PHY555 Energy & Environment
PC 7 Wind energy

1 Betz limit

1. mass conservation for an incompressible fluid : $vS = \text{cst}$
2. Two ways to calculate the force applied by S onto the wind : momentum conservation and pressure forces, estimated from Bernouilli $v^2/2 + p/\rho = \text{cst}$. Lead to $\frac{1}{2}\rho S(v_2^2 - v_1^2) = \rho S v(v_2 - v_1)$ hence $v = (1+a)/2$
3. $P = Fv = \frac{(1+a)(1-a^2)}{2}\frac{1}{2}\rho S v^3$ is maximal for $a = 1/3$, leading to $P = \frac{16}{27} \times \frac{1}{2}\rho S v^3$ and $F = \frac{4}{9}\rho S v_1^2$

2 Blade design

1. $dF_{L,D} = C_{L,D}(\alpha) \times \frac{1}{2}\rho v_{\text{apparent}}^2 \times \text{cord}(r)dr$
2. $v_{\text{apparent}} = v/\cos\chi = 2v_1/3\cos\chi$. Normal force: $\frac{1}{3}\rho\frac{v_1^2}{\cos\chi} \times \text{cord}(r)dr \times (C_D(\alpha(r))\cos\chi + C_L(\alpha(r))\sin\chi)$. Tangent : $\frac{1}{3}\rho\frac{v_1^2}{\cos\chi} \times \text{cord}(r)dr \times (C_L(\alpha(r))\cos\chi - C_D(\alpha(r))\sin\chi)$
3. $\alpha = \alpha_0\, \forall r \Rightarrow \beta(r) = \alpha_0 + \arctan\frac{r\Omega}{v}$
4. Normal force = Betz force $\frac{1}{3}\rho\frac{v_1^2}{\cos\chi} \times \text{cord}(r)drC_L(\alpha(r))\sin\chi = \frac{4}{9}\rho 2\pi r dr v_1^2$ leads to $\text{cord}(r) = \frac{8\pi}{3C_L}\frac{v_1}{\Omega}\frac{1}{\sqrt{1+\left(\frac{3\Omega r}{2v_1}\right)^2}}$.

   The torque is $dM = r \times dF_T$ and the power $P = \Omega\int dM = \frac{16}{27} \times \frac{1}{2}\rho S v^3$