PHY555 Energy & Environment
PC 2 Radiative balance - guidlines

1 Orders of magnitude

1. Show that the incoming solar energy flux is $F_\odot = 340\,\mathrm{W/m^2}$, as indicated in Fig. 1.

$$I_\odot = \frac{P_T}{4\pi R_T^2} = \frac{1}{4}\frac{R_S^2}{d_s^2}\sigma T_s^4$$

2. Without any atmosphere and with an albedo of $R = 0.3$, what would be the average temperature T_0 on Earth ?

$$T_0 = \left(\frac{(1-R)F_\odot}{\sigma}\right)^{1/4} \sim 255\,\mathrm{K}$$

3. Greenhouse effect: consider now the atmosphere as a simple greybody with absorptivity $\mathcal{A} = 0.8$ in the infrared region, and transparent for visible radiations. Evaluate the average temperature T taking this greenhouse effect into account.

$$T = \frac{T_0}{\left(1-\frac{\mathcal{A}}{2}\right)^{1/4}} = 290\,K$$

4. As a convention, energy flux towards the surface are counted as positive, and energy flux towards space are counted as negative. Estimate the change in net energy flux at the top of the atmosphere dF_P if the surface temperature changes by an amount dT, all other parameters remaining constant. Express and estimate the Planck feedback parameter α_P, defined as $dF_P = \alpha_P dT$.

$$\alpha_P = -4\left(1-\frac{\mathcal{A}}{2}\right)\sigma T^3 = -3.3\,\mathrm{W/m^2/K}$$

5. Consider that for some reason, the net energy flux at the top of the atmosphere is increased by an amount F_0 (radiative forcing). If the climate system equilibrates this imbalance only by changing the surface temperature, by how much would the surface temperature vary as a response to this forcing ?

$$F_0 + \alpha_p dT = 0 \Rightarrow dT = -F_0/\alpha_P$$

2 CO2 Radiative forcing

1. The atmosphere is actually not equally absorptive at all wavelength. Noting $\mathcal{A}_\lambda$ the absorptivity of the atmosphere at a given wavelength λ, express the radiation emitted from the ground and transmitted through the atmosphere.

$$I_{\text{transmitted}} = \int d\lambda B(T,\lambda)\,\mathcal{A}_\lambda$$

2. Changing the amount of CO_2 in the atmosphere changes the absorptivity in the spectral region where CO_2 is opaque, and changes therefore the net energy flux at the top of the atmosphere. In this question, we will estimate the atmospheric absorptivity as a function of CO_2 concentration.

   (a) $\frac{1}{I_\lambda(z)} \frac{d}{dz} I_\lambda(z) = -n_{CO_2}(z)\sigma(\lambda)$

   (b) $\ln \frac{I_\lambda(\infty)}{I(0)} = \ln(1 - \mathcal{A}_\lambda) = c_{CO_2} n_0 \sigma_\lambda L$

   (c) Picture qualitatively the change in absorptivity when CO_2 concentration varies. In which spectral region is the difference most significant ?

   Regions with partial absorption.

3. We approximate the CO_2 absorption cross section as

$$\sigma(\lambda) = \sigma_0 \exp\left(-r\left|\frac{1}{\lambda} - \frac{1}{\lambda_0}\right|\right) \tag{1}$$

   Express the spectral range $\Delta\lambda$ where atmospheric absorptivity is significant.

$$\frac{\Delta\lambda}{\lambda_0^2} \simeq \frac{2}{r} \log\left(\sigma_0 c_{CO_2} n_0 L\right) \text{ with } \begin{cases} \sigma_0 = 10^{-19}\,\text{cm}^2 = 10^{-23}\,\text{m}^2 \\ \lambda_0 = 15\,\mu\text{m} \\ r = \frac{\ln 10^{-4}}{100} = 0.09\,\text{cm} \end{cases}$$

4. Express the change in net energy flux at the top of the atmosphere resulting from the change in atmospheric absorptivity when CO2 concentration is doubled. Compare your result to the 3.73 W/m² value report in the IPCC AR6 WG1

$$\frac{2\lambda_0^2}{r} \log 2 \times B(\lambda_0, T_g) = +6.9\,\text{W/m}^2$$

5. CO2 concentration has increased from 270 to 410 ppm since the preindustrial era. Estimate the corresponding radiative forcing. Compare your result to the IPCC estimate

   CO2 has not doubled, it has increased by a factor 1.5. Keeping the scaling law $F \propto \log c/c_{ref}$, we expect the forcing to be $3.73 \times \log 1.5/\log 2 = 2.18$, which is in very good agreement with CO2 contribution to the total anthropogenic forcing.

6. If the climate system equilibrates the corresponding forcing only by changing the surface temperature, by how much would the surface temperature since the XIXth century ?

   Temperature variation : $dT = 2.7/3.3 = 0.8$, less than actually observed, because we didn't account for any feedback.

3 Feedback loops

1. Consider that a change dT in the temperature surface leads to feedback on the climate system (change in albedo, water vapour pressure...), which lead to a radiative forcing $F_{\text{FB}} = \alpha_{\text{FB}} dT$. Show that the temperature change in response to a forcing F_0 is now given by

$$dT = \sum dT_i = -\frac{F_0}{\alpha_P} + \frac{F_0}{\alpha_P}\frac{\alpha_{FB}}{\alpha_P} - \frac{F_0}{\alpha_P}\left(\frac{\alpha_{FB}}{\alpha_P}\right)^2 + \frac{F_0}{\alpha_P}\left(\frac{\alpha_{FB}}{\alpha_P}\right)^3 + ... = -\frac{1}{\alpha_P + \alpha_{FB}} F_0 \tag{2}$$

2. The value of climate feedback parameters are given in the IPCC report as shown below. Estimate the climate sensitivity, defined as the surface temperature increase if the CO2 concentration is doubled.

   $\alpha_P + \alpha_{FB} = -1.2 W/m^2/K$, so if $CO2$ is doubled, $F = +3.73 W/m2$ and $dT = -3.73/1.2 = +3.1\,K$, quite good estimate.