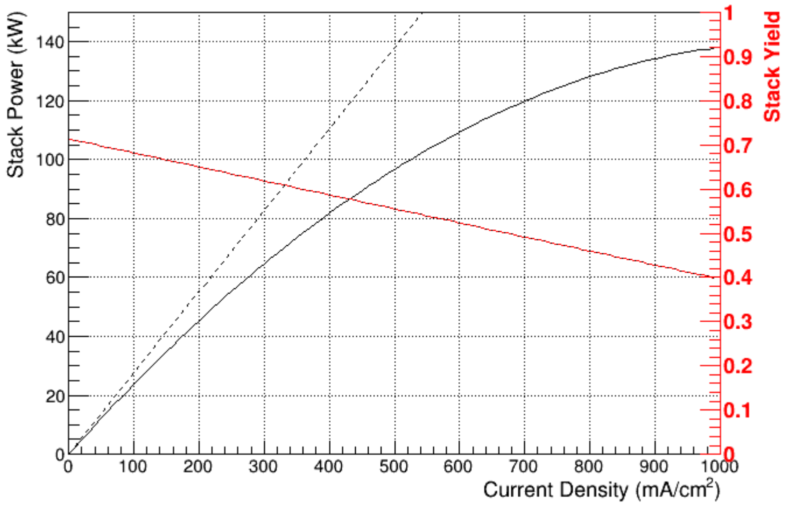
3. *Draw the stack power and yield as function of current density.*

The cell voltage reads: $U_{cell} \approx 0.9\,\text{V} - 0.4\,\text{V} \times \left(\frac{I_s}{1000\,\text{mA/cm}^2} \right)$

and the stack power is $P_{\text{stack}} = n_{\text{cell}} \times I_S \times S_{\text{cell}} \times U_{\text{cell}}$,

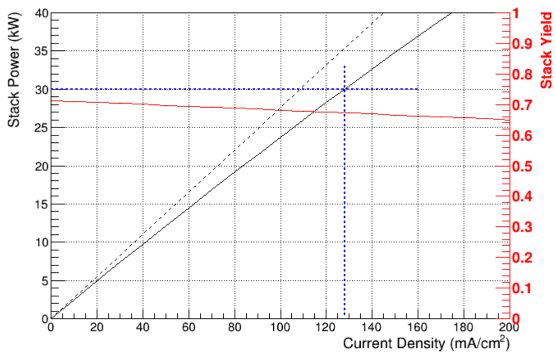
for a yield: $\eta_{\text{cell}} = \frac{E}{P_{\text{ci}}} = \frac{2 \times N_A \times e \times U_{\text{cell}}}{P_{\text{ci}}}$

The following plot is obtained:



4. *What intensity would be needed to operate the car at 100 km/h?. Estimate the hydrogen consumption for* $100\,\text{km}$.

A zoom on the previous graph shows that a power of $30\,\text{kW}$ corresponds to a current density of $I_S = 128\,\text{mA/cm}^2$, thus a total current of $I = I_S \times S_{\text{cell}} = 95\,\text{A}$ per cell. The corresponding yield would be ~65%. The hydrogen consumption is:



$$\frac{dm}{dt} = \frac{M_{H_2} \times I}{2 N_A e} \approx 0.36\,\text{g/s}$$

corresponding to a fuel consumption of $1.3\,\text{kg}/100\,\text{km}$

5. *Compute the hydrogen density at this pressure, and estimate the autonomy of the car.*

From the law of perfect gases, $\frac{P}{\rho} = \frac{RT}{M_{H_2}}$, thus $\rho = \frac{P M_{H_2}}{RT} = 56{,}5\,\text{kg/m}^3$.

The reservoir can therefore contain up to $6.9\,\text{kg}$, corresponding to an autonomy of $\sim 500\,\text{km}$

Part 3. Hydrogen production and transportation (5 points)

1. *Production*

   Gasification of fossil fuel can in principle operate with coal or methane, but produces a significant amount of CO_2, and requires heat: The balance equation is

   $$C_nH_m + 2nH_2O \Rightarrow nCO_2 + (2n + m/2)H_2$$

   Partial combustion does not require external heating, but produces comparatively less hydrogen, with a balance equation:

   $$C_nH_m + n/2O_2 + nH_2O \Rightarrow nCO_2 + (n + m/2)H_2$$

   So partial combustion produces more carbon dioxide per produced hydrogen. Both methods produce large amount of greenhouse gases and are therefore not a solution for global warming.

   Electrolysis, if using renewable energies, can be a clean source of hydrogen. The yield can reach, in recent installations, up to 80%.

2. *Compute the compression energetic cost (per mole) under the two following assumption:*
   - *Adiabatic compression*
   - *Isothermal compression (i.e. the compression heat is recovered, e.g. in a co-generation unit)*

   What fraction of the combustion energy does it correspond to?

   Compression is done in a turbo-compressor, so the relevant variable is enthalpy (not internal energy). For adiabatic compression, $Q=0$ thus $W=\Delta H=C_p\Delta T$. Using Laplace formula $T^{\gamma}P^{1-\gamma}=C_{\mathrm{ste}}$ one finds:

   $$W_{\mathrm{adiabatic}}=C_pT_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right)$$

   and the gas temperature at the end of the compression is 1936 K.

   For di-hydrogen $W_{\mathrm{adiabatic}}=46.7\,\mathrm{kJ/mol}$

   For isothermal compression, heat is removed from the system, $Q<0$, and $\Delta H=Q+W=0$ (as temperature is unchanged).

   thus $W_{\mathrm{isothermal}}=-Q=-T\Delta S=-T\,C_v\ln\left(\frac{T^{\gamma}P_2^{1-\gamma}}{T^{\gamma}P_1^{1-\gamma}}\right)=T\,R\ln\left(\frac{P_2}{P_1}\right)=16.2\,\mathrm{KJ/mol}$.

   This corresponds to respectively 17% and 5.6% of the combustion energy of hydrogen. It is therefore quite important to use co-generation in hydrogen compression to avoid loosing too much energy.

3. *Liquefaction cost:*

   The removed heat by elementary cooling by temperature variation $\mathrm{d}T$ reads

   $\delta Q=C_p\mathrm{d\,T}$.

   If performed by a Carnot machine with yield $\eta=\frac{T_0}{T}-1$, the corresponding elementary work is

$$\delta W = -\eta \times \delta Q = -C_p \left(\frac{T_0}{T} - 1 \right) \mathrm{d}T$$

Corresponding to a total work

$$W_{\text{cool}} = \int_{T_0}^{T_{\text{vap}}} \delta W = -C_p \log \left[\left(\frac{T_{\text{vap}}}{T_0} \right) - (T_{\text{vap}} - T_0) \right] = 14.9\,\text{kJ/mol}$$

The liquefaction being done at constant temperature, the liquefaction work is simply:

$$W_{\text{liq}} = \left(\frac{T_0}{T_{\text{vap}}} - 1 \right) \times L_v = 12.4\,\text{kJ/mol}$$

Corresponding to a total cost of cooling + liquefaction of $27.3\,\text{kJ/mol} = 3.8\,\text{kWh/kg}$.

Industrial processes have an efficiency of ~ 25%, meaning that ~40% of the energy of hydrogen is used in the liquefaction process.

4. *To what fraction of the energy stored in transported hydrogen does it correspond to?*

   Gaseous Hydrogen density at 250 bars is $\rho = \frac{P M_{H_2}}{RT} = 20{,}2\,\text{kg/m}^3$,

   whereas liquid hydrogen density is $70{,}8\,\text{kg/m}^3$. Tankers can carry:

   - Liquid tanker: $1770\,\text{kg} \Rightarrow 2.1\,10^{11}\,\text{J}$, equivalent to 6075 l of gasoline
   - Gaseous tanker: $504\,\text{kg} \Rightarrow 6.1\,10^{10}\,\text{J}$, equivalent to 1732 l of gasoline

   For a 600 km delivery distance (1 200 km round trip), the trailer would use $12 \times 40 = 480\,\text{l}$ of gasoline, which would correspond to ~8% of energy content (liquid hydrogen) and ~28% of energy content (gaseous) hydrogen. In addition, for gaseous transport, the tank is never completely emptied: the tanks are usually emptied up to a pressure of ~ 40 bars, corresponding to a delivered fraction of ~80%.

5. *Personal conclusions*

   Different elements can be discussed here:

   - Overall efficiency of Hydrogen car not so bad. With tank at 700 bars, autonomy is reasonable
   - This would be a clean (carbon-free) energy only if the hydrogen is produced by electrolysis from renewable electricity, which currently is marginal (~ 2%)
   - Compression, liquefaction and transportation eat up a very significant fraction of store energy, degrading the yield by a factor of ~ 2 (transport by pipes could be an alternative, but requires a large investment cost)