Corrections

Part 1. Housing Energy Consumption (4 points)

1. Energy losses though the envelope.

We have $\vec{j}_Q = -\lambda \vec{\nabla} T \Rightarrow J_Q = \dfrac{\Delta T}{R}$.

Assuming a flat root, the wall surface is $S_w = (7+10) \times 5 \times 2 = 170\,\mathrm{m}^2$ and the roof surface $S_r = 7 \times 10 = 70\,\mathrm{m}^2$. More realistically the roof surface can be taken to be $S_r \sim 100\,\mathrm{m}^2$, and the wall surface could be increased a bit.

For the thermal resistances given in the subject and a wall and roof areas of respectively $\sim 170\,\mathrm{m}^2$ and $\sim 100\,\mathrm{m}^2$ one obtains:

	Month	January	February	March	April	May	June	July	August	September	October	November	December
	Average Temperature (°C)	-10,3	-6,3	-3,4	3,5	9,4	13,3	16	15,1	10,3	5,6	-2,4	-6,9
	Duration	31	28	31	30	31	30	31	31	30	31	30	31
Walls	J_Q (W/m²)	9,9	8,5	7,5	5,2	3,2	1,9	1,0	1,3	2,9	4,5	7,2	8,7
Walls	Power (W)	1677	1448	1282	887	549	326	172	223	498	767	1225	1482
Walls	Consumption (kWh)	1247,8	973,2	953,9	638,8	408,8	234,9	127,8	166,1	358,5	570,7	881,9	1103,0
Roof	J_Q (W/m²)	6,3	5,4	4,8	3,3	2,1	1,2	0,6	0,8	1,9	2,9	4,6	5,5
Roof	Power (W)	627,4	541,8	479,7	331,9	205,6	122,1	64,2	83,5	186,3	286,9	458,2	554,6
Roof	Consumption (kWh)	466,8	364,1	356,9	239,0	152,9	87,9	47,8	62,1	134,1	213,5	329,9	412,6

The annual energy losses through the envelope, ignoring the ground, amounts to $\sim 10\,500\,\mathrm{kWh/yr}$. This was obtained without taking into account the boundary layer. When adding 0,17 to the resistance to take that layer into account, the total consumption goes down to $\sim 10\,000\,\mathrm{kWh/yr}$.

2. Energy losses through the ventilation

Energy losses can be expressed as function of the mass flow $\dot{m}$: $P = (1-\eta) \times \dot{m} \times c \times \Delta T$

where $\eta = 0,8$ is the efficiency of the heat recovery system. Indeed, a fraction η of the heat available in the air is recovered, thus only a fraction $1-\eta$ needs to be brought by the heating system. The mass flow is taken to be 0,6 volume of the house per hour, the volume being approximately $V \sim 7 \times 10 \times 5 = 350\,\mathrm{m}^3$: $\dot{m} = 0,6 \times \rho_{\mathrm{air}} \times V / 1\,\mathrm{hr}$. With the values in the subject, one obtains the following losses:

Month	January	February	March	April	May	June	July	August	September	October	November	December
Power loss (W)	413,1	356,7	315,8	218,5	135,3	80,4	42,3	55,0	122,7	188,9	301,7	365,1
Consumption (kWh)	307,3	239,7	235,0	157,3	100,7	57,9	31,5	40,9	88,3	140,6	217,2	271,7

Total losses through the ventilation system: $\sim 1\,900\,\mathrm{kWh/yr}$

3. *Total energy needs:*

Compared to classical houses, the losses though the envelope are reduced by ~30%, and corresponds therefore to about $\sim 7\,000\,\text{kWh/yr}$. The total energy needs are about $\sim 10\,000\,\text{kWh/yr}$.

Part 2. Thermal Solar Collectors (8 points)

1. Power reaching the collector

The power reaching the collector is:

$$P_{\text{sun}} = w \times L \times f \times \frac{\Omega_s}{\pi} \times \sigma T_{\text{sun}}^4 \approx 179\,\text{W} \text{ (Eq. 2.1)}$$

Without concentration the temperator of the collector will be way too low to allow for a thermodynamical machine to work.

2. Coolant temperature

2.a. The absorbed power reads

$$P_A = a \times P_{\text{sun}} = a \times w \times L \times f \times \frac{\Omega_s}{\pi} \times \sigma T_{\text{sun}}^4 \text{ (Eq. 2.2)}$$

The radiated power is (under approximation of emissivity equal to absorbency):

$$P_R = a \times w \times L \times \sigma T_{\text{max}}^4 \text{ (Eq. 2.3)}$$

And the balance gives:

$$T_{\text{max}} = T_{\text{sun}} \left(f \times \frac{\Omega_s}{\pi} \right)^{1/4} = 354\,\text{K} \approx 81\,°\text{C} \text{ (Eq. 2.4)}$$

2.b. Coolant + fluid: The balance is now for a slice $[z, z + \mathrm{d}z]$:

$$\mathrm{d}P_{\text{cool}} = \mathrm{d}P_A - \mathrm{d}P_R = aP_{\text{sun}} \times \left(1 - \left(\frac{T_{\text{cool}}}{T_{\text{max}}} \right)^4 \right) \times \frac{\mathrm{d}z}{L} \text{ (Eq. 2.5)}$$

2.c. Distribution of temperature: the idea is to make an energy balance between times t and $t + \mathrm{d}t$ for a slice $[z, z + \mathrm{d}z]$:

The energy entering the control volume during $\mathrm{d}t$ can be expressed as:

$$\mathrm{d}U = P_{\text{cool}} \frac{\mathrm{d}z}{L} \times \mathrm{d}t \text{ (Eq. 2.6)}$$

Under assumption of equilibrium (permanent regime), this energy is transmitted to the fluid, leading to an increase of temperature:

$$\mathrm{d}U = \dot{m} \times \mathrm{d}t \times c \times (T(z + \mathrm{d}z) - T(z)) = \dot{m} \times c \times \frac{\mathrm{d}T}{\mathrm{d}z} \mathrm{d}z \, \mathrm{d}t \text{ (Eq. 2.7)}$$

Equating the two expressions, one obtains:

$$\dot{m} \times c \times \frac{\mathrm{d}T}{\mathrm{d}z} = a \times w \times f \times \frac{\Omega_s}{\pi} \times \sigma T_{\text{sun}}^4 \times \left(1 - \left(\frac{T_{\text{cool}}}{T_{\text{max}}} \right)^4 \right) \text{ (Eq. 2.8)}$$

Which can be rewritten as: $\dfrac{z_s}{T_{\text{max}}} \dfrac{\mathrm{d}T}{\mathrm{d}z} = 1 - \left(\dfrac{T(z)}{T_{\text{max}}} \right)^4$ (Eq. 2.9)

15

with:
$$z_s = \frac{\dot{m} \times c \times L \times T_{max}}{a\, P_{sun}} = \frac{\dot{m} \times c \times T_{max}}{a \times w \times f \times \dfrac{\Omega_s}{\pi} \times \sigma T_{sun}^4} \approx 21.4\,\text{m} \quad \text{(Eq. 2.10)}$$

2.d. We define $u = z/z_s$ and $y = T/T_{max}$. We then have

$$\frac{1}{T_{max}} \frac{dT}{dz} = \frac{1}{T_{max}} \frac{dT}{du} \times \frac{du}{dz} = \frac{1}{z_s} \times y(u) \quad \text{(Eq. 2.11)}$$

So that Eq. 2.9 can be rewritten simply as: $\dfrac{dy}{du} = 1 - y^4(u)$ (Eq. 2.12)

2.e. For $T_{max} = 354\,\text{K}$, thus $T(0)/T_{max} = 0.91$, which from the graph in Fig. 4 corresponds to $u_0 = 1.139$. Moreover $z_s = 21.4\,\text{m}$ (Eq. 2.10). One obtains then $u(L) = u_0 + L/zs = 1.256$ which corresponds to $y(L) = 0.942$. Finally one has: $T(L) = 334\,\text{K} = 61\,°\text{C}$
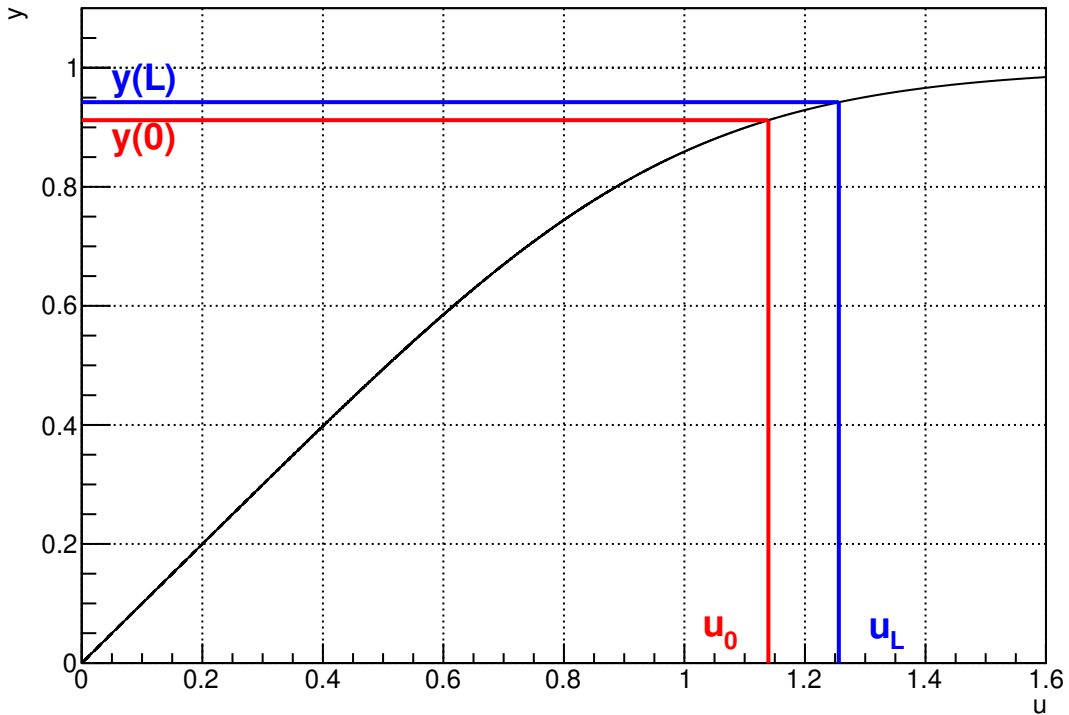
3. *Overall collection Efficiency*

3.a. Using the same energy balance as before, but taking into account the whole collector, the power given to the fluid is:

$$P_{cool} = (T(L) - T(0)) \times \dot{m} \times c \quad \text{(Eq. 2.13)}$$

To be compared to the incident solar power:

$$P_{sun} = w \times L \times f \times \frac{\Omega_s}{\pi} \times \sigma T_{sun}^4 \quad \text{(Eq. 2.14)}$$

The net effiency is the ratio of the two:

$$\eta_{cool} = \frac{P_{coll}}{P_{sun}} = \frac{(T(L) - T(0)) \times \dot{m} \times c}{w \times L \times f \times \frac{\Omega_s}{\pi} \times \sigma T_{sun}^4} = \ldots = a \frac{T(L) - T(0)}{T_{max}} \frac{z_s}{L} \quad \text{(Eq. 2.15)}$$

3.b. Assuming $a=1$, one obtains: $\eta_{coll} \approx 26\%$

4. *Total collected energy:*

The solar irradiance gives the total received energy per house:

$$P_{house}=5.45\,GJ/m^2/year \times w \times L=272\,GJ/house/year$$

Inserting the collection efficiency ($\eta_{coll} \approx 32\%$), we find a collected energy of $87\,GJ/house/year$. For the whole community (52 houses), this corresponds to $\sim 4500\,GJ/year$ for a litterature value of $\sim 4000\,GJ/yr$

Part 3. Inter-seasonal Thermal Heat Storage (8 points)

1. Dimensionning

The average ground temperature (avereged over the years using the values from Fig. 2) is $T_{avg}=3.6°C$. The total capacity is for 52 houses:

$$V\times c\times(T_{max}-T_{avg})=6000\,GJ$$

corresonding to one and half year of heat production. Note that a significant fraction of the heat stored during the summer is used during the next winter, to the whole storage will take several years to fully load (typically 3-4 years).

2. Diffusion equation

The diffusion equation reads:

$$\left.\begin{array}{l}\mu c\dfrac{\partial T}{\partial t}+\mathrm{div}\,\vec{j}_Q=0\\[2mm]\vec{j}_Q=-\lambda\vec{\nabla}T\end{array}\right\}\;\Rightarrow\;\dfrac{\partial T}{\partial t}=\dfrac{\lambda}{\mu c}\nabla^2 T,\quad D=\dfrac{\lambda}{\mu c}\quad\text{(Eq. 3.1)}$$

3. Dimensionless parameter

Dimentionless parameter: $u=\dfrac{x}{\sqrt{Dt}}$ (Eq. 3.2)

Propagation distance scales as the square root of the time. For the values given in the table, one obtains $D=8.7\times10^{-7}\,m^2/s$ and the following diffusion time curve:
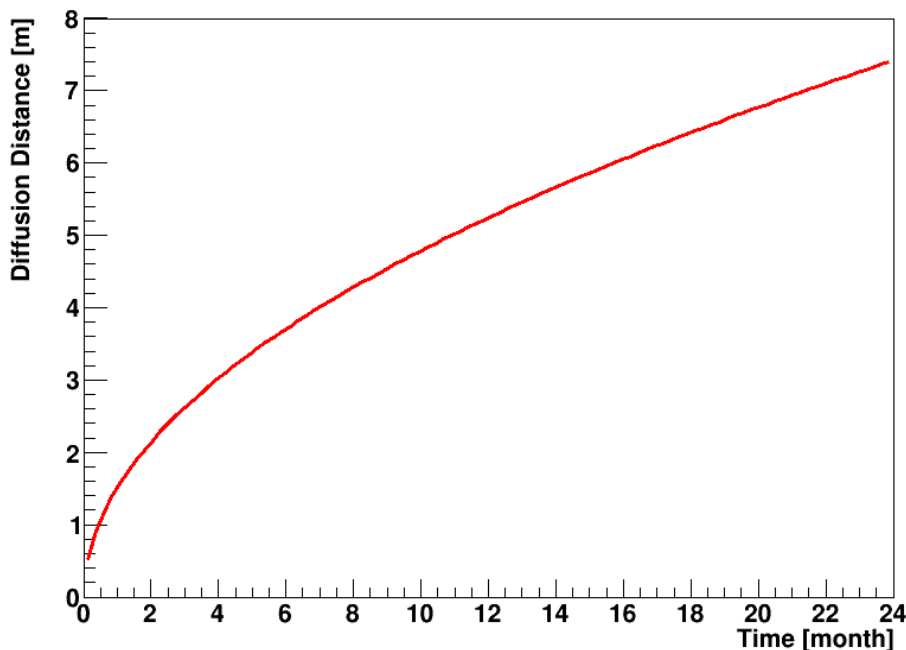

Figure 9: Diffusion distance as function of time

Orders of magnitudes

The following conclusions can be derived:

- In order for the temperature to be ~ homegeneous in the borehole after a few

monthes, the distance between the boreholes should be of the order of ~2 m

- The heat does not really leak out of the storage in the first years (the diameter of the storage is ~ 35 m)
- Boreholes should be > 6 m from the edges of the borehole so that heat stays inside
- For one day, the diffusion distance is only ~20 cm, so that the temperature during the charging time is strongly inhomogeneous

4. Homogeneous equation

$$\left.\begin{array}{l}\dfrac{\partial T}{\partial x}=\dfrac{\partial T}{\partial u}\dfrac{\partial u}{\partial x}=\dfrac{1}{\sqrt{Dt}}\dfrac{\partial T}{\partial u}\\[2mm]\dfrac{\partial T}{\partial t}=\dfrac{\partial T}{\partial u}\dfrac{\partial u}{\partial t}=\dfrac{-u}{2t}\dfrac{\partial T}{\partial u}\end{array}\right\}\quad\Rightarrow\quad\dfrac{\partial^2 T}{\partial u^2}=-\dfrac{u}{2}\dfrac{\partial T}{\partial u}\quad\text{(Eq. 3.3)}$$

5. Homogeneous solution

Solution:

$$g'=-\frac{u}{2}\times g\quad\Leftrightarrow\quad\frac{\mathrm{d}g}{g}=-\frac{u\,\mathrm{d}u}{2}\quad\Leftrightarrow\quad g(u)=C\times\exp\left(-\frac{u^2}{4}\right)\quad\text{(Eq. 3.4)}$$

The temperature is then, after inserting the proper boundary conditions:

$$T(u)=T_0+C\times\int_0^u\exp\left(-\frac{v^2}{4}\right)\mathrm{d}v=T_0+(T_1-T_0)\times\mathrm{erf}\left(\frac{u}{2}\right)=T_0+(T_1-T_0)\times\mathrm{erf}\left(\frac{-x^2}{4Dt}\right)\quad\text{(Eq. 3.5)}$$

6. Heat flux and thermal effusivity:

The heat flux is:

$$j_Q=-\lambda\frac{\partial T}{\partial x}=-\lambda\frac{\partial T}{\partial u}\frac{\partial u}{\partial x}=\frac{-\lambda}{\sqrt{Dt}}(T_1-T_0)\exp\left(\frac{-x^2}{4Dt}\right)\quad\text{(Eq. 3.6)}$$

In particular, the heat extracted from the tubes ($x=0$) decreases with time as:

$$j_Q(x=0)=\frac{\lambda}{\sqrt{Dt}}(T_0-T_1)=\sqrt{\lambda\mu c}\frac{(T_0-T_1)}{\sqrt{t}}=e(T_0-T_1)\quad\text{(Eq. 3.7)}$$

The term $e=\lambda/\sqrt{D}=\sqrt{\lambda\mu c}$ is called **thermal effusivity**, and characterises the ability of one body to exchange energy.

The energy stored in the BTES is the integral over time of the heat flux. It varies as:

$$Q=2e(T_0-T_1)\times\sqrt{t}\quad\text{(Eq. 3.8)}$$

7. For the soil and the insulation layer, we obtain the following values:

		Soil	XPS
Thermal Conductivity	(W/m/K)	2	0,03
Massive heat capacity	(kJ/kg/K)		1,5
Density	(kg/m^3)		30
Volumic heat capacity	(kJ/m^3/K)	2300	45
Diffusion Coefficient	(m^2/s)	8,70E-07	6,67E-07
Effusivity	(J/K/m^2/s$^{1/2}$)	2144,8	36,7

The values of the diffusion coefficients are pretty similar, but the effusivity is a factor 60 smaller in the insulation layer. A high value of thermal effusivity indicates that the medium can absorb a large heat content without changing its temperature. A low value indicates that the temperature will change rapidly when two bodies are put in contact. The diffusion coefficient only indicates how fast the equilibrium is reached: for XPS insulation, the thermal equilibrium is reached rather fast, but the amount of energy going through the insulator is 60 times less than that going through the same thickness of soil.

8. Various comments:

- Energy injected into the BTES seems to be rather stable with time, instead of square root. As we reach equilibrium, this is more or less expected, but in the transition phase the injected energy should scale as square root of time

- Energy extracted from the BTES rises during the first years, as the total energy loaded in the BTES is too low: the system needs several years to operate in a stable manner.

- Average temperature rises with time and becomes ~ rather stable after 3-4 years: this is the time needed to fully load the BTES.

- Annual oscillations of temperature correspond to the loading/unloading cycle, whereas the average trend corresponds to the multi-year loading

Part 4. Alternative Heat Pump heating (5 points)

1. Pressure-enthalpy diagram

    1.a. different sides of the solid line

    1.b. For an ideal gas: $dH = CdT$ inpependantly of pressue. Isothermal lines are vertical in single phase regions. In phase transition regions, temperature is also constant.

    1.c. The horizontal lines give the enthalpy differences. One obtains about $200\,\text{kJ/kg}$, which is about 10 times less than water.

2. Heat Extraction

    2.a. The goal is to extract heat from the cold source and give it to the hot one. The cycle needs to be operated in the direct (anti-clockwise) direction, from 2 to 1.

    2.b. Individual transformations:

- Isentropic compression: 3 → 2
- Isentropic expansion: 1 → 4
- Isobaric heating: 4 → 3
- Isobaric cooling: 2 → 1

    2.c. Relation between enthalpy & work:

$$\Delta H = W + \underset{=0}{\underbrace{Q}} = W \quad \text{(Eq. 4.1)}$$

    2.d. Vapor fraction at point 4:

A constant enthalpy evolution corresponds to a vertical line on the diagram. One has during a phase change:

$h = (1-x) \times h_{\text{liquid}} + x \times h_{\text{gas}}$ where x is the gaz fraction. Here, with $h = -210\,\text{kJ/kg}$, $h_{\text{liquid}} = -250$ and $h_{\text{gas}} = -40$, one finds $x = 0.19$, compatible with the value read out directly from the graph.

    2.e. Filled table:

	1	2	3	4
T (K)	15°C	30°C	5°C	-10°C
p (bar)	5.8	5.8	2	2
h (kJ/kg)	-210	-2	-25	-210
Phase	Liquid	Gaz	Gaz	20% gaz, 80% liquid

    2.f. Work and heat are read horizontally, from the enthalpy difference for each transformation:

$$W(3 \rightarrow 2) = 23\,\text{kJ/kg}, \quad Q(2 \rightarrow 1) = 208\,\text{kJ/kg}, \quad \eta = \frac{Q(2 \rightarrow 1)}{W(3 \rightarrow 2)} \approx 9.0 \quad \text{(Eq. 4.2)}$$

The Carnot efficiency for the same temperatures would be:

$$\eta_{\text{Carnot}} = \frac{T_H}{T_H - T_L} = \frac{293}{293 - 263} = 9.7 \quad \text{(Eq. 4.3)}$$

2.g. In the previous question, we counted to work effectively delivered to the fluid. Efficiency of the compressor ~0.4 – 0.7 has to be taken into account in the overall efficiency.

2.h. Raising the curve from -10°C to 0°C reduces the compressing work (3→2). As a crude approximation, the temperature difference is scaled by 2/3, so is the work. The CoP is increased by roughly 3/2, reaching a maximum of ~13.5 and a realistic value around 7~8.

3. 21 000kWh = 77 GJ. CoP : 2360/77= 30.

4. Coolants used in heat pumps such as R134a are toxic for the ozone layer and strong greenhouse gases (factor 1430 more than CO_2 for R134a). In contrast, Drake Landing uses only water.