PHY555 Energy & Environment
PC 5 Nuclear energy

Reminders

- A nucleus composed of Z proton and N neutrons has a mass number $A = Z + N$.
- The binding energy of nucleons B depends on Z and N. The mass of the nucleus is lower than the mass of its constituants as
$$m_A c^2 = Z m_p c^2 + N m_n c^2 - B(Z, N) \tag{1}$$
where $m_p c^2 = 938.3$ and $m_n c^2 = 939.6\,\mathrm{MeV}$
- All nuclei have about the same density $2.3 \times 10^{17}\,\mathrm{kg/m^3}$. The typical radius of a nucleus thus scales with the mass number as
$$R \simeq 1.2\,\mathrm{fm} \times A^{1/3} \tag{2}$$
- A nucleus is *fissile* if it can undergo nuclear fission when submitted to thermal neutrons. By contrast, the fission of a *fissionable* nucleus requires high energy neutrons.

1 Orders of magnitude

1. Show that the typical energy scale involved in a nuclear reaction is about ten million times larger than that of a chemical reaction.
2. A nuclear reactor burns ^{235}U nuclei to heat up the primary circuit to about 300°C. Estimate the minimal mass of ^{235}U required to run a reactor during one year.
3. The natural abundance of ^{235}U is 0.7%, and that of ^{238}U is roughly 99.3%. Estimate the minimal mass of natural uranium required to run a reactor during one year.
4. Estimate the corresponding missing mass.

2 Nuclear energy - the Bohr-von Weizsacker model

A way to model the nuclear material is to assume that it behaves like a very high density incompressible fluid (Model of Bohr-von Weizsacker –1936): a nucleus comparable a "droplet". To separate a nucleus in all its components (nucleons), one needs to provide the binding energy given by:

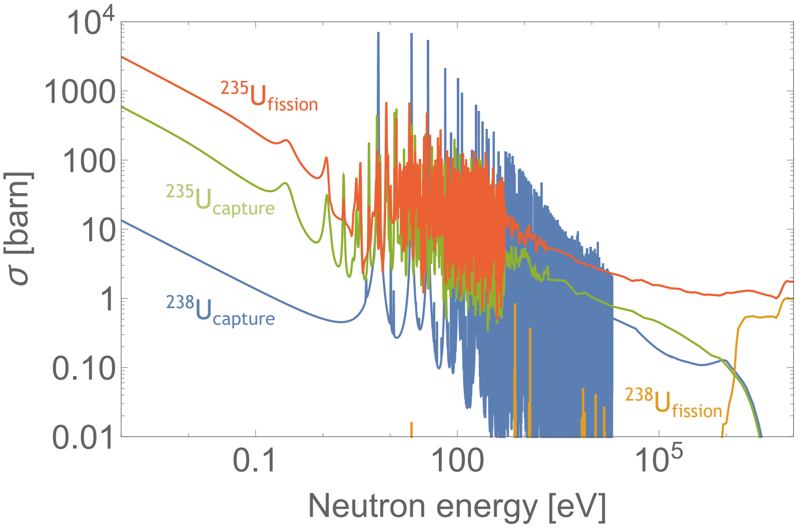
$$B(Z, N) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} + a_p \frac{(-1)^Z + (-1)^N}{A^{1/2}}$$

with $a_V = 16\,\mathrm{MeV}$, $a_S = 18\,\mathrm{MeV}$, $a_C = 0.72\,\mathrm{MeV}$, $a_A = 23\,\mathrm{MeV}$ and $a_p = 12\,\mathrm{MeV}$.

1. Interpret the different terms in the equation of Bohr-von Weizsacker
2. Assuming $N = Z$ for simplicity, estimate the most stable nucleus
3. Show that as Z increases, stability actually favors an imbalance $N > Z$.
4. What is the energy released by the symmetric fission of a ^{236}U nucleus?
5. Considering a simplified model where the strong interaction acts only at distances shorter than the nucleus radius size, show that most nucleus cannot undergo spontaneous fission.
6. Explain why odd nuclei (^{233}U, ^{235}U, ^{239}Pu...) are fissile while even nuclei (^{238}U, ^{240}Pu ...) are only fissionable.

3 Conventional reactor design

Among fissile materials, only ^{235}U is stable enough to exist in nature. The graph below shows the cross section for fission and neutron capture for ^{235}U and ^{238}U.



1. Uranium fission produces neutrons at about 2 MeV. Why use ^{235}U (which generated about 2.5 neutrons per fission) rather than ^{238}U (which generated about 2.8 neutrons per fission)?

2. Express the probability η for a neutron interacting with an uranium nucleus to trigger a fission. Explain why nuclear reactors require uranium to be enriched and neutrons to be slowed down.

3. To slow down neutrons, a moderator is introduced. The moderator is a nucleus of mass number A_m, with a large scattering cross section σ^s and a small absorption cross section σ^c. It can be shown that the average energy loss per collision can be expressed as

$$\xi = \left\langle \log \frac{E_{\text{after collision}}}{E_{\text{before collision}}} \right\rangle = 1 + \frac{(A_m - 1)^2}{2A_m} \ln\left(\frac{A_m - 1}{A_m + 1}\right)$$

   For a mixture (or a molecule), the effective logarithmic energy loss can be expressed as

$$\bar{\xi} = \frac{\Sigma \sigma_i^s \xi_i}{\Sigma \sigma_i^s}$$

   Explain why heavy water ^2H_2O is the a most appreciated moderator.

4. The fission of ^{235}U leads in 6.4% of cases to the formation of Iodine 135, triggering the following decay chain

$$^{135}\mathrm{I} \underset{9.5\,\mathrm{h}}{\rightarrow} {}^{135}\mathrm{Xe} \underset{13.1\,\mathrm{h}}{\rightarrow} {}^{135}\mathrm{Cs} \underset{3{,}3\,\mathrm{Myr}}{\rightarrow} {}^{135}\mathrm{Ba}$$

   Xenon 135 can also be produced directly from fission of ^{235}U with probability 0.23%.

   The capture cross section of Iodine 135 is negligible, while that of Xenon 135 is about 3 Mb.

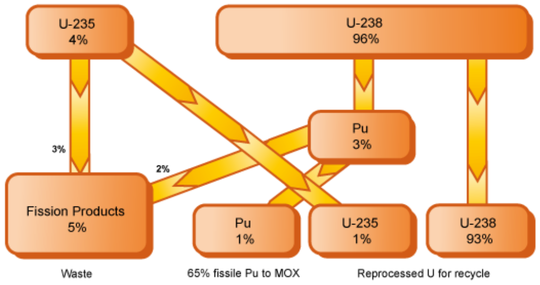
   (a) Why is Xenon 135 called a "neutron poison"?
   (b) Taking $\phi = 10^{13}\mathrm{cm}^{-2}.\mathrm{s}^{-1}$ as the neutron flux, write the kinetic equations governing the time evolution of Iodine 135 and Xenon 135 concentration.
   (c) Express the Xenon to Uranium ratio during the steady operation of the reactor.
   (d) Explain why is it difficult to restart a reactor few hours after shutdown.

4 Plutonium: wastes or resource?

Some neutrons are absorbed by ^{238}U and trigger the following decay chain, leading to the formation of fissile ^{239}Pu with lifetime > 24 000yr.

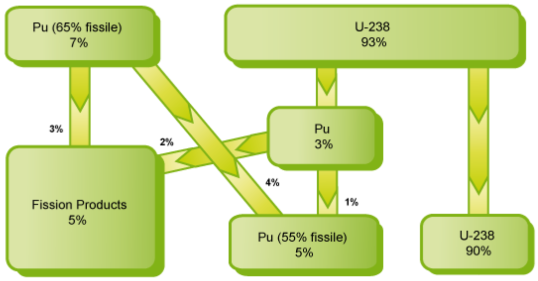
$$^{238}\mathrm{U} + n \to {}^{239}\mathrm{U} \underset{23\,\mathrm{min}}{\to} {}^{239}\mathrm{Np} \underset{2{,}3\,\mathrm{day}}{\to} {}^{239}\mathrm{Pu}$$

Reaction in Standard UO$_2$ Fuel



Basis: 45,000 MWd/t burn-up, ignores minor actinides

Reaction in MOX Fuel



Basis: 45,000 MWd/t burn-up, ignores minor actinides

1. In open cycle reactors, the initial fuel (UOX) is composed of 4% ^{235}U and 96% ^{238}U. After 1 year, the fuel is composed of 94% Uranium, 1% Plutonium and 5% fission products. How much Plutonium does such a reactor produce after n years ?

2. Mono-recycling recovers most of the Plutonium produced by burning UOX, and produces a 7% Pu - 93% ^{238}U fuel (MOX). After 1 year, the fuel is composed of 90% Uranium, 5% Plutonium and 5% fission products. How much net Plutonium does such a reactor produce after n years ?

3. Fast breeder: an alternative approach to the ^{235}U route is to consider taking advantage of Plutonium formation from $^{238}\mathrm{U} + n$, and use ^{239}Pu as fuel. We note
    - p_b the probability that a neutron interaction triggers the decay of ^{238}U to ^{239}Pu.
    - p_f the probability that a neutron interaction triggers ^{239}Pu fission, leading to the production of ν neutrons.
    - p_c the probability that a neutron interaction transforms ^{239}Pu into ^{240}Pu.

    and consider that these are the only three possible interactions for a neutron.

    (a) The breeding condition corresponds to sustaining the ^{239}Pu population. Show that this condition leads to

    $$p_b = \frac{1}{2}, \qquad p_f = \frac{1}{2\left(1 + \sigma^c_{239}/\sigma^f_{239}\right)}, \qquad p_c = \frac{\sigma^c_{239}/\sigma^f_{239}}{2\left(1 + \sigma^c_{239}/\sigma^f_{239}\right)}$$

    (b) Show that a U/Pu reactor cannot operate with slow neutrons, but only with fast neutrons.

    (c) Estimate the fraction of plutonium in the fuel of such a reactor.

    (d) Discuss the status of Plutonium and depleted Uranium in such breeder reactor as compared to their status in conventional technology.

Data

$\langle\sigma\rangle$	fission	capture		fission	capture		ν
	Thermal			Fast			
^{235}U	40	4					2.5
^{238}U	0	1		0	0.3		2.8
^{239}Pu	90	50		1.85	0.5		2.9

σ	capture	scattering
^{1}H	0.2	20
^{2}H	0.0003	3.4
^{16}O	0.0002	3.8