PHY555 Energy & Environment
PC 2 Radiative balance

The purpose of this tutorial is to understand the radiative energy balance presented in the recent IPCC assesment report (Chapter 7), and to evaluate the impact of increasing CO_2 concentration in the atmosphere. We will introduce some of the main notions to perform the radiative energy balance of Earth: radiative forcing, feedback parameters and climate sensitivity.

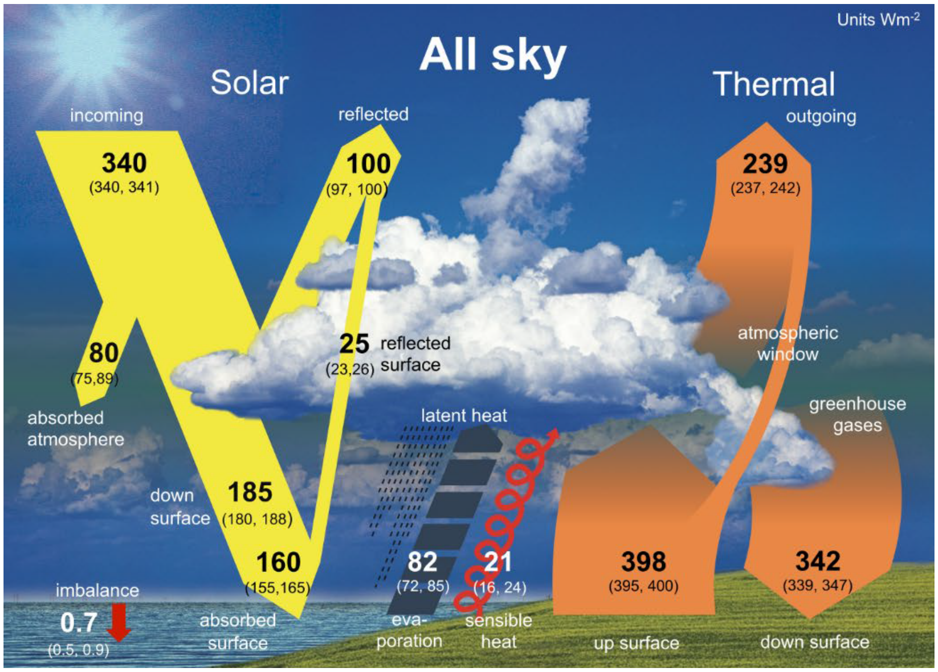


Figure 1: Earth energy balance, IPCC AR6, Chap 7 (2021)

Reminders on blackbody radiation

1. The spectral density of energy flux emitted by a surface at temperature T is given by the Planck law :

$$B(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp \frac{hc}{\lambda k_B T} - 1} \tag{1}$$

2. The total intensity radiated by a unit surface of blackbody is given by the Stefan law

$$\begin{aligned} F &= \int_0^\infty d\lambda \, B(\lambda, T) \\ &= \int_0^\infty d\lambda \, \underbrace{\frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp \frac{hc}{\lambda k_B T} - 1}}_{B(\lambda,T)} = \sigma T^4 \end{aligned} \tag{2}$$

where $\sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2} = 5.67 \times 10^{-8}\,\mathrm{W/m^2/K^4}$ is the Stefan constant.

3. The ability of a system to absorb radiation is equal to the ability of this system to emit this radiation, according to the Kirchhoff law:

$$\mathcal{A}_\lambda = \epsilon_\lambda \,\forall\lambda \tag{3}$$

   where $\mathcal{A}_\lambda$ is the absorptivity of the system (ie the absorbed fraction of incident power at wavelength λ) and ϵ_λ is the emissivity of the system (ie the ration between the thermal emission of the system and the emission of a blackbody at the same temperature)

1 Orders of magnitude

We first recover the main orders of magnitude from basic principles.

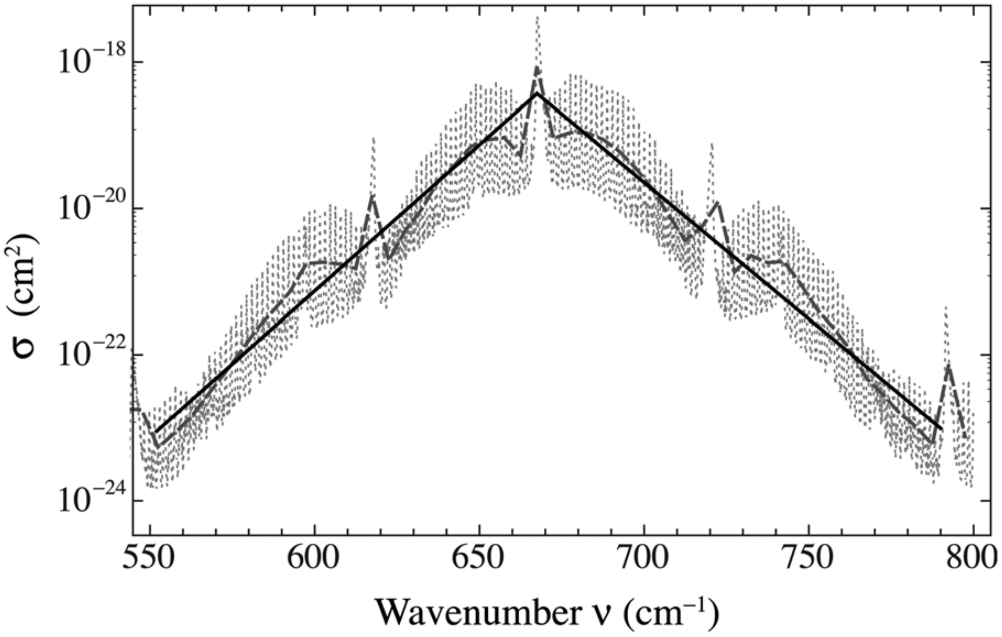
1. Show that the incoming solar energy flux is $F_\odot = 340\,\mathrm{W/m^2}$, as indicated in Fig. 1.
2. Without any atmosphere and with an albedo of $R = 0.3$, what would be the average temperature T_0 on Earth ?
3. Greenhouse effect: consider now the atmosphere as a simple greybody with absorptivity $\mathcal{A} = 0.8$ in the infrared region, and transparent for visible radiations. Evaluate the average temperature T taking this greenhouse effect into account.
4. As a convention, energy flux towards the surface are counted as positive, and energy flux towards space are counted as negative. Estimate the change in net energy flux at the top of the atmosphere dF_P if the surface temperature changes by an amount dT, all other parameters remaining constant. Express and estimate the Planck feedback parameter α_P, defined as $dF_P = \alpha_P dT$.
5. Consider that for some reason, the net energy flux at the top of the atmosphere is increased by an amount F_0 (radiative forcing). If the climate system equilibrates this imbalance only by changing the surface temperature, by how much would the surface temperature vary as a response to this forcing ?

2 CO2 Radiative forcing

1. The atmosphere is actually not equally absorptive at all wavelength. Noting $\mathcal{A}_\lambda$ the absorptivity of the atmosphere at a given wavelength λ, express the radiation emitted from the ground and transmitted through the atmosphere.
2. Changing the amount of CO_2 in the atmosphere changes the absorptivity in the spectral region where CO_2 is opaque, and changes therefore the net energy flux at the top of the atmosphere. In this question, we will estimate the atmospheric absorptivity as a function of CO_2 concentration.
   (a) The absorption cross-section $\sigma(\lambda)$ of a single CO_2 is given below. Neglecting the radiation emission by the atmosphere, show that the intensity emitted from the ground at a given wavelength decreases upon its propagation in the atmosphere as

$$\frac{1}{I_\lambda(z)}\frac{d}{dz}I_\lambda(z) = -n_{\mathrm{CO_2}}(z)\sigma(\lambda) \tag{4}$$

   where $n_{\mathrm{CO_2}}(z)$ is the density of CO_2 molecules at altitude z.



(b) We assume that CO_2 is well mixed with other gases, and that the atmosphere density decreases with altitude as

$$n_{\text{air}}(z) = n_0 \exp\left(-\frac{z}{L}\right)$$
$$\text{with } L = \frac{RT_g}{Mg} \simeq 8\,\text{km and } n_0 = 2.5\,10^{25}\,\text{m}^{-3}$$

Show that the absorptivity of the atmosphere at wavelength λ is

$$\mathcal{A}_\lambda = 1 - \exp\left(-c_{CO_2} n_0 L \sigma(\lambda)\right) \tag{5}$$

where c_{CO_2} is CO_2 concentration in the atmosphere.

(c) Picture qualitatively the change in absorptivity when CO_2 concentration varies. In which spectral region is the difference most significant ?

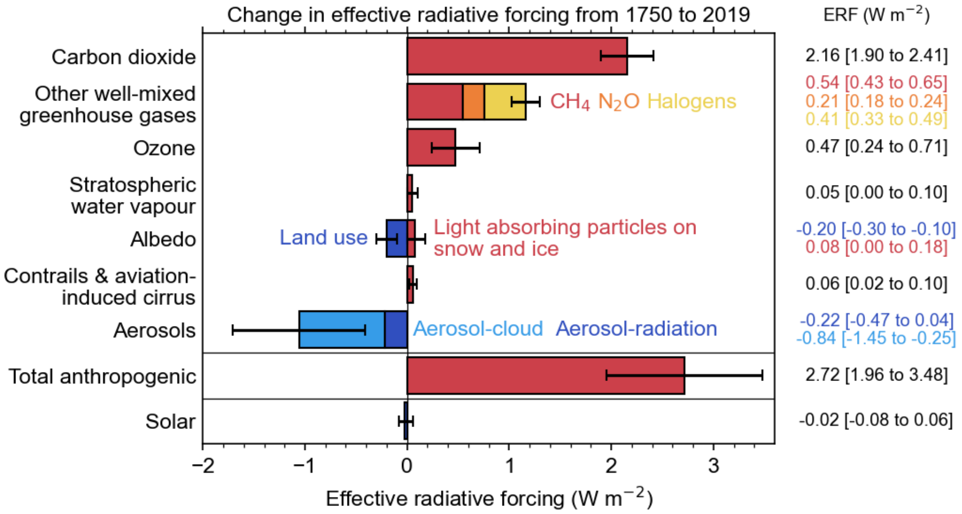
3. We approximate the CO_2 absorption cross section as

$$\sigma(\lambda) = \sigma_0 \exp\left(-r\left|\frac{1}{\lambda} - \frac{1}{\lambda_0}\right|\right) \tag{6}$$

Express the spectral range $\Delta\lambda$ where atmospheric absorptivity is significant.

4. Express the change in net energy flux at the top of the atmosphere resulting from the change in atmospheric absorptivity when CO2 concentration is doubled. Compare your result to the 3.73 W/m² value report in the IPCC AR6 WG1

5. CO2 concentration has increased from 270 to 410 ppm since the preindustrial era. Estimate the corresponding radiative forcing. Compare your result to the IPCC estimate



6. If the climate system equilibrates the corresponding forcing only by changing the surface temperature, by how much would the surface temperature since the XIXth century ?

3 Feedback loops

1. Consider that a change dT in the temperature surface leads to feedback on the climate system (change in albedo, water vapour pressure...), which lead to a radiative forcing $F_{\mathrm{FB}} = \alpha_{\mathrm{FB}} dT$. Show that the temperature change in response to a forcing F_0 is now given by

$$dT = -\frac{1}{\alpha_P + \alpha_{FB}} F_0 \tag{7}$$

2. The value of climate feedback parameters are given in the IPCC report as shown below. Estimate the climate sensitivity, defined as the surface temperature increase if the CO2 concentration is doubled.