

PHY 555 Energy and environment

Final exam Friday, December 11th 2020

Lecture notes allowed, duration 3h

- Note 1: All parts of the problem are independent, and results given in previous questions can be used without being demonstrated.
- Note 2: Quality of the redaction of justification of calculation will be taken into account in the evaluation. Answers can be written in French or in English.
- Note 3: The physical constants needed in the problem are grouped in tables at the end of the subject.
- Note 4: You don't need to do everything to have 20/20 ! Start with the topics you feel most familiar with.
- Note 5: There is no need to copy the question text. Just identify them by their numbers.
- Note 6: Lecture notes and small class documents (notes and slides) are allowed as well as French-English dictionary. No other document is allowed.
- Note 7: Documents mentioned above might be consulted in a digital form on a tablet/computer, but the device (and other devices with connection capabilities, such as phones, calculator, ...) must be put in flight mode. No Wifi connection is allowed.

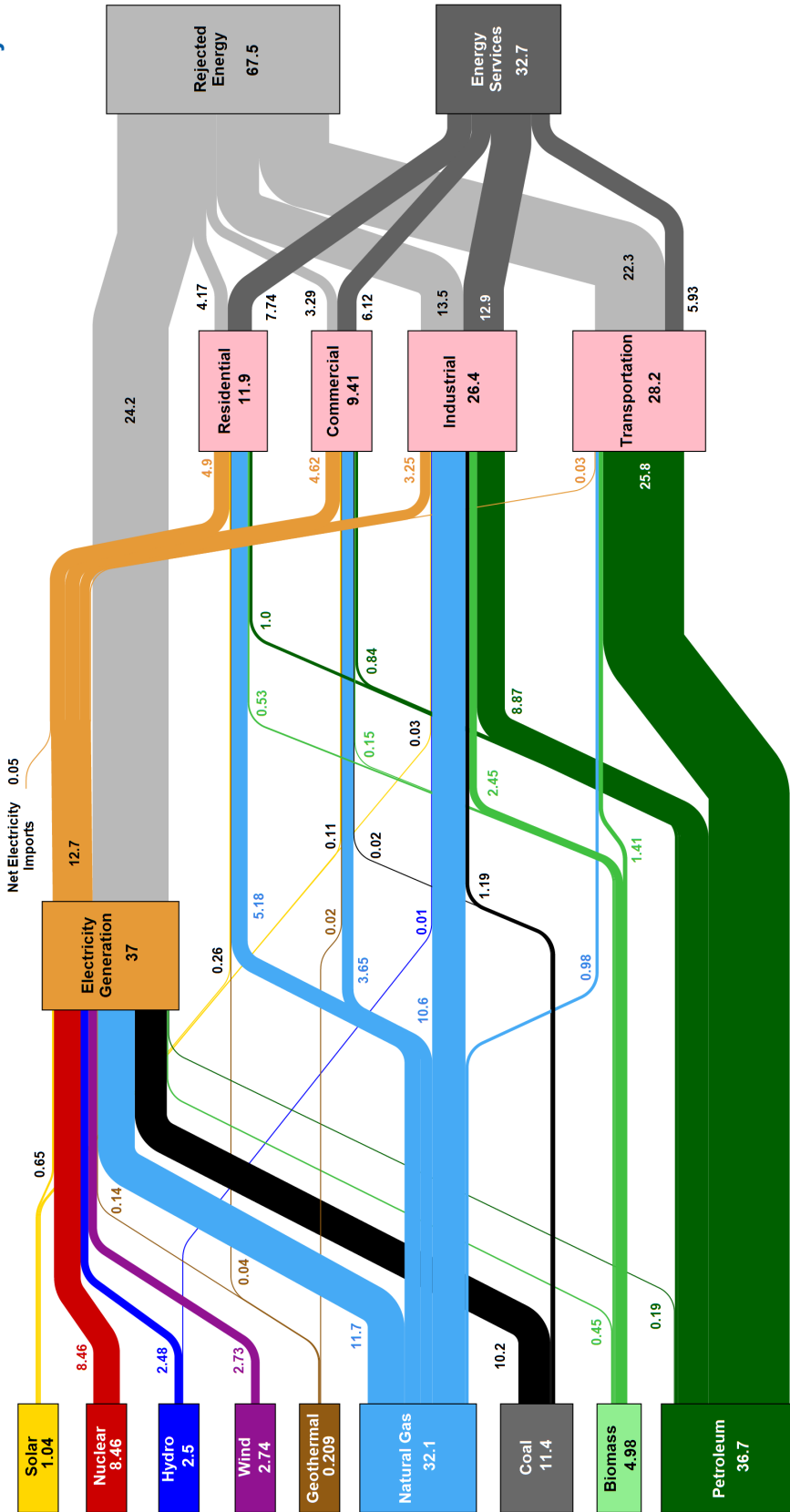
1 Energy balance

The Sankey graph in Figure 1 shows the primary, final and useful energy consumed in the United States over the year 2019. Values are given in *Quad*. 1 Quad is defined as a quadrillion of British Thermal Unit (BTU), 1 BTU being the imperial equivalent of a calorie (ie the energy required to increase the temperature of one pound of water by one degree Fahrenheit). In decent units, 1 Quad roughly equal to 10^{18} J or 25 Mtoe.

Estimate and **comment** the following quantities (half of the points will be attributed for the correct numerical values, the other half on relevant comments) :

1. Average conversion efficiency of power plants
2. Share of renewable sources in the electrical mix
3. Average car engine efficiency
4. Average final energy per capita
5. Surface occupied by solar panels
6. CO₂ emission from fuel combustion, in MtCO₂, considering that 25% of oil is not actually consumed, but exported.

Estimated U.S. Energy Consumption in 2019: 100.2 Quads



Source: LBNL March, 2020. Data is based on DOE/EIA MER (2019). If this information or a reproduction of it is used, credit must be given to the Lawrence Livermore National Laboratory. The data is based on the Energy Information Administration's (EIA) Energy Flow Accounts (EFA) reports consumption of renewable resources (i.e., hydro, wind, geothermal and solar) for electricity in BTU-equivalent values by assuming a typical fossil fuel plant heat rate. The efficiency of electricity production is calculated as the total retail electricity delivered divided by the primary energy input into electricity generation. End use efficiency is estimated as 65% for the residential sector, 65% for the commercial sector, 21% for the transportation sector and 49% for the industrial sector, which was updated in 2017 to reflect DOE's analysis of manufacturing. Totals may not equal sum of components due to independent rounding. LBNL-MI-410527

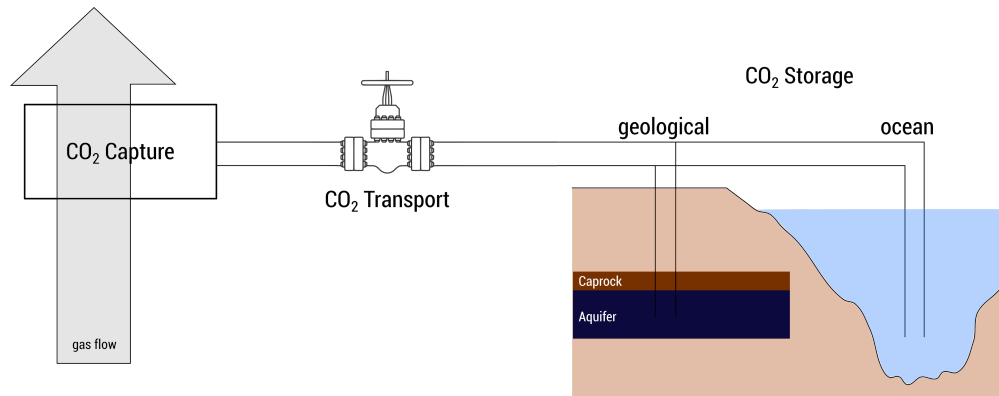
Figure 1: Estimated U.S. energy consumption in 2019, Lawrence Livermore National Laboratory.

2 Problem: CO₂ capture and storage

As the concentration of CO₂ increases in the atmosphere, strategies evolve to attenuate climate change and prevent global warming for reaching values above +2°C as compared to the pre-industrial period. According to the special report of the IPCC on a “Global Warming of 1.5 °C”, it will not be sufficient to reduce emissions, and active CO₂ capture will be mandatory:

“All pathways that limit global warming to 1.5°C with limited or no overshoot project the use of carbon dioxide removal (CDR) on the order of 100–1000 GtCO₂ over the 21st century. CDR would be used to compensate for residual emissions and, in most cases, achieve net negative emissions to return global warming to 1.5°C following a peak (high confidence).” (IPCC, SR15, SPM (2019)).

Several approaches are considered to achieve CDR. In this problem, we will focus on Carbon Capture and Storage (CCS) and investigate the three main steps of the process: the selective removal of CO₂ from a gasflow, the transportation of liquified CO₂ and the sequestration of CO₂ either in the ocean or underground.



2.1 Orders of magnitude

1. Knowing that the atmospheric pressure at sea level is 1 bar ($=10^5$ Pa), show that the mass of the atmosphere is around 5×10^{18} kg.
2. Estimate the number of air moles in the atmosphere, considering that the air is essentially di-nitrogen and di-oxygen molecules.
3. In the pre-industrial period, carbon dioxide represented 280 ppm (parts per million) of the atmosphere composition. Estimate the carbon content of the atmosphere. Express your answer in gigatonne of carbon GtC (be careful not to confuse GtC with GtCO₂ !)
4. The total anthropic CO₂ emissions until 2018 are estimated to 2200 GtCO₂. By the end of 2017, carbon dioxide concentration raised to 405 ppm. Estimate the fraction of carbon from anthropic emissions that has been removed from the atmosphere. Comment the result.
5. One of the main natural carbon sink is the Ocean. Assuming that the first ~ 100 m of the Ocean are in chemical equilibrium with the atmosphere and considering the vapour pressure diagram provided in appendix, estimate the concentration in g/m³ of CO₂ dissolved in surface waters.
6. Evaluate the amount of CO₂, in GtCO₂, currently stored in the surface waters of the Ocean.
7. Comment on the evolution of this storage in the context of climat change.

2.2 CO₂ capture

In this section, we will estimate the energy required to extract a mole of CO₂ directly from a gas stream. All gases will be considered as ideal.

1. Consider N moles of an ideal gas undergoing an isothermal compression from pressure p_i to p_f . Show that the entropy of the system changes by

$$\Delta S = NR \log \frac{p_i}{p_f} \quad (1)$$

where $R = 8.314 \text{ J/K/mol}$ is the ideal gas constant.

2. Consider two compartments with identical temperature T and pressure p , containing initially N_1 moles of gas 1 and N_2 moles of gas 2 respectively. The wall between the compartments is opened, and gases can mix. Estimate the entropy created in the process.
3. Estimate the minimal amount of work required to separate a mixture of N_1 moles of gas 1 and N_2 moles of gas 2 into two separate pure systems at constant temperature T and pressure p .
4. Consider a gas containing a molar fraction x of CO_2 . In the low concentration regime, show the minimal energy $W_{1 \text{ mole CO}_2}$ required to extract 1 mole of CO_2 from such a gas can be written as:

$$W_{1 \text{ mole CO}_2} \simeq -RT (\log x - 1) \quad (2)$$

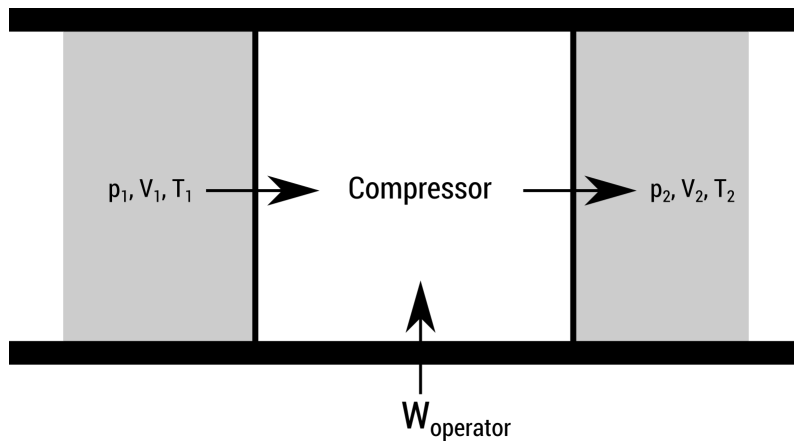
5. Direct Air Capture consists in capturing CO_2 from the ambient atmosphere.
 - (a) Using previous results, evaluate in kWh the minimal energy W_0 required per kilogram of captured CO_2 from ambient air.
 - (b) Real processes for CO_2 capture require much more work than the thermodynamic limit evaluated in the previous question. We denote η the ratio between the thermodynamic limit W_0 and the quantity of energy actually required to extract a kilogram of CO_2 . Furthermore, the energy used to capture the CO_2 might produce emission itself, which have to be taken into account in the balance. Considering that this energy is provided by a power source emitting I tons of CO_2 per kWh, estimate the energy cost per net kilogram of captured CO_2 .
 - (c) The following table is reproduced from "Economic and energetic analysis of capturing CO_2 from ambient air" by K. House et al, PNAS, vol. 108 (2011). Considering an efficiency η as small as 0.05, evaluate the cost of the required for air capture in \$ per t CO_2 avoided for each of these technologies.

	Conventional coal	Natural gas-fired combined cycle	Advanced nuclear	Biomass	Hydro
Cost [\$/MWh]	95	84	110	107	115
CO_2 intensity [t CO_2 /MWh]	1	0.4	0	0.2	0

6. Rather than Direct Air Capture, it is also possible to capture CO_2 in the exhaust gases of a combustion process. Consider for instance a combustion process under atmospheric conditions, assuming every dioxygen molecule reacts with a carbon atom to form a CO_2 molecule. Using previous results, estimate the energy required to isolate 1 kg CO_2 from the exhaust gas of this reaction. Comment this result.

2.3 CO_2 Transport

To transport the pure CO_2 from the capture site to the storage site, the gas has to be turned into a condensed phase, which requires a compression step. To describe this process, we consider the compressor as an open system (see figure below). A volume V_1 of gas is injected at pressure p_1 and temperature T_1 and a volume V_2 is extracted at pressure p_2 and temperature T_2 . This process is considered adiabatic and reversible.



1. Evaluate the work W_{operator} that has to be provided to the compressor in order to perform a compression from p_1 to p_2 for 1 kg of CO_2 . Express your answer as a function of T_1 , p_1 , p_2 and the adiabatic factor γ .
2. A first option is to compress directly CO_2 from atmospheric pressure to 11 MPa. Estimate the energy required for this process. In which state is the CO_2 at the end of the process ?
3. A second option is to compress the CO_2 to 20 bar at 25°C , then to cool it down until it liquifies. Evaluate the energy $W_{2,\text{compress}}$ required to to compress 1 kg CO_2 from 1 bar to 20 bar. Estimate the amount heat to be then removed to cool down and liquify the system. Deduce the minimal amount of work $W_{2,\text{cool}}$ for this stage, and express the total energy W_2 required for the second option.
4. Comment on these two options.
5. Under liquid form with a viscosity η , the CO_2 is now injected in a pipeline of radius a to be transported to the storage site. We note $\mathbf{v} = v(r, \theta, z, t)\mathbf{u}_z$ the fluid velocity field, assumed to be monodirectional. For the sake of simplicity, we will treat CO_2 as an incompressible fluid. We remind the Navier-Stokes equation

$$\rho \left(\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \text{grad}) \mathbf{v} \right) = -\text{grad}p + \eta \Delta \mathbf{v} \quad (3)$$

- (a) Performing a simple balance, show that the velocity field and specific mass of the fluid satisfy the continuity equation:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = 0 \quad (4)$$

- (b) Show that the stationary velocity field depends only on r .
(c) Show that the pressure field depends only on z , and decreases linearly over the pipeline length
(d) Deduce that the velocity field follows the Poiseuille's law:

$$v_z(r) = \frac{1}{4\eta} \frac{\partial p}{\partial z} \times (r^2 - a^2) \quad (5)$$

- (e) Show that the volumic flow rate Q follows de Hagen-Poiseuille's law:

$$Q = -K \frac{\partial p}{\partial z} \quad (6)$$

and give the expression of K .

- (f) Evaluate the power dissipated when maintain a volumic flow rate Q in a pipeline of radius a and length L . Comment your result.

2.4 CO₂ storage

In this last section, we consider two approaches for permanently storing the captured CO₂ : either underground, or deep in the Ocean. In both cases, for simplicity, we will consider CO₂ to be immiscible in water.

2.4.1 Geological storage

CO₂ can be stored in a geological storage. To do so, the CO₂ is injected in a aquifer, a permeable rock initially filled with water, covered by an impermeable layer (cap rock) which prevents CO₂ from diffusing back to the surface. Special care has to be given not to fracture the cap rock, which can happen for pressures exceeding 200 bar.

In this section, we consider a geological storage at a depth 1500m, with an average pressure of $p_0 = 150$ bar. We will estimate the mechanical constraints resulting from the injection of a volume $V_{\text{CO}_2} = 10^9 \text{ m}^3$ of CO₂ in a closed aquifer of $V_0 = 5 \times 10^{11} \text{ m}^3$.

1. At high pressures, water and rocks can be considered as compressible, with a compressibility defined for a pure body as:

$$\chi = \frac{1}{\rho} \frac{\partial \rho}{\partial p} \quad (7)$$

where ρ is the specific mass and p the pressure. We note ρ_{w0} and ρ_{r0} the specific mass at pressure p_0 of water and rocks respectively. Show that for pressure close to p_0 , the specific mass changes linearly with the pressure increase.

2. Considering a compressibility for water of $\chi_w = 5 \cdot 10^{-10} \text{ Pa}^{-1}$ and for rocks of $\chi_r = 4 \cdot 10^{-10} \text{ Pa}^{-1}$, and a porosity (ie a volumic fraction of water in the aquifer) of $\phi = 20\%$, what is the average pressure reached in the geological formation once the whole volume of CO₂ is injected ? Comment this value.
3. We admit that the velocity field of water in a porous medium obeys Darcy's law

$$\mathbf{v} = -\frac{k}{\eta} \nabla p \quad (8)$$

where $k = 0.1 \text{ m}^2$ is the permeability of the medium and $\eta = 10^{-3} \text{ Pa.s}$ is the viscosity of the fluid. Adapting the continuity equation eq.(4) to the current problem and considering only small pressure variations, show that the pressure field follows the equation

$$\frac{\partial p}{\partial t} = \frac{k}{\bar{\chi} \mu} \Delta p \quad (9)$$

Give the expression of the quantity $\bar{\chi}$ and comment the result.

4. The injection rate in the well is adjusted such that the pressure 1m away from the injection point is kept at 160 bar throughout the process. What is the typical it will take for pressure to increase significantly 1km away from the injection point ?

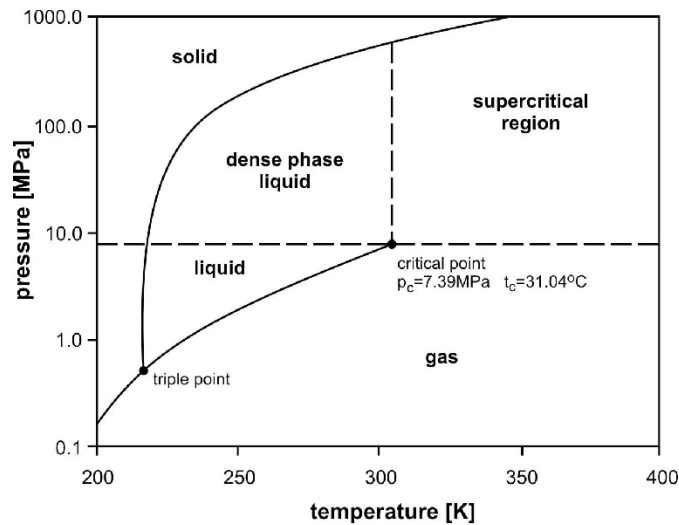
2.4.2 Ocean storage

Finally, we consider the possibility of storing liquid CO₂ in the depth of the Ocean.

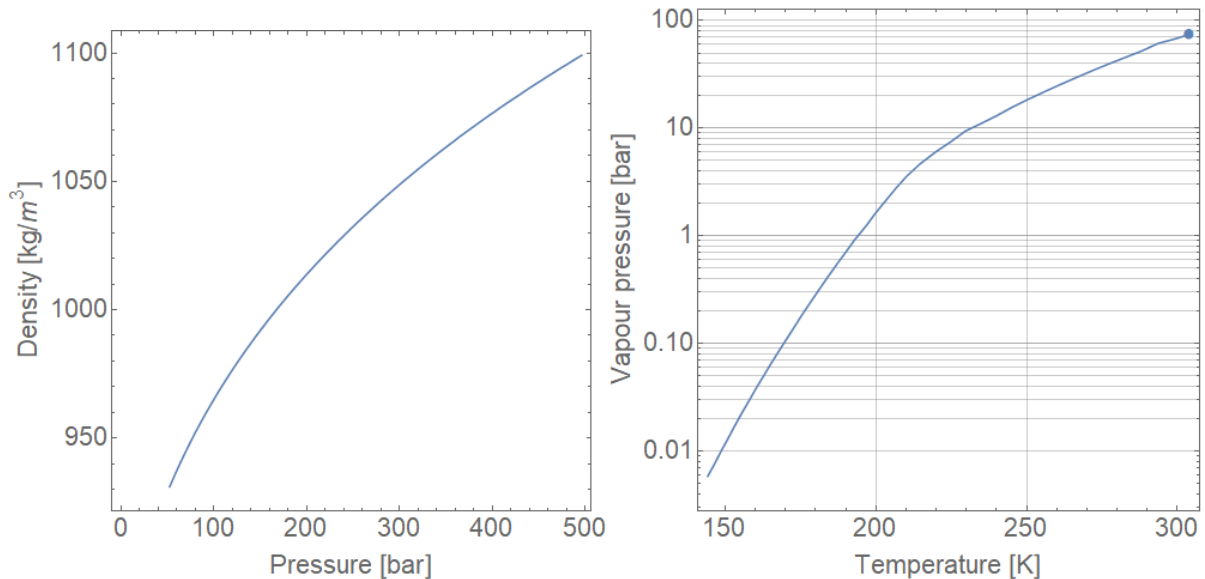
1. Considering the vapour pressure diagram for CO₂ provided in appendix, estimate the depth below which CO₂ can naturally exist as a liquid.
2. In reality, Ocean storage projects inject CO₂ at depth below 3000m. Comment this observation, based on the previous question and the documents provided in appendix.

Appendix

- Earth radius : 6 400 km
- Population in the US : 330 million
- Emission factors in kgCO₂ per TJ : Natural Gas = 56 100, Crude Oil = 73 300, Coal = 94 600, Wood = 112 000 [ref 2006 IPCC Guidelines for National Greenhouse Gas Inventories]
- Avogadro number : $\mathcal{N} = 6 \times 10^{23} \text{ mol}^{-1}$. Molar mass for dinitrogen $m_{\text{N}_2} = 28 \text{ g/mol}$, dioxygen $m_{\text{O}_2} = 32 \text{ g/mol}$, water $m_{\text{H}_2\text{O}} = 18 \text{ g/mol}$ and carbon $m_{\text{C}} = 12 \text{ g/mol}$.
- Thermodynamic quantities of CO₂
 - Latent heat at -20°C : 330 kJ/kg
 - Heat capacity : at constant pressure $C_p = 37.1 \text{ J/mol/K}$, at constant volume $C_v = 28.8 \text{ J/mol/K}$
 - Phase diagram :



- Density and vapour pressure :



- Vector analysis in cylindrical coordinates,

$$\text{grad}f = \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \mathbf{e}_\theta + \frac{\partial f}{\partial z} \mathbf{e}_z \quad (10)$$

$$\text{div} \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \quad (11)$$

$$\Delta(f(r)) = \frac{1}{r} \frac{d}{dr} \left(r \frac{df}{dr} \right) \quad (12)$$

$$\text{div}(f \mathbf{A}) = f \text{div} \mathbf{A} + (\mathbf{A} \cdot \text{grad}) f \quad (13)$$

$$(\mathbf{A} \cdot \text{grad}) \mathbf{B} = \left(A_r \partial_r + \frac{A_\theta}{r} \partial_\theta + A_z \partial_z \right) \mathbf{B} \quad (14)$$