École Polytechnique
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PHY 555 – Energy and Environment

PC7 – Wind Energy

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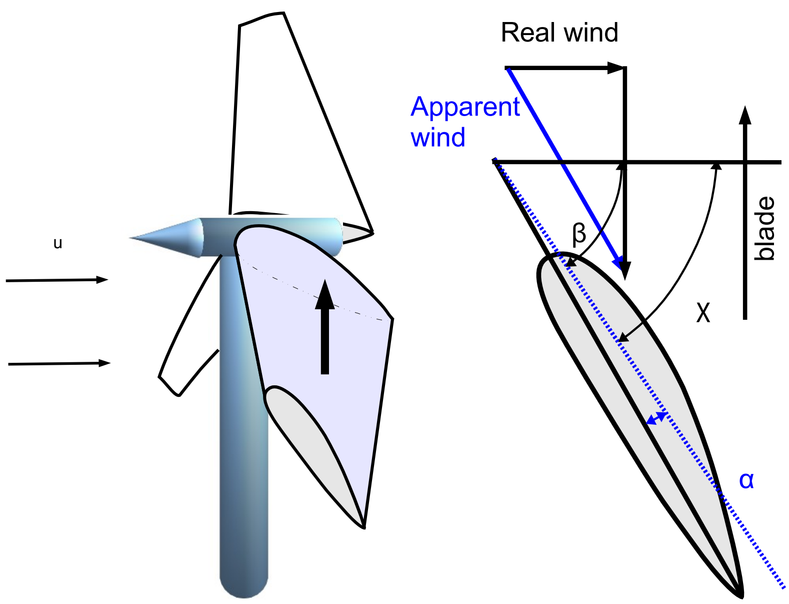
1 – Ideal wind turbine: the Betz limit

Simple physical considerations allows determining the maximum fraction of the wind power a turbine can extract when it is rotating in a plane perpendicular to the wind (windmill, modern turbines). The actual wind turbine never reach this limit, called the Betz Limit (1919).

1. Let's consider a steady flow of air through the rotor. Air is assumed to be a homogeneous and incompressible fluid. The rotor is modelled as a uniform disk that turns the wind thrust into rotational energy (without specifying how). Why is it impossible to extract all the wind power? (Help:. Consider a cylindrical tube of the same diameter enclosing the rotor.)
2. Calculate the thrust on the rotor as a function of the upstream wind speed $v_1 = u$ and the downstream wind speed v_2 far away from the rotor, and connect it to the pressure difference across the rotor. (Far from the rotor, one can consider the upstream and downstream pressures in the wind identical to the atmospheric pressure.) What is the wind speed immediately in front and behind the rotor?
3. Expressing the wind power that is transferred to the turbine and the wind thrust (normal to the rotor) as function of the speed ratio $a = v_2 / v_1$ and v_1. Determine the maximal fraction c_P of wind power that can be extracted by a wind turbine can extract from the wind. Then determine the thrust on the rotor.

2 – Blade Shape of an ideal turbine, torque and power

1. Recall the expression for the lift force F_L and drag force F_D on a wing (air-plane wing, turbine blade) with an aerodynamic profile. How do these forces vary with the angle of attack α (angle



between the blade and the wind direction). What limits the maximum lift?

2. Express the tangential $\mathrm{d}F_T(r)$ and normal $\mathrm{d}F_N(r)$ forces acting on a surface element $c \times \mathrm{d}r$ of the blade located at a distance r from the axis of rotation as function of the torsion angle of the blade $\beta(r)$ formed by the relative wind and the chord c of the element of the blade profile. (The apparent wind angle on the blade at a distance r from the axis will be noted χ. The "chord" c is the width of the blade at a distance r)
3. We define the ratio of blade tip speed λ (tip-speed ratio) as the ratio between the speed of the tip of the blade and that of the true wind. This amount is generally limited by considerations of interaction between the blades (turbulent streaks) and noise. (For modern fast bladed wind turbines λ varies between 6 and 10). For a true wind velocity u, is it preferred to maintain the angle of attack α constant over the entire length of the blade. How should the twist angle β of the blade vary with the distance r to the axis r?
4. We now consider that the rotor is an ideal wind turbine that reaches the Betz limit. That is to say, all the thrust on the rotor is fully converted into rotational energy, and this for every element of the blade. Then the normal force on each element of the blade must be equal to the wind thrust on a corresponding ring of radius r and a width $\mathrm{d}r$ of the rotor. Determine the optimum width of the blade (chord) versus r. (It will be placed under conditions in which we can neglect the drag force.) Calculate the torque exerted by the blades on the rotor axis, then the transmitted power. Is the result surprising?
5. Calculate the optimal shape of the blades, still keeping a constant angle of attack, but taking into account the effect of the drag force. Calculate the total torque and show that the power is equal to $P_{\text{ideal}} \times (1 - \lambda . C_D / C_L)$.