PC 7
Wind Energy

PHY 555 – Energy & Environment

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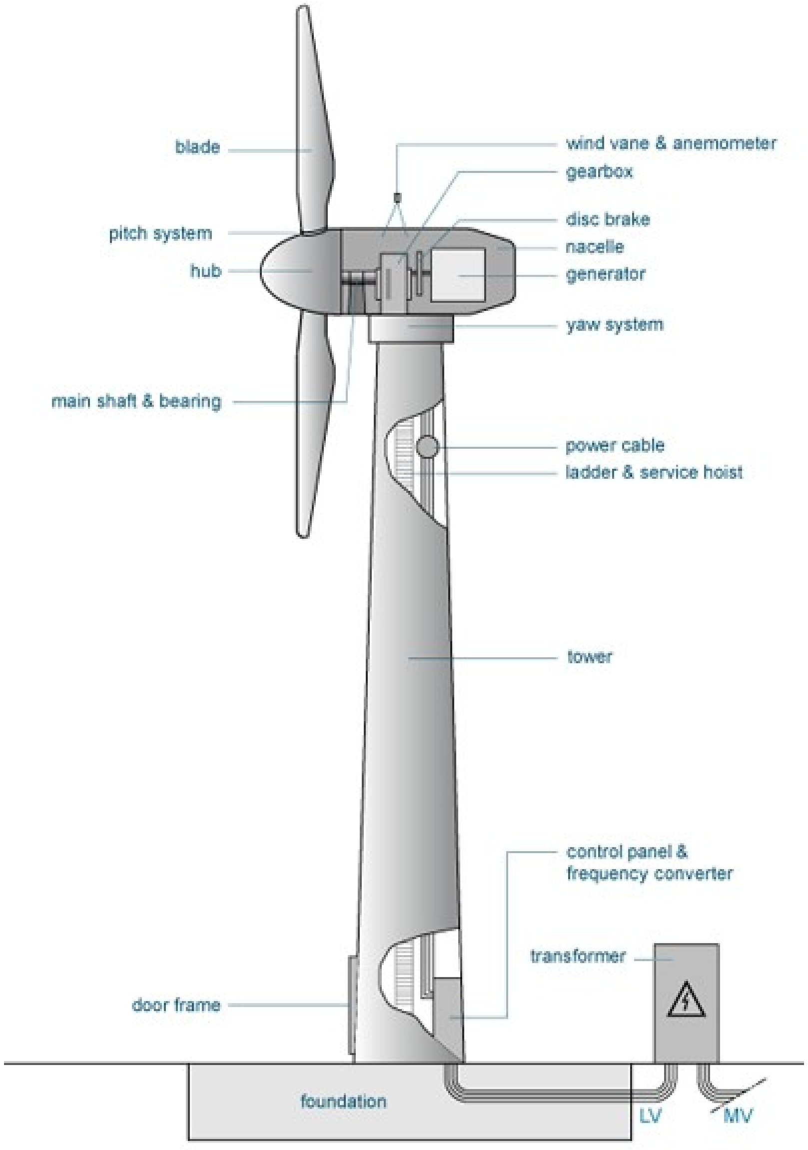
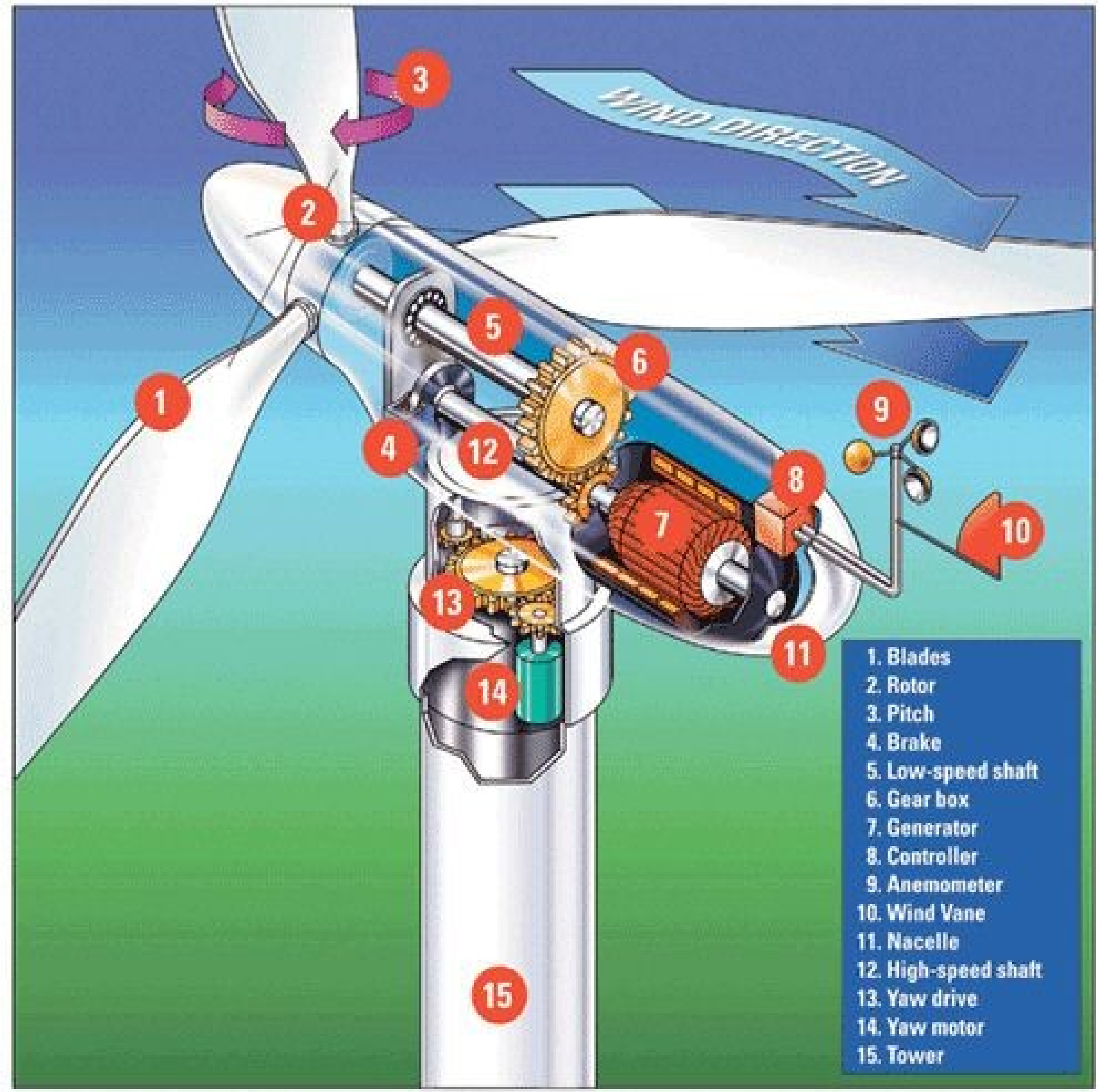
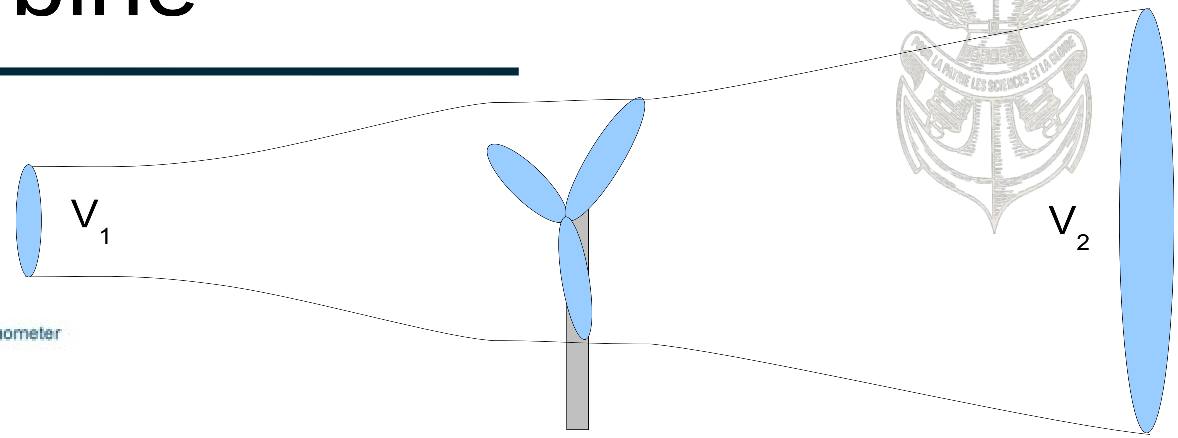
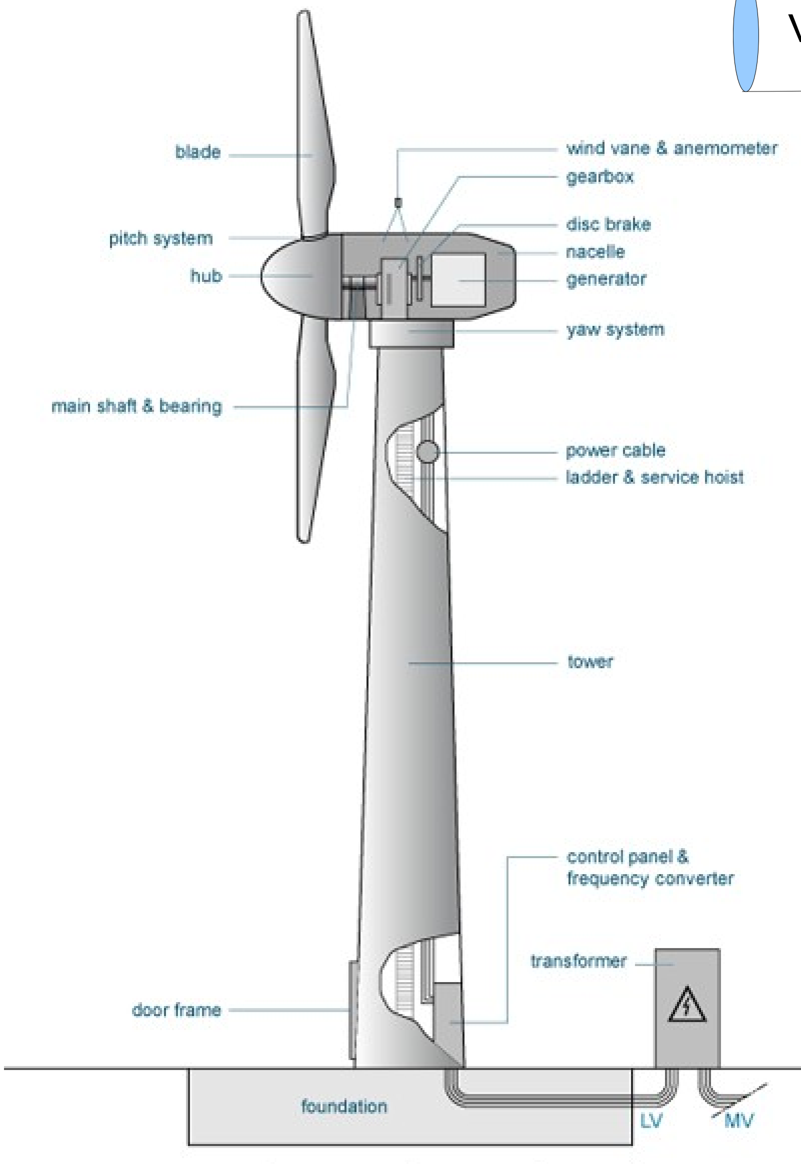
Horizontal Axis Wind Turbine

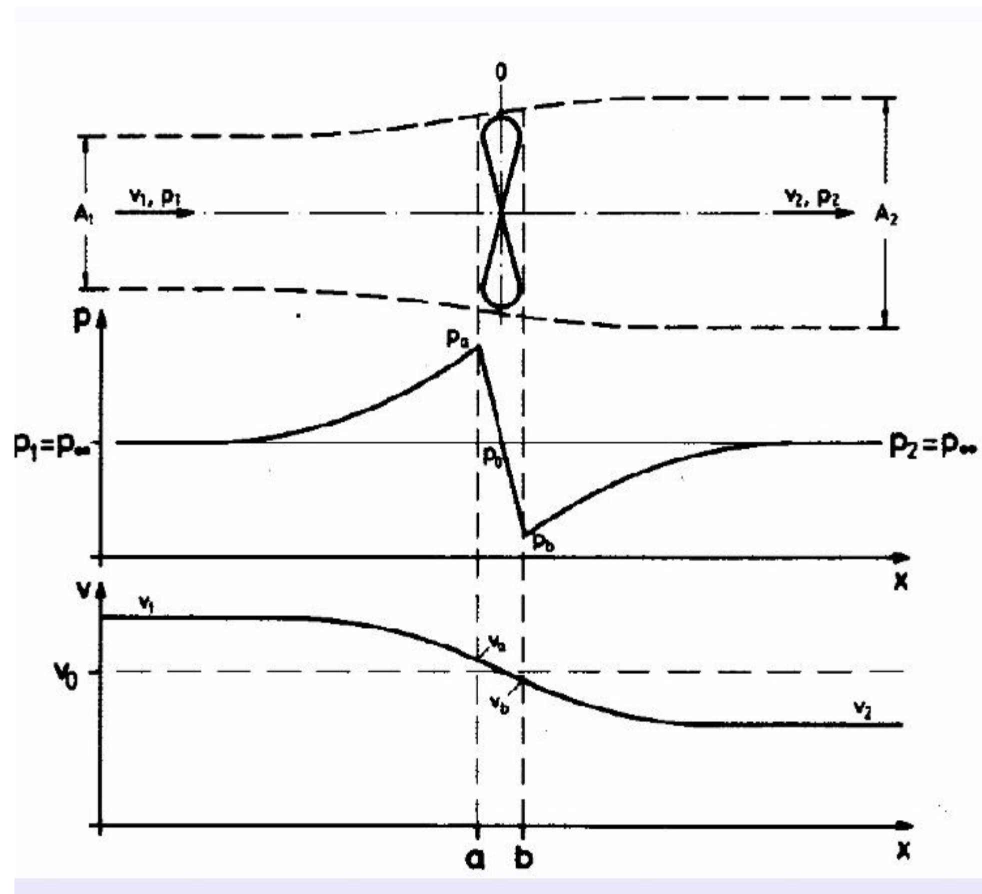




Yield of a wind turbine

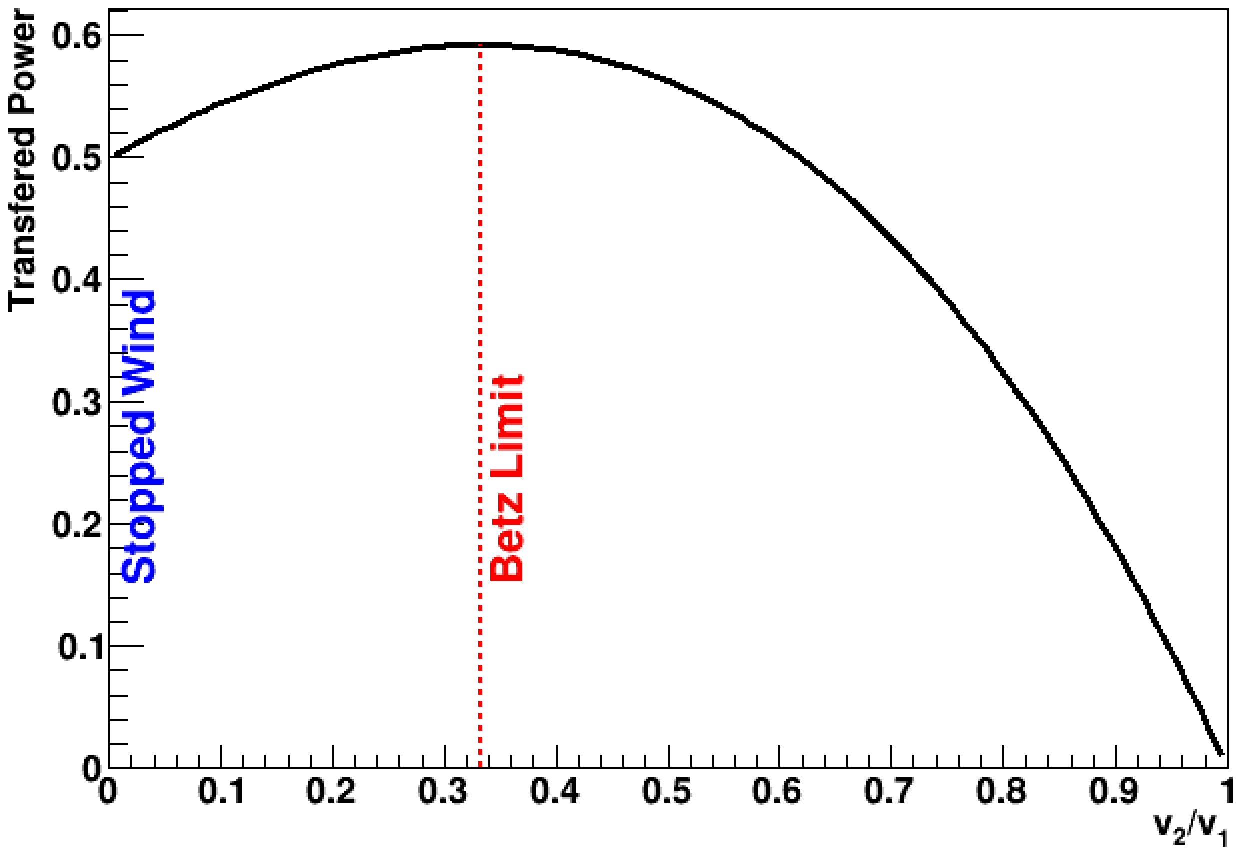




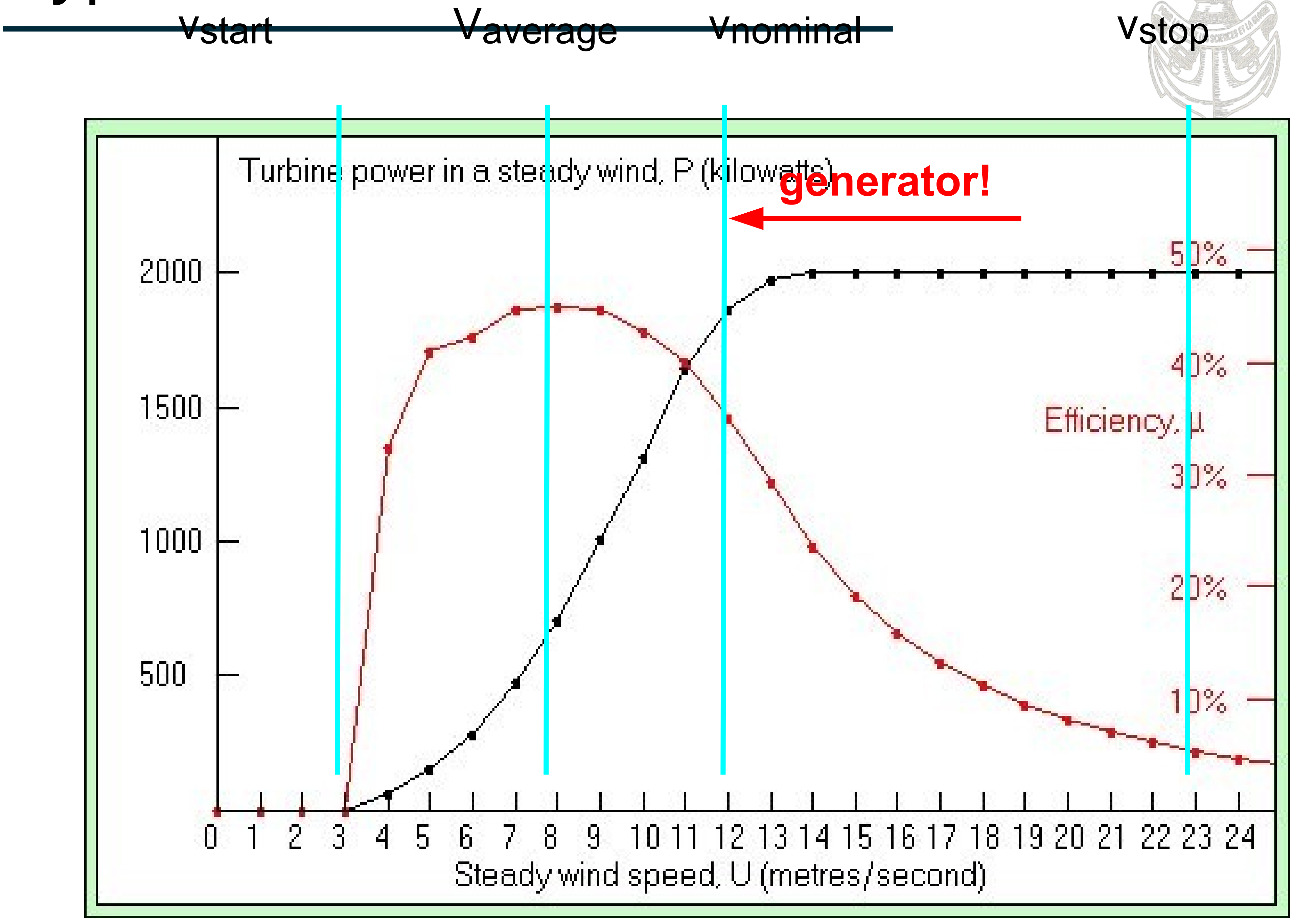


Betz's Limit

- Maximal power obtained for $v_2/v_1 = 1/3$.
- In that cas, 16/27 of the wind power is extracted



Typical Power Curve of a wind turbine



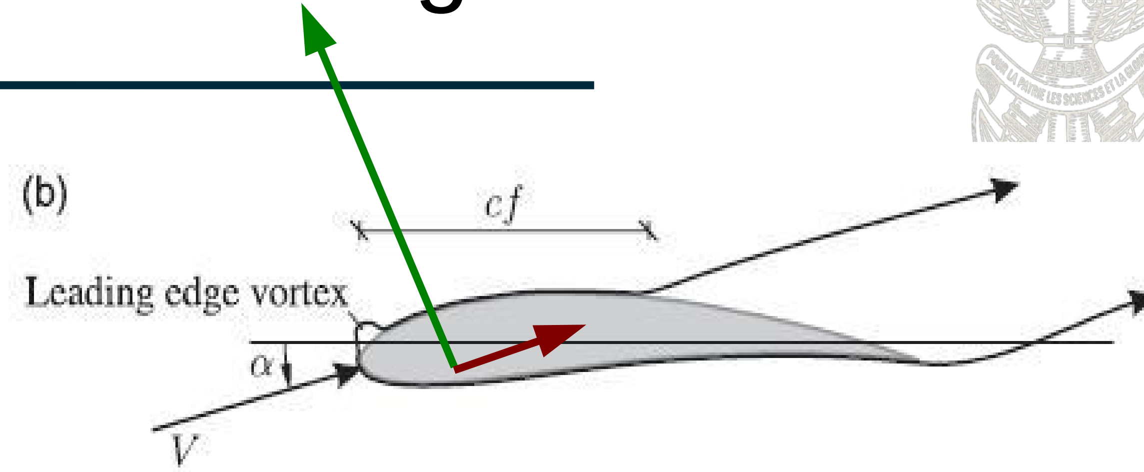

KENT HILLS
WIND FARM
MAIN ENTRANCE

Speed at end of blade

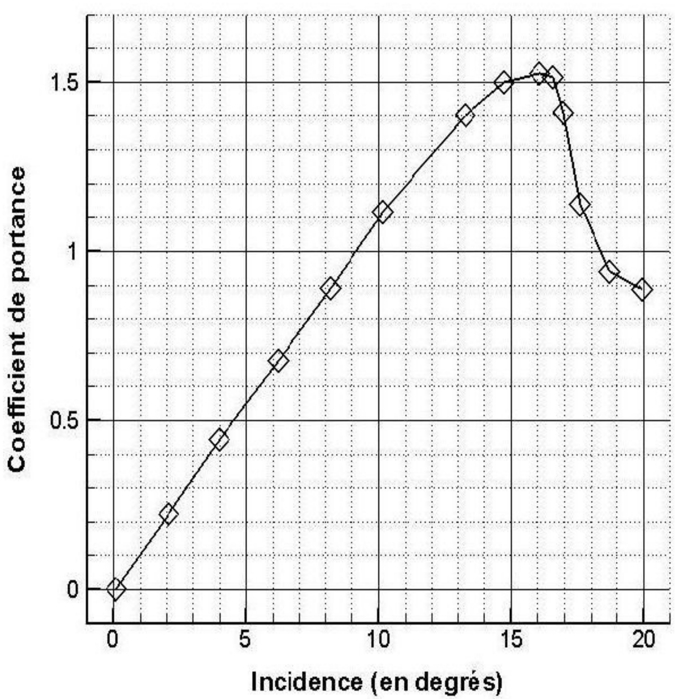
Lift force versus attack angle

$$F_L = \frac{1}{2}\rho A v^2 C_L(\alpha)$$

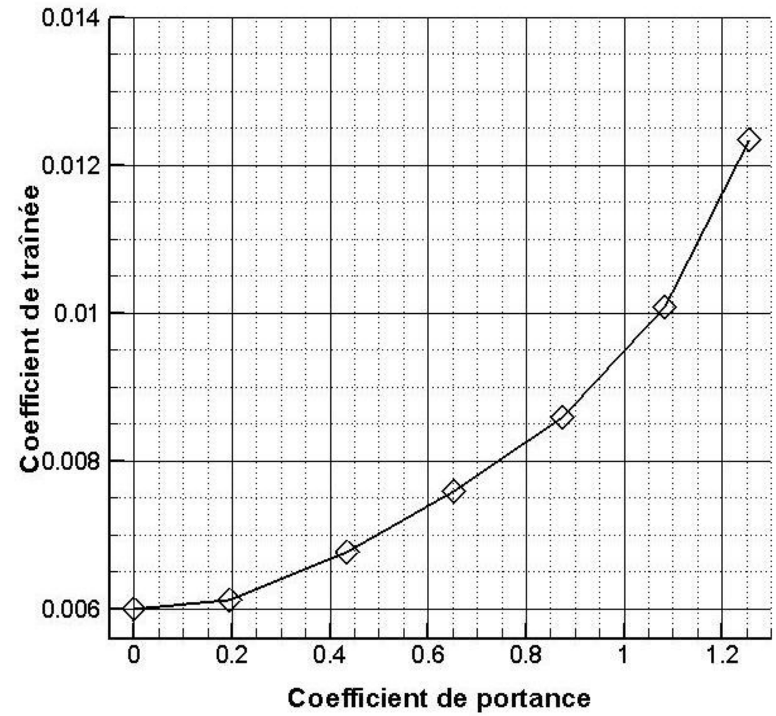
$$F_D = \frac{1}{2}\rho A v^2 C_D(\alpha)$$



C_L

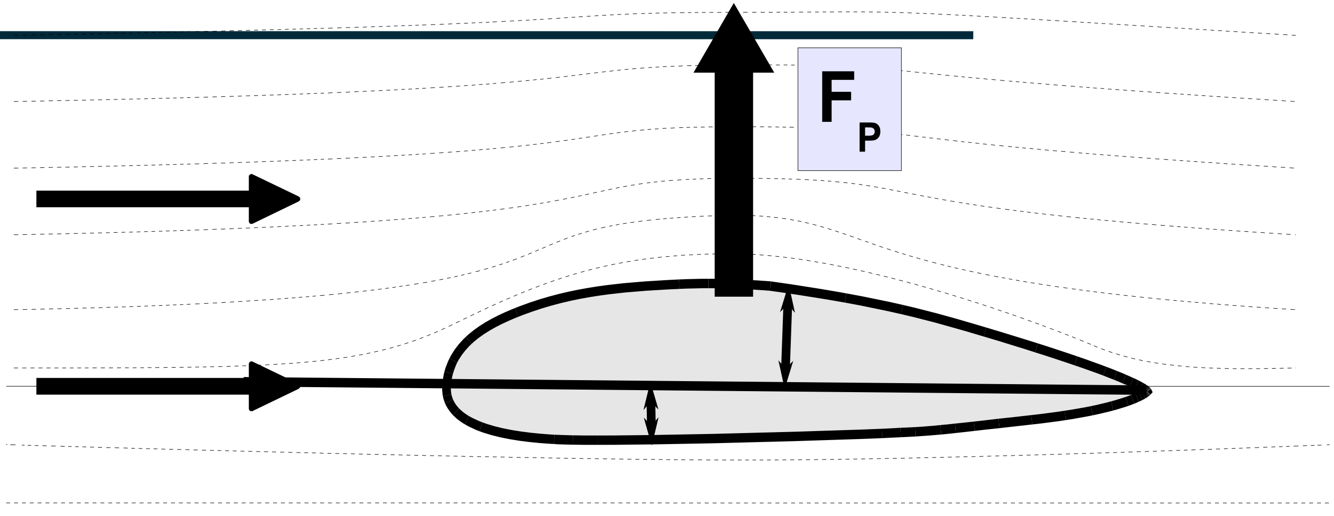


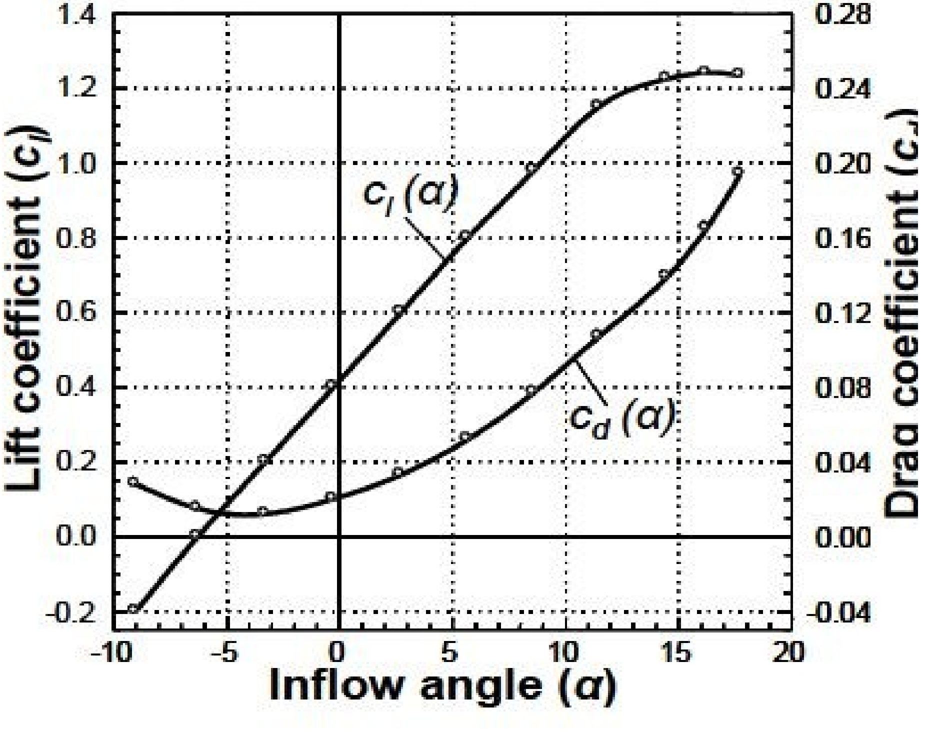
C_D



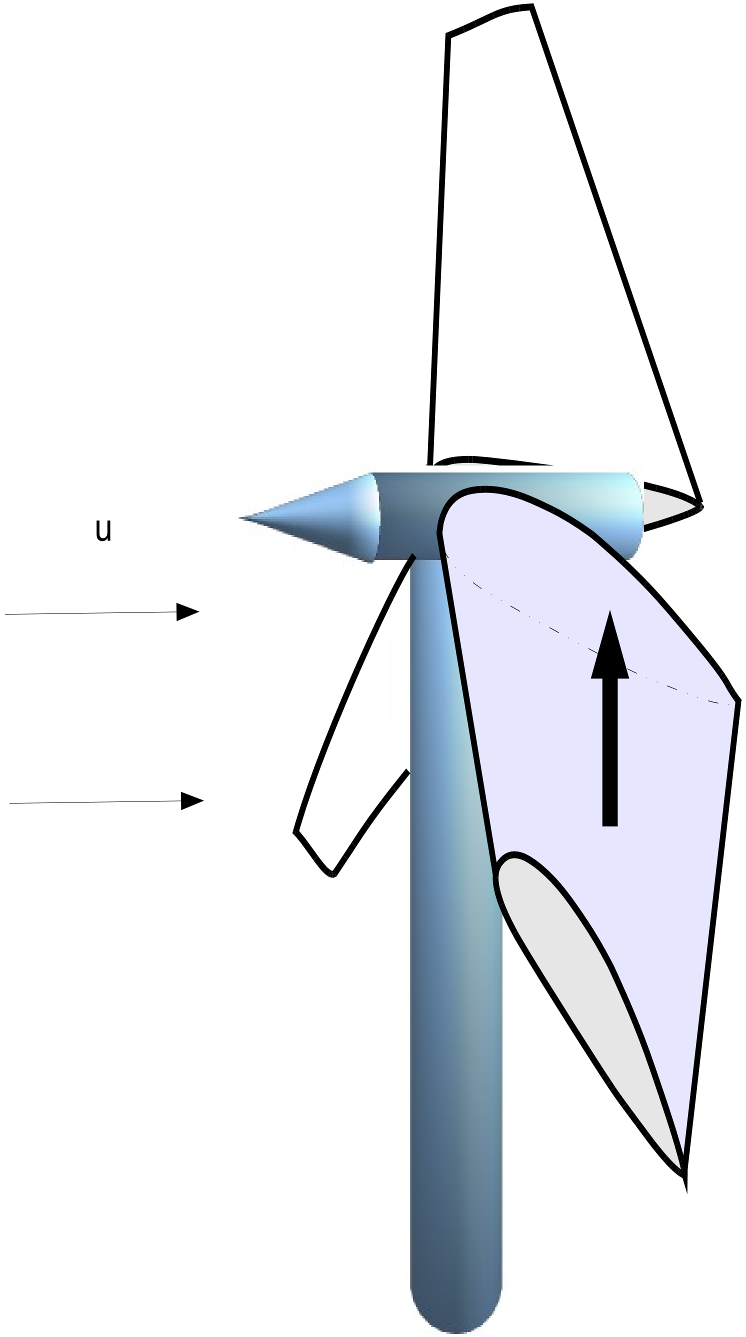
C_L

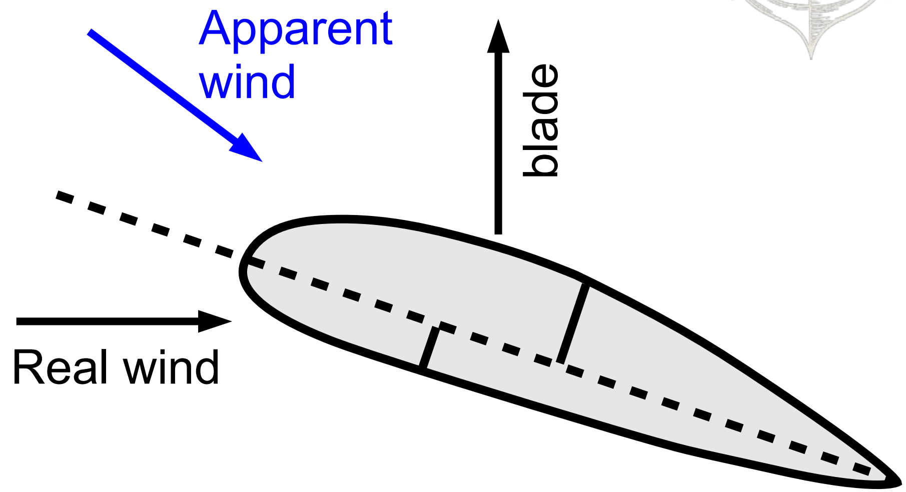
Lift force at zero angle





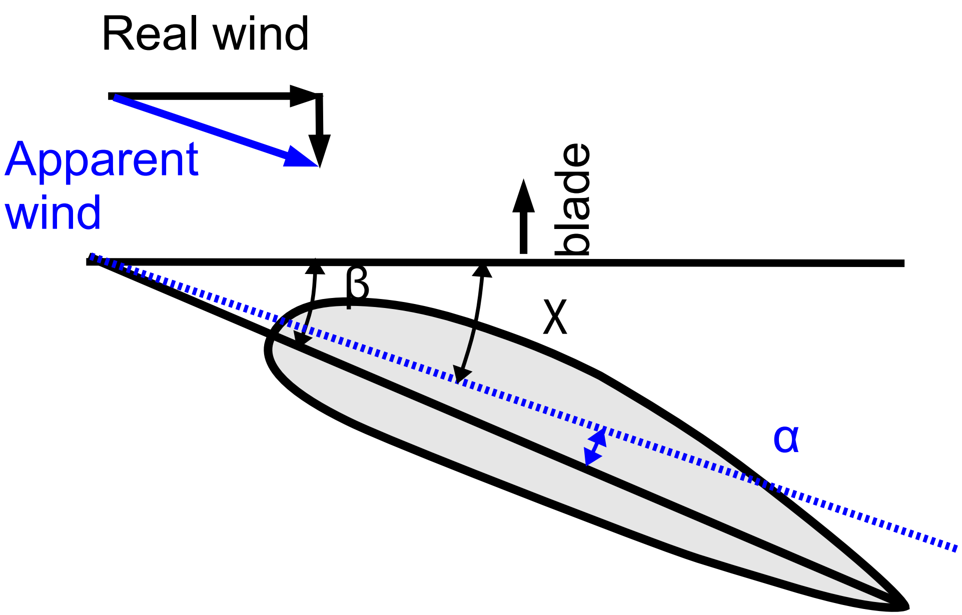
Twisting



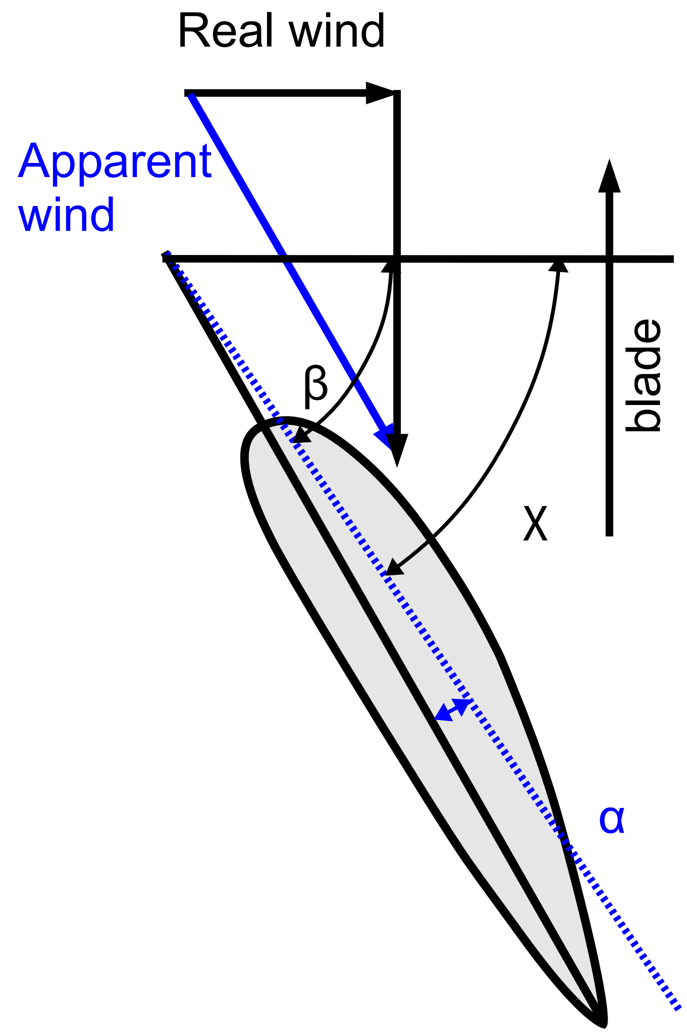


Twisting of the blade

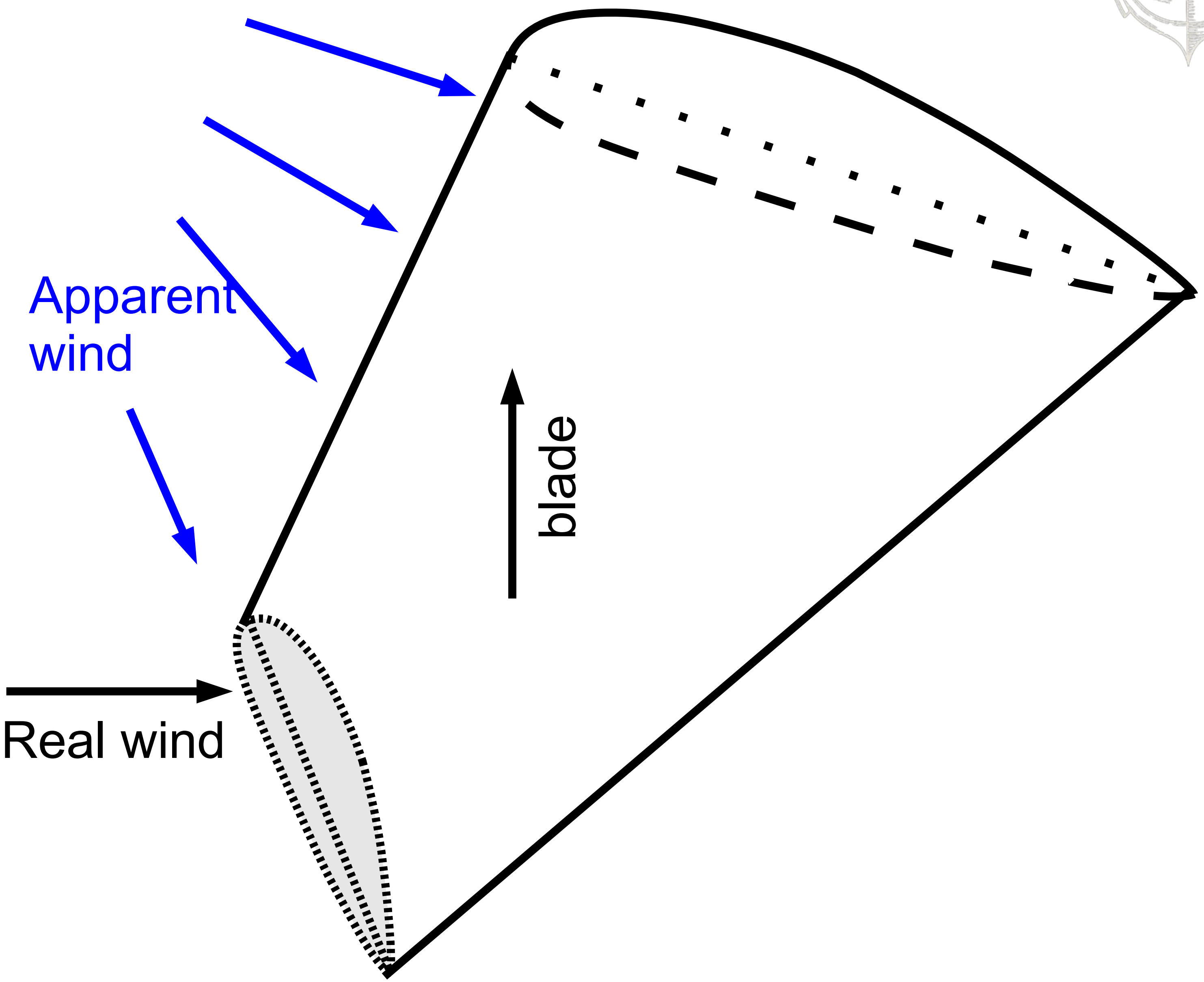
Close to rotor



End of blade



Twisting

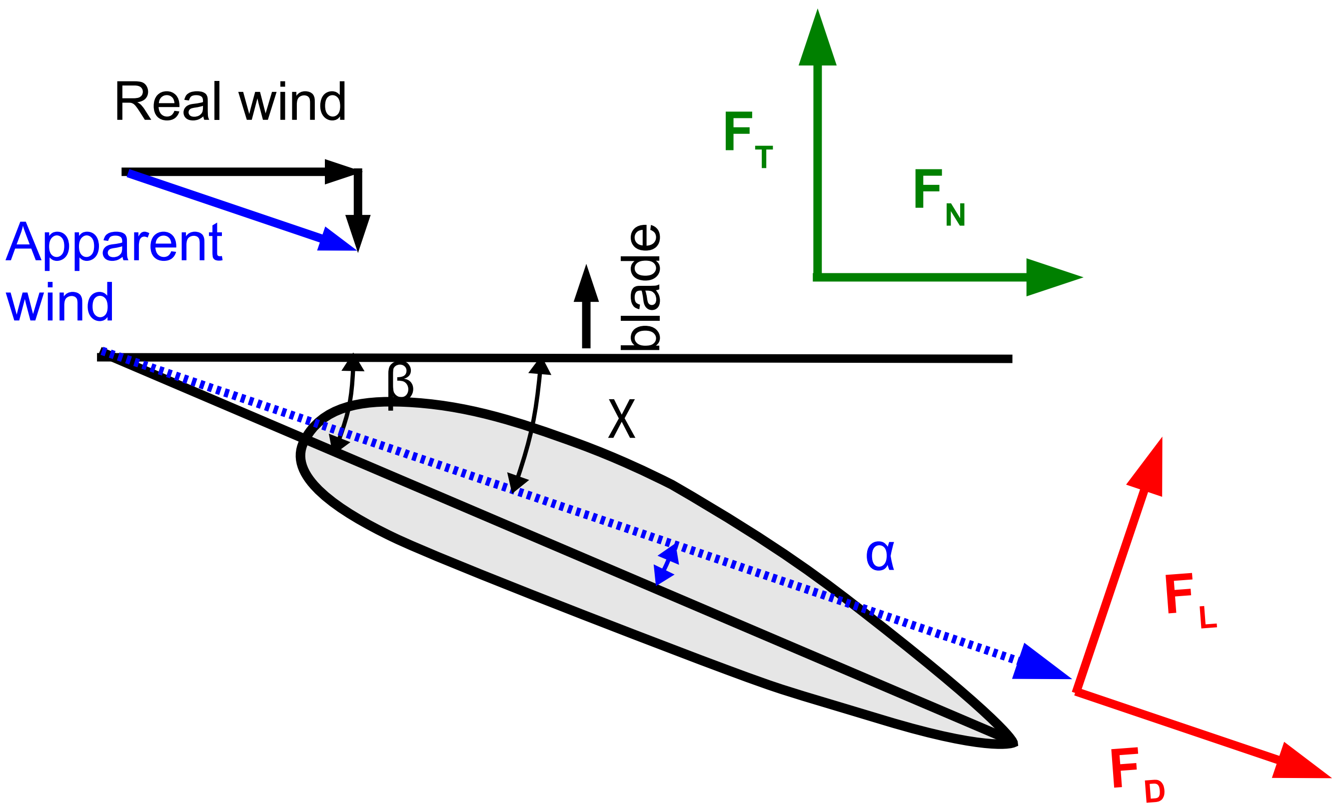


Twisting

$$\beta(r) = \chi(r) + \alpha_0$$

- Twisting Angle :

$$\beta(r) = \arctan\left(\frac{\omega r}{v}\right) + \alpha_0$$



Expression of forces

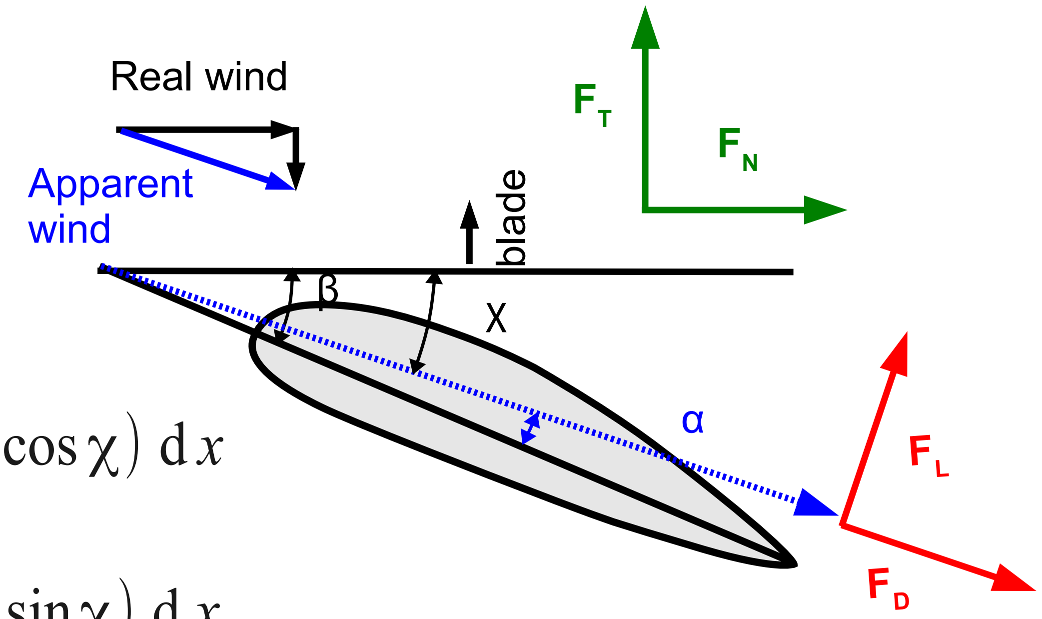
- Expression of forces

$$F_N = \frac{1}{2}\rho S V_R^2 \left(C_L(\alpha)\sin\chi + C_D(\alpha)\cos\chi\right)$$

$$F_T = \frac{1}{2}\rho S V_R^2 \left(C_L(\alpha)\cos\chi - C_D(\alpha)\sin\chi\right)$$

$$V_R = v/(\cos\chi)$$

- Inserting the apparent wind :



$$\mathrm{d}F_N = \frac{1}{2}\rho c \frac{v^2}{\cos^2\chi}\left(C_L(\alpha)\sin\chi + C_D(\alpha)\cos\chi\right)\mathrm{d}x$$

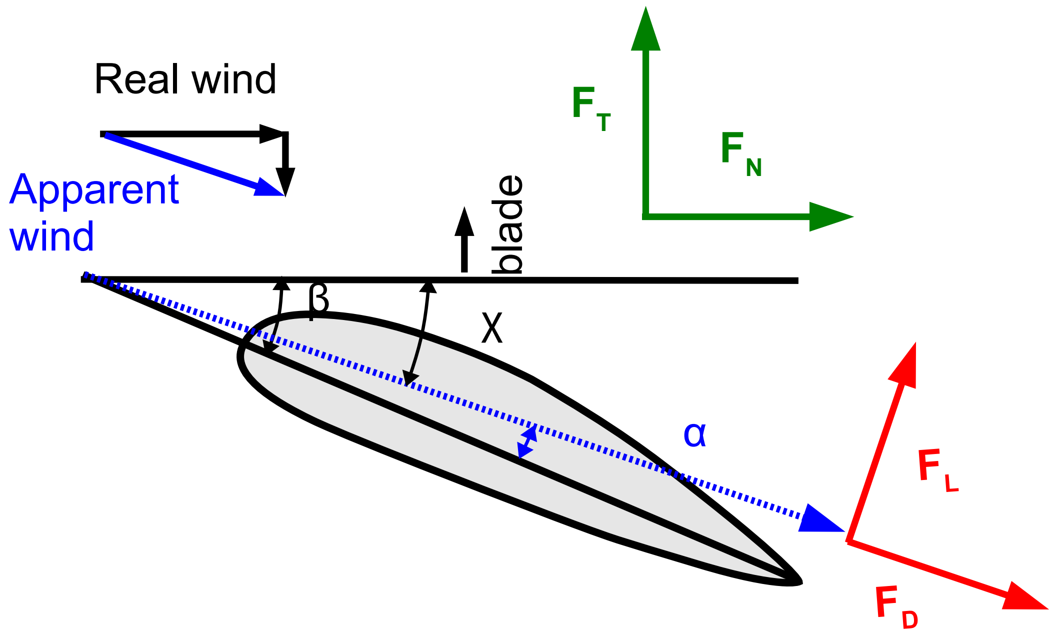
$$\mathrm{d}F_T = \frac{1}{2}\rho c \frac{v^2}{\cos^2\chi}\left(C_L(\alpha)\cos\chi - C_D(\alpha)\sin\chi\right)\mathrm{d}x$$

Chord

- Equating the normal thrust (in Betz regime) with the normal force obtained from the lift:

$$\mathrm{d}F = \frac{8\pi}{9}\rho v_1^2 r\,\mathrm{d}r, \qquad v_1 = \frac{3}{2}v$$

$$\mathrm{d}F_N = \frac{1}{2}\times\frac{4}{9}\rho C_L v_1^2 \frac{\sin\chi}{\cos^2\chi} c(r)\,\mathrm{d}r$$

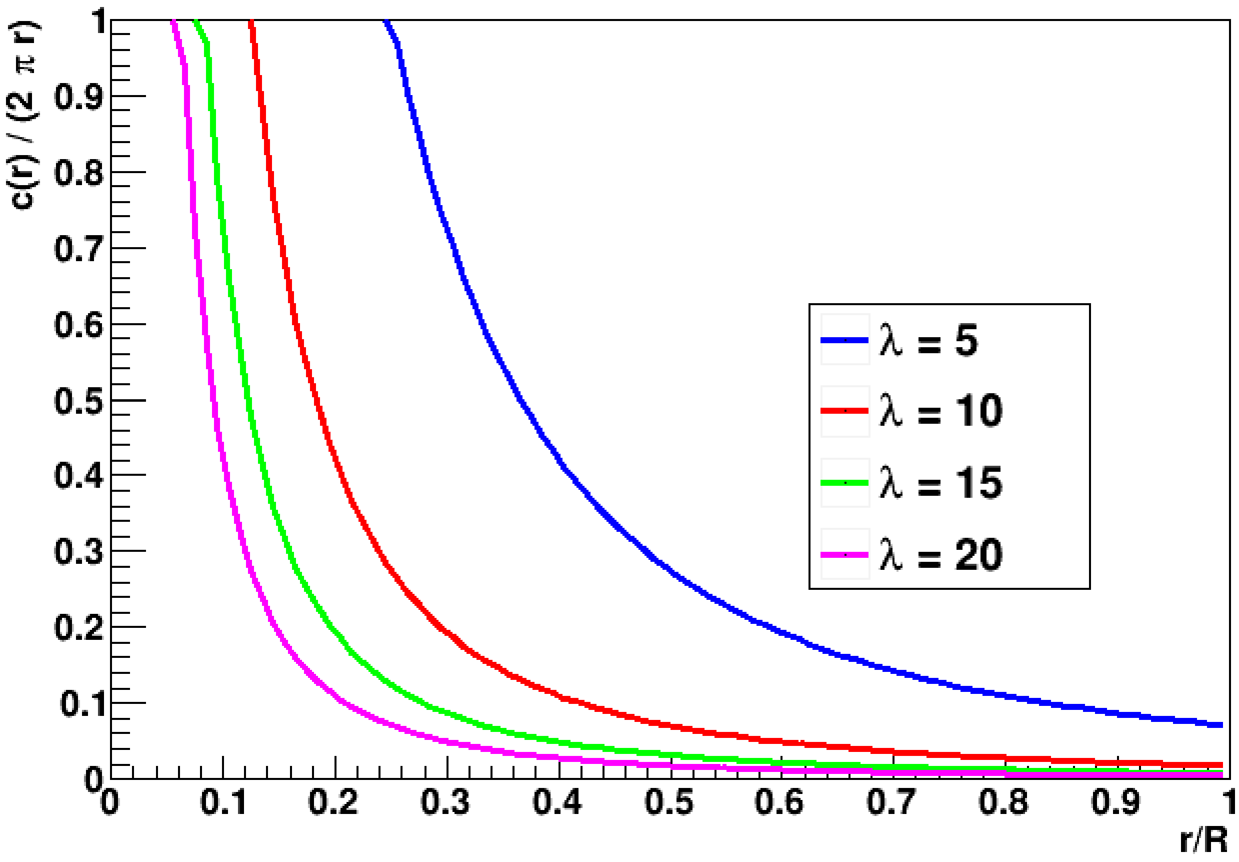


- The chord is then obtained :
  - ~ Hyperbolic profile

$$c(r) = \frac{8\pi}{3\lambda C_L} R\cos\chi = \frac{8\pi R}{3\lambda C_L\sqrt{1+\left(\frac{3\lambda r}{2R}\right)^2}}, \qquad \lambda = \frac{\omega R}{v_1} = \frac{2}{3}\frac{\omega R}{v}$$

Shape of the blade

- Getting thinner away from the rotor
- Covers only a small fraction of the disk (few %)
- The higher the tip-speed ratio, the thinner the blade



Chord and Momentum

- From the chord, the tangential force can be obtained:

$$\mathrm{d}F_T = \cot\chi \times \mathrm{d}F_N = \rho v_1^2 \frac{16\pi R}{27\lambda}\mathrm{d}r$$

- Integrating it gives he dynamical momentum

$$M = \int r \times \mathrm{d}F_T \frac{\rho S v_1^3}{2} \times \frac{16 S}{27\omega}$$

- Which is compared to the power transferred to the rotor:

$$P = M \times \omega = \frac{16}{27} \times \left(\frac{1}{2}\rho S v_1^3\right)$$

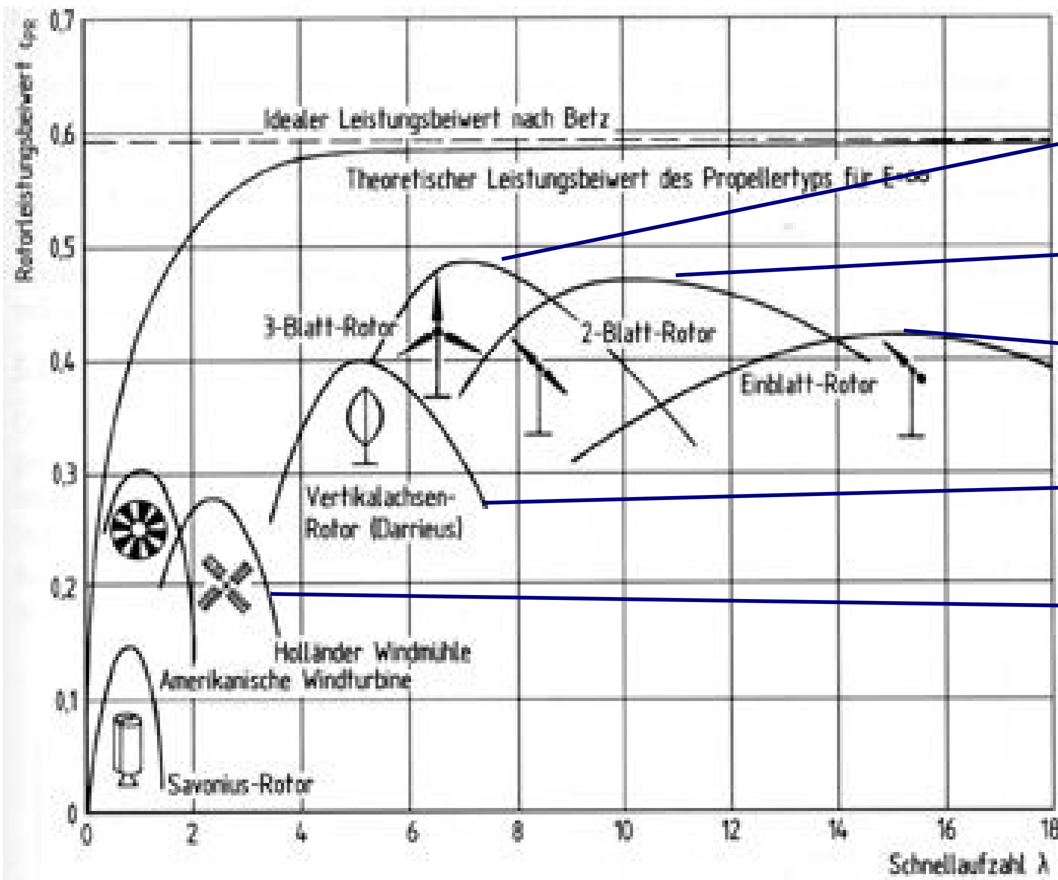
- The obtained shape of the blades gives exactly the Betz limit!

Yield of turbines

$$\lambda = \frac{\omega R}{U}$$

- Speed ratio at end of blades (*tip-speed-ratio) :*
  U   speed of the wind (far away)
  ω   angular rotation speed
  R   blade length

$C_P = P/P_0$



3 blades

2 blades

single blade

Darreius

Traditional Mill

λ

Turbines with 1 or 2 blades

- Lighter but faster, and thus noisier

Traditional Mill (low speed)

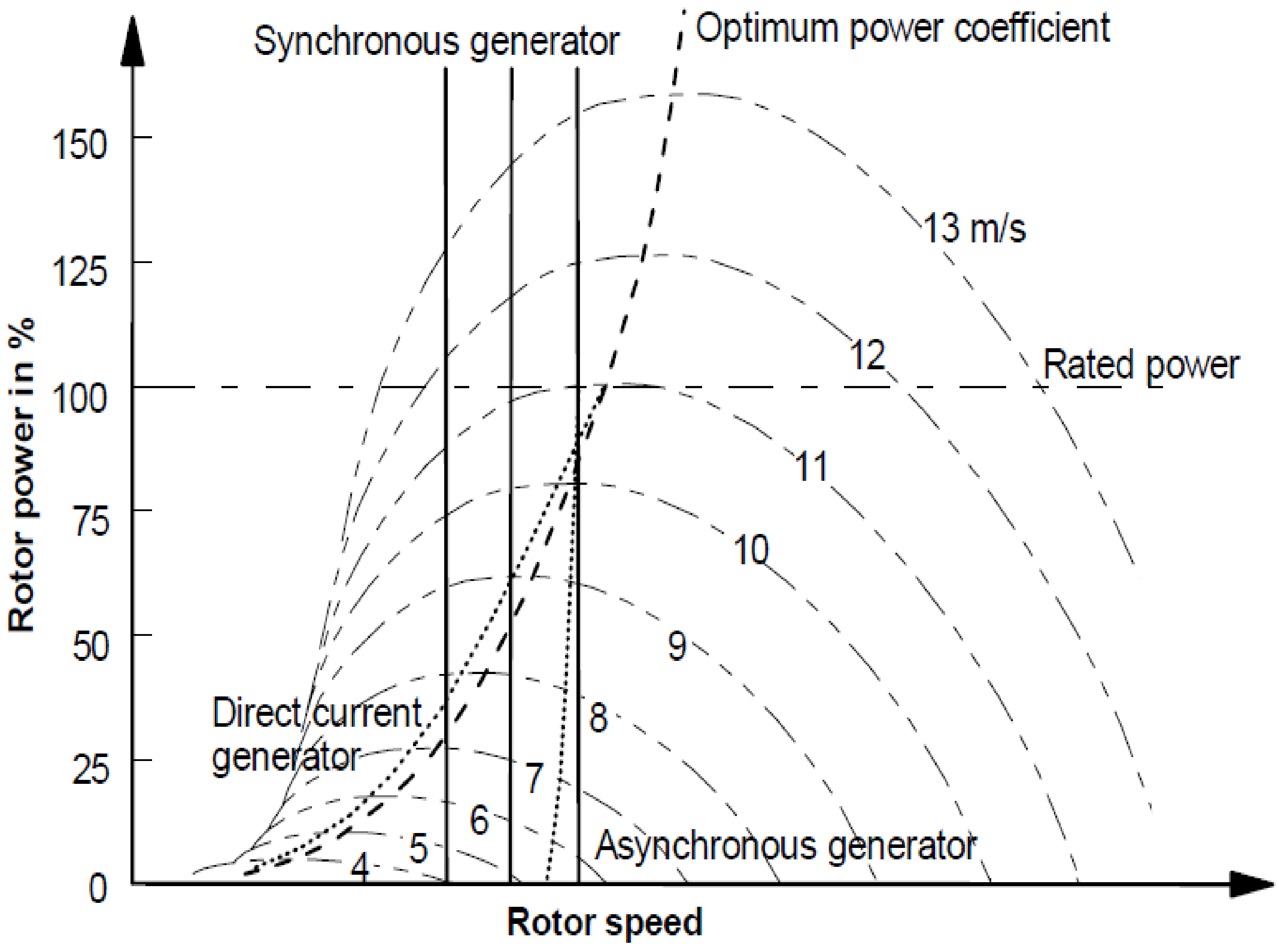
Darreius Rotor

Large size Wind Turbines

Accidents

Power versus wind speed

- The rotation speed of the turbine depends on the wind speed



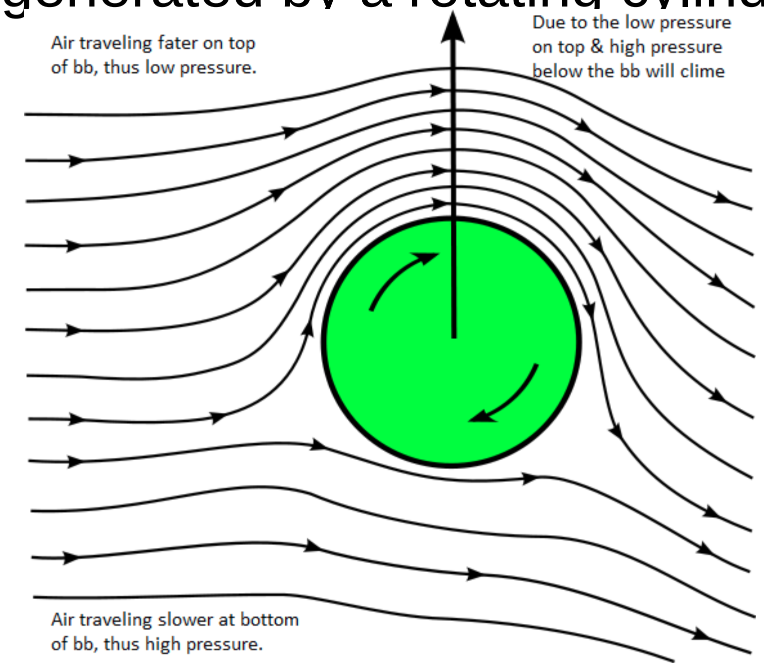
Innovative – or fancy – projects – I

- Vertical axis wind turbine (StatoEolien: fixed stator channelling the wind on a rotor in an optimal way)
  - Diameter 4 meters, 2 meters height, it rotates at 7 km/h wind (versus 15 km/h for a blade turbine) and supports higher winds 150 km/h
  - Pros : Silence insensitive to direction and strength of the wind
  - Can be installed on the roof of a building, cost 15,900 euros (~ 10 000 euros after tax credit), including installation.
  - More expensive that usual turbines
  - ~ 150 MWh/year

GUAL Industrie (http://www.gual-industrie.com/)

Innovative – or fancy – projects – II

- Magnus effect turbine :
  - Lift generated by a rotating cylinder

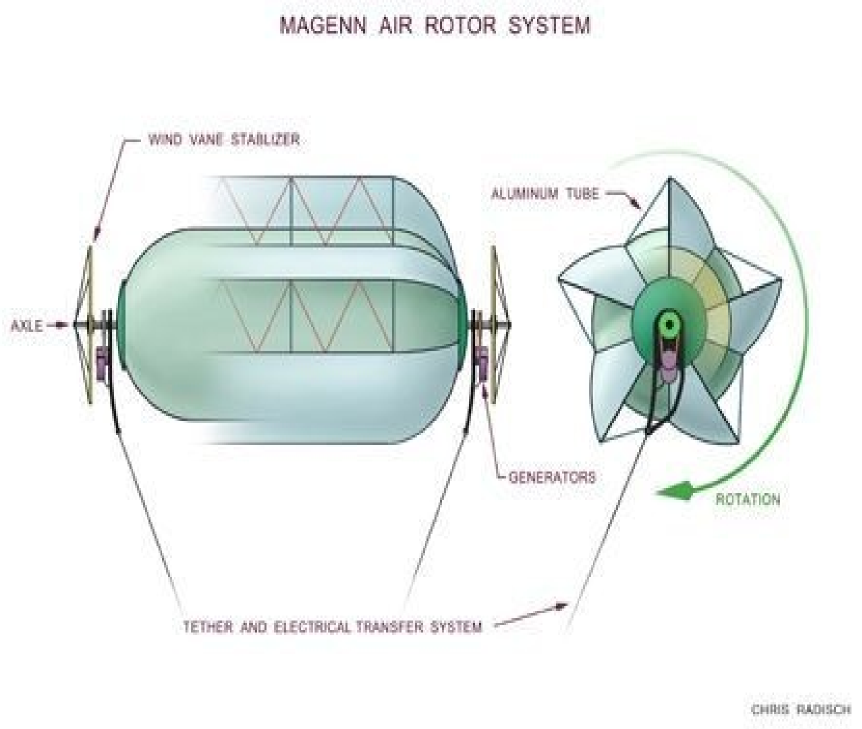


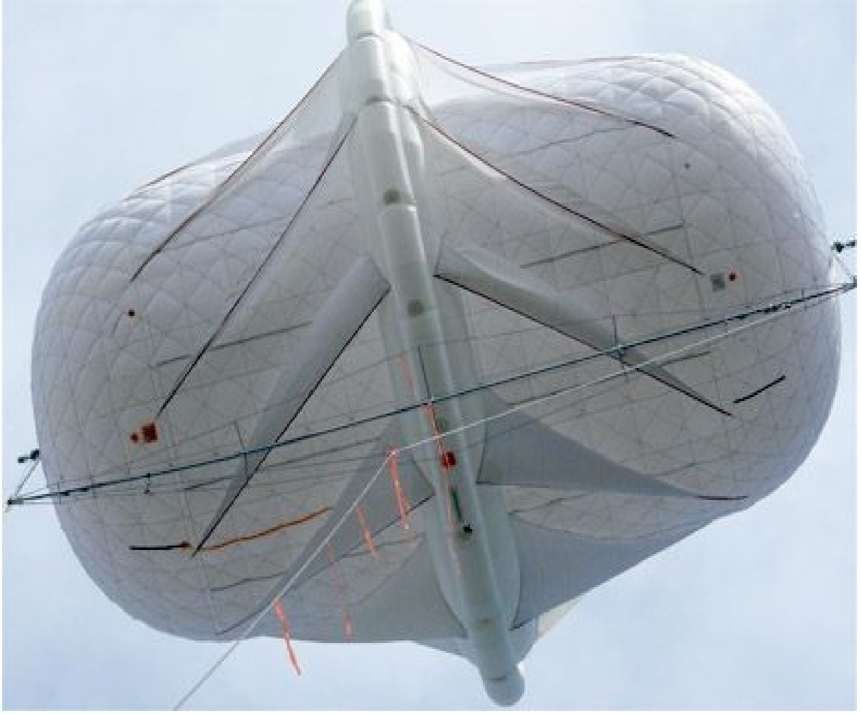
e.g.: http://www.mecaro.jp/eng/index.html

- Pros :
  - almost silent.
  - Lift can be controlled by changing the rotation speed

Innovative – or fancy – projects – III

- Inflatable Turbine (http://www.magenn.com/)
  - Capturing high-altitude jet stream (regular winds)
  - No foundation, tower, ...
  - ~ 1 MW
  - But the problem is to bring back the power to the ground ...





Innovative – or fancy – projects – III