Evaluation PHY555 Energy & Environment

Friday 21st December 2019

Part II, Lecture notes allowed, duration 2h

Note 1: All parts of the problem are **independent.**

Note 2: Quality of the redaction of justification of calculation will be taken into account in the evaluation. Answers can be written in **French** or in **English.**

Note 3: The physical constants needed in the problem are grouped in tables at the end of the subject (Part 5., page 12)

Note 4: The total grading is on 25 and will be truncated. **You don't need to do everything to have 20/20 ! Start with the topics you feel most familiar with.**

Note 5: There is no need to copy the question text. Just identify them by their numbers.

Note 6: **Lecture notes and small class documents (notes and slides)** are allowed as well as **French-English dictionary**. No other document is allowed.

Note 7: Documents mentioned above might be consulted in a **digital form** on a tablet/computer, but the device (and other devices with connection capabilities, such as phones, calculator, ...) must be put in **flight mode**. **No Wifi connection** is allowed.

Problem: Drake Landing Solar Community

The Drake Landing Solar Community (DLSC) is a proof of concept for solar heating located in the town of Okotoks, Alberta, Canada. It consists of 52 individual homes, constructed in 2007. DLSC is heated by a district system designed to store abundant solar energy underground during the summer months and distribute the energy to each home for space heating needs during winter months. Key features of DLSC are in particular[1]:

- 12th year of reliable operation with no unscheduled interruptions in heating delivery operations;
- 100% solar fraction in the 2015-2016 heating season, meaning all the heat required by the houses for space heating was supplied by solar energy;
- Consistent solar fractions above 90% over the last 5 years, with an average of 96% for the period 2012-2016;
- High solar fraction of 92% even during the very cold winter of 2013-2014;
- Very low electricity usage, with coefficient of performance (COP) above 30. This means that for every kWh of electricity, the system delivers more than 30 kWh of heat;

Figure 1: Aerial View of Drake Landing Solar Community

1 https://www.dlsc.ca/

Part 1. Housing Energy Consumption (4 points)

In this part, we will estimate the energy consumption of the houses. DLSC houses are typically 7×10 meters in size, and 5 meters high (for two floors and $140\,\mathrm{m}^2$ living area). Temperature profile in Okotoks is given in Fig. 2.

Month	Average Temperature (°C)
January	-10,3
February	-6,3
March	-3,4
April	3,5
May	9,4
June	13,3
July	16
August	15,1
September	10,3
October	5,6
November	-2,4
December	-6,9

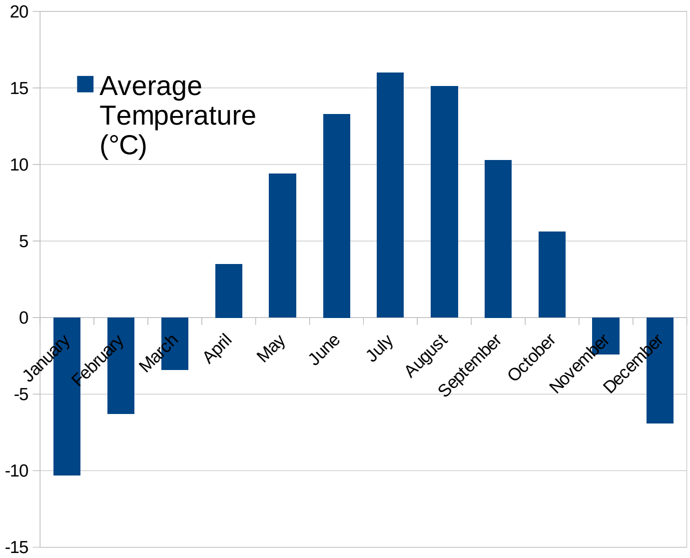

Figure 2: Temperature profile in Okotoks

1. Energy losses through walls: Construction regulations in Alberta impose a minimal thermal resistance of the external walls of $R_\mathrm{w}=2.97\,\mathrm{K\,m^2/W}$ and of the roofs of $R_\mathrm{r}=4.67\,\mathrm{K\,m^2/W}$. Estimate, for an internal temperature of the house $T_\mathrm{i}=19\,°\mathrm{C}$ the energy losses through the envelope across the year, and the corresponding yearly average.
2. Safety regulations impose a minimum coefficient of air renewal of 0.6 volume per hour, with a minimum of $25\,\mathrm{m^3/h/person}$. Assuming that the envelope is relatively air-tight, the air flow is brought by a double-flux ventilation system with 80% energy recovery efficiency. Estimate the energy losses through the ventilation system.
3. DLSC homes had more stringent energy requirements than other conventional homes built at that time, with 30% better energy efficiency en the envelope. Estimate the average needs per year for heating.

Part 2. Thermal Solar Collectors (8 points)

The key power source of the Drake Landing heating system is solar collectors installed on the roofs of garages (Fig. 3) and totalling $2300\,\mathrm{m}^2$. The goal of this second part is to estimate the total energy gathered by this system.

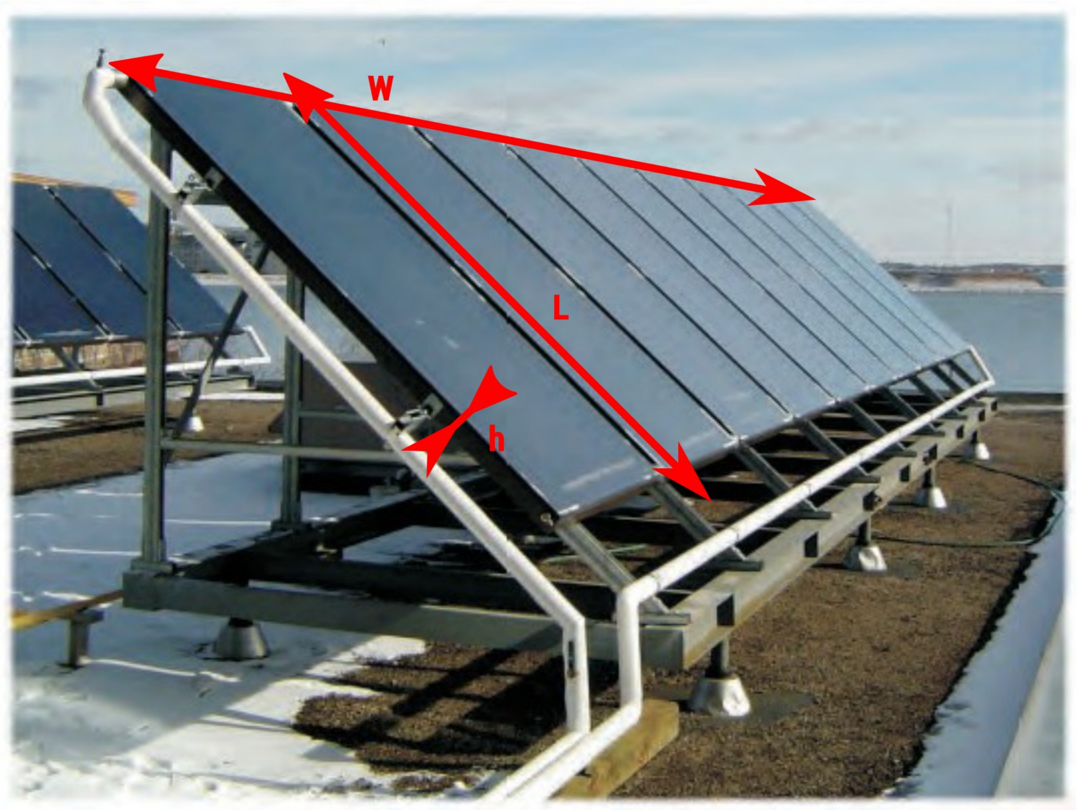

Figure 3: Solar collectors

1. The collector as is modelled as a pipe with rectangular cross-section (width $w=20\,\mathrm{m}$, height $h=8\,\mathrm{cm}$) and length $L=2.5\,\mathrm{m}$. It is described as a black-body with absorptivity a. Through the collector flows a coolant (50% Propyle glycol) of massive heat capacity $c_{cool}=3.6\,\mathrm{kJ/kg/K}$, at a mass flow rate $\dot{m}=0.3\,\mathrm{kg/s}$. The system is placed under sunlight. We note T_{sun} the temperature of the Sun and $\Omega_s=6\times10^{-5}\mathrm{sr}$ its solid angle as viewed from the Earth. The presence of the atmosphere decreases the power reaching the ground by a factor $f\simeq0.73$. Show that the power reaching the collector can be expressed as:

$$P_{sun}=w\times L\times f\times\frac{\Omega_s}{\pi}\times\sigma T_{sun}^4 \quad \text{(Eq. 2.1)}$$

   and explain in one sentence with this system cannot be used to provide work, but only heat

2. Coolant temperature:

   2.a. We first consider the collector alone under sunlight. Perform a radiation balance on the collector, and deduce the equilibrium temperature T_{max} for the collector under sunlight.

   2.b. The collector is still under sunlight, but it now also provides a power P_{cool} to a coolant fluid (in this case, water) passing through the pipe. Assuming that the collector and the coolant are locally at the same temperature $T(z)$, show that the elementary power $\mathrm{d}P_{cool}$ provided to the coolant between position z and $z+\mathrm{d}z$ takes the form

$$dP_{cool} = P_{sun} \times a \times \left(1 - \left(\frac{T(z)}{T_{max}}\right)^4\right) \times \frac{dz}{L} \text{ (Eq. 2.2)}$$

2.c. The temperature of the collector at location z is $T(z)$, and we assume that the collector and the coolant are locally at the same temperature. Perform a local energy balance to show that the temperature distribution follows the equation:

$$\frac{z_s}{T_{max}} \frac{dT}{dz} = 1 - \left(\frac{T(z)}{T_{max}}\right)^4 \text{ (Eq. 2.3)}$$

and give the expression of z_s

2.d. Let $u \rightarrow y(u)$ the function defined by the first order differential equation $y'(u) = 1 - y(u)^4$ with $y(0) = 0$ and u is dimentionless distance ($u \sim z/z_s$). Show that the coolant temperature can be expressed as

$$T(z) = T_{max}\, y\left(u_0 + \frac{z}{z_s}\right) \text{ (Eq. 2.4)}$$

where u_0 is such that $y(u_0) = \frac{T(0)}{T_{max}}$

2.e. The graph of $y(u)$ is shown in Fig. 4 for $u \in [0,1]$. Comment the figure. Considering that the coolant enters the collector at a temperature $T(0) = 50\,°C$, estimate the output temperature $T(L)$. Perfect absorption of the incident sunlight ($a = 1$) is assumed in this question.

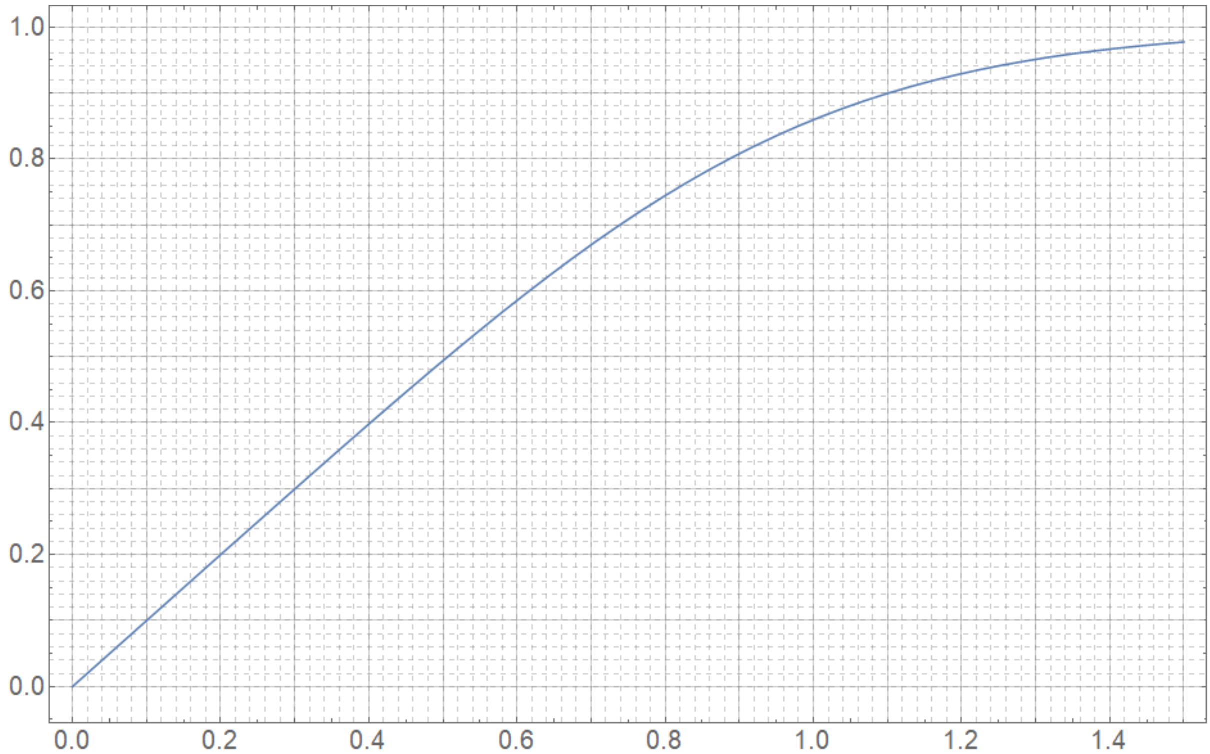

Figure 4: Graph of the function $u \rightarrow y(u)$ *solution of the differential equation* $y'(u) = 1 - y(u)^4$.

3. The overall collection efficiency is defined as

$$\eta_c = \frac{P_{\text{cool}}}{P_{\text{sun}}} \quad \text{(Eq. 2.5)}$$

Where P_{cool} is the total power provided to the coolant over his passage through the collector.

3.a. Perform a global energy balance on the the coolant to express P_{cool} and show that:

$$\eta_c = a \frac{T(L) - T(0)}{T_{max}} \frac{z_s}{L} \quad \text{(Eq. 2.6)}$$

where T_{max} and z_s are the quantities introduced in the previous question.

3.b. Estimate the overall efficiency of the system assuming ideal absorptivity

4. The total solar irradiance at Drake Landing is about $5.45\,\text{GJ}/\text{m}^2/\text{yr}$. Assuming that the average efficiency is given by the value calculated in question 3.b, Estimate the total collected energy.

Part 3. Inter-seasonal Thermal Heat Storage (8 points)

Drake Landing Solar Comumunity uses a borehole thermal energy storage (BTES) system as inter-seasonal thermal heat storage. A BTES is an underground structure for storing large quantities of solar heat collected in summer for use later in winter (Fig. 5). It is basically a large, underground heat exchanger.

A BTES consists of an array of boreholes resembling standard drilled wells. After drilling, a plastic pipe with a "U" bend at the bottom is inserted down the borehole. To provide good thermal contact with the surrounding soil, the borehole is then filled with a high thermal conductivity grouting material.

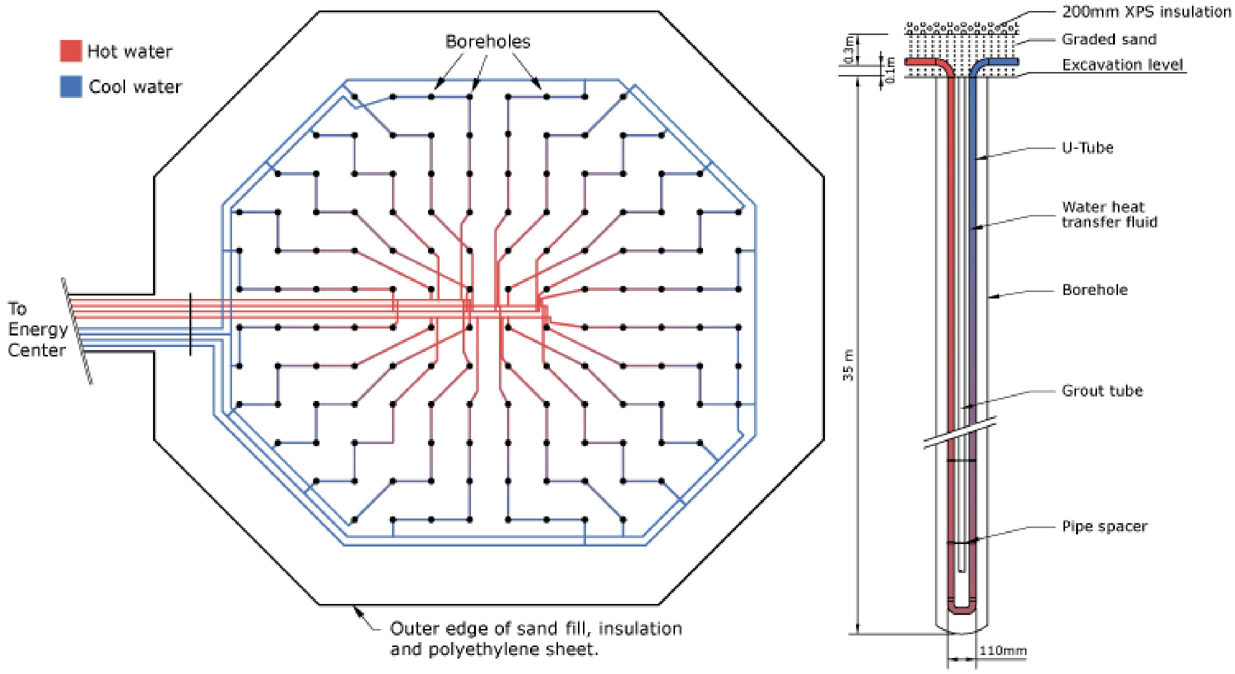

Figure 5: Borehole aerial view (left) and side view (right)

The BTES in the Drake Landing Solar Community (DLSC) consists of 144 boreholes, each stretching to a depth of 37 meters and planned in a grid with 2.25 meters between them. The BTES field covers 35 metres in diameter. At the surface, the U-pipes are joined together in groups of six that radiate from the center to the outer edge, and then connect back to the Energy Centre building. The entire BTES field is then covered in a layer of insulation and then soil – with a landscaped park built on top. Numerical values of constants relevant to the BTES are given in Part 5.

In this part of the problem, we will investigate how the thermal energy can be stored in, or restored from the BTES. For that, we will consider diffusion of heat in **transient regime** (out of equilibrium). For simplicity, we will restrict ourself to a one-dimentional problem and assume that the main results remains valid in a 2D, cylindrical geometry.

1. Dimensionning: how much energy can the BTES contain? (take the annual average temperature as origin). To how many years of solar production of the SLDC does this correspond to?
2. Recall the diffusion equation for a body of thermal conductivity λ, heat capacity c and mass density μ. Express the diffusion coefficient D as function of λ, c, and μ.

3. Using dimensional arguments, recall how the diffusion distance scales with time? How far does the heat diffuse in the storage in one day? in one month? in one year? What can we conclude concering the geometry of the storage and the spacing of the boreholes?

4. We define the dimentionless parameter $u = x / \sqrt{D \times t}$. Homogeneous solution: assuming that the temperature depends only on u , show that the diffusion equation can be rewitten as:

$$\frac{\partial^2 T}{\partial u^2} = -\frac{u}{2}\frac{\partial T}{\partial u} \quad \text{(Eq. 3.1)}$$

5. We consider the case where a body is put in contact with a thermostat at temperature T_0 at time $t=0$ (Fig. 6). The body tempeature far from the contact is T_1 . Heat starts then to diffuse towards $x \geq 0$:

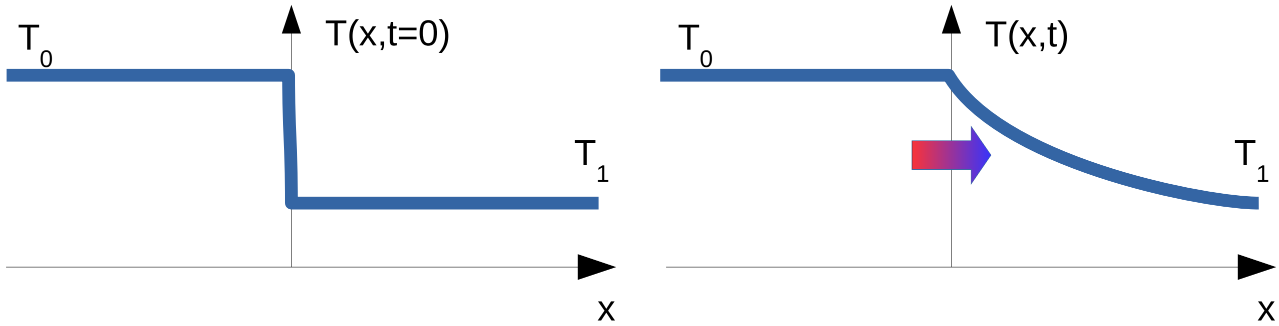

Figure 6: Transient diffusion regime

Solve the equation 3.1 with these boundary conditions. One can use the intermediate function $g(u) = \frac{\partial T}{\partial u}$. We define the "error function": $\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-v^2)\, dv$. Show that the temperature as function of $x \geq 0$ and $t \geq 0$ can be expressed as:

$$T(x,t) = T_0 + (T_1 - T_0) \times \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \quad \text{(Eq. 3.2)}$$

6. Compute the heat flux at $x=0$ as function of time. Show that it can be expressed as:

$$j_Q(x=0) = e\frac{(T_0 - T_1)}{\sqrt{t}} \quad \text{(Eq. 3.3)}$$

where e is called the *thermal emissivity*. Express e as function of λ, c, and μ . In what units is it expressed? How does the energy stored in the BTES evolve with time? How does the energy losses outside of the BTES scale with time?

7. Compare the diffusion coefficients of the storage layer (soil) and insulation layer (XPS), as well as their effusivities. What quantity caracterises a good thermal insulator?

8. Evolution of BTES storage: Fig. 7 shows the evolution over the years of the energy injected and extracted from the BTES storage (left), associated with the storage core temperature (right). Comment the shape of the two curves in the light of the calculations performed above.

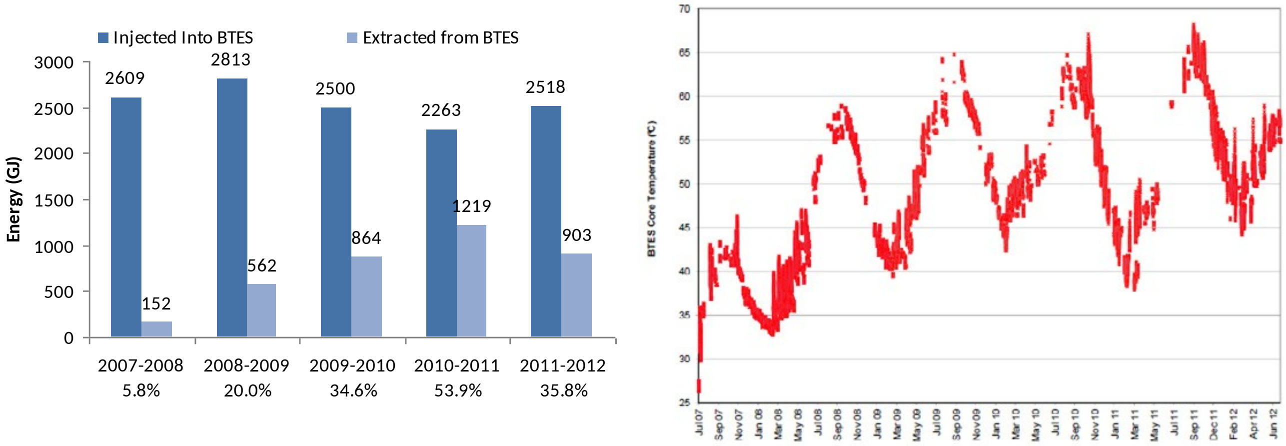

Figure 7: ***Left:*** *Energy injected into and extracted from BTES;* ***Right:*** *BTES core temperature. From B. Sibbitt et al, 2012.*

Part 4. Alternative Heat Pump heating (5 points)

In this last part, we will compare the heating system of Drake Landing to a more usual alternative based on a simple heat pump. In a heat pump, a refrigerant ($C_2H_2F_4$, usually called R134a) is operated to take heat for an external cold source (the typical winter temperature at Drake landing in $-10\,°C$) and to give heat to a hot source (assuming houses are kept at $19\,°C$).

1. Let's consider a the heat pump using R134a as refrigerant. The Pressure-Enthalpy diagram of R134a molecule is shown in Fig. 8. In order to get more familiar with this diagram,
   - 1.a. Indicate the regions where the fluid is under liquid phase, under vapor phase, and under a mixed phase.
   - 1.b. In regions where the fluid can be treated as ideal, how is an isothermal line supposed to look like? And in the wet steam region?
   - 1.c. Using the cycle for the R134a coolant, estimate the latent heat at 293K and comment this value.
2. Heat extraction: Using the cycle shown on Fig. 8:
   - 2.a. In which direction should be cycle be operated during summer to extract heat?
   - 2.b. Identify which transition corresponds to each of the following transformation :
     - Isentropic compression
     - Isentropic expansion
     - Isobaric heating
     - Isobaric cooling
   - 2.c. What is the relation between enthalpy and work over an isentropic transformation? What is the relation between enthalpy and heat over an isobaric transformation?
   - 2.d. Estimate the vapor fraction at point 4.
   - 2.e. Copy and fill the following table

	1	2	3	4
T (K)				
p (bar)				
h (kJ/kg)				
Phase				

   - 2.f. The heat pump efficiency is defined as the ratio between the heat provided by the fluid to the hot source and the work provided by the compressor onto the fluid. Estimate and comment the value of this efficiency.
   - 2.g. The real overall Coefficient of Performance for the heat pump (ie the ratio between the output heat to the electricity delivered to the system) is around 5. Where does the discrepancy with the previous estimate come from?
   - 2.h. Using a simple ground-air heat exchanger ("Canadian Well"), it is possible to increase the temperature of the air used as a cold source from -10°C to 0°C. Estimate quantitatively by how much would the CoP be improved by such a measure.

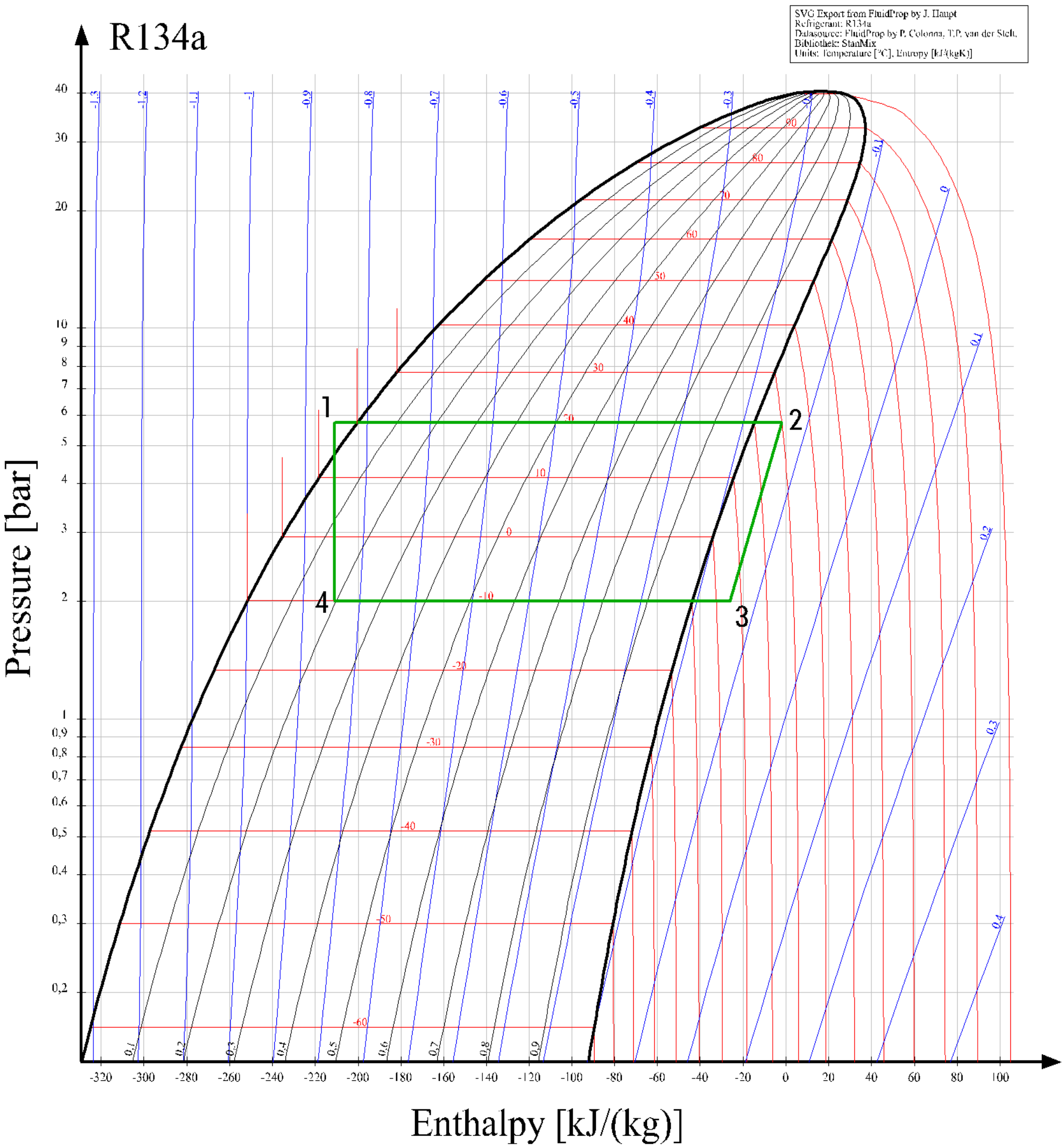

Figure 8: Pressure-Enthalpy diagram of R134a. Red lines represent isothermal transformations (temperature expressed in °C), blue lines represent isentropic transformations.

3. The total heating need of Drake Landing was 2360 GJ in 2016-2017, and the electrical consumption was 21 600 kWh. Comment on this value.
4. besides the higher CoP, what are the advantages of the Drake Landing installation as compared to a usual heat pump system?

Part 5. Constants relevant to the problem

Generic Energy Data

Ton Oil Equivalent	1 t.o.e. = 41 855×10^6 J
Sun temperature	T_{sun} = 5800 K
Sun Solid Angle	$\Omega_s = 6 \times 10^{-5}$ sr
Air density	ρ_{air} = 1.2 kg/m^3
Dry air heat capacity	c_{air} = 1,007 kJ/kg/K

Constants

Stefan constant	$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}
Gravity at Earth	g = 9.81 m s^{-2}
Avogadro's number	$N_A = 6.02 \times10^{23}$ mol^{-1}
Absolute Zero	T_0 = -273,15 °C
Planck constant	h = 6,626 0 755×10^{-34} J.s
Perfect gas constant	R = 8,314 J mol^{-1} K^{-1}
Electronvolt	1eV = 1.6×10^{-19} J

Borehole Heat Storage System

Storage shape	Cylindrical
Storage diameter	35 m
BTES Storage Volume	34 000 m^3
Max Storage Temperature	74 °C
Number of boreholes	144
Number of boreholes in series	6
Borehole depth	35 m
Borehole spacing	2.25 m
Borehole diameter	150 mm
Ground type	Soil
Thermal conductivity	2.0 W/m/K
Volumetric heat capacity	2.3 MJ/m^3/K)
Injection power	2 530 GJ/year
Extraction power	1 370 GJ/year
Charging temp	~50-65 °C
Insulation thickness	0,2 m
Insulation material	XPS (Extruded polystyrene foam)
Insulation thermal conductvivity	0.03 W/m/K
Insulation density	30 kg/m^3
Insulation heat capacity	1500 J/kg/K