École Polytechnique
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PHY 555 – Energy and Environment

PC1 – Orders of Magnitude Solution Strategies

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1 – Everyday life

1. Energy expended: If we use simply black body radiation ($A\sigma T^4$) for a human body, we get a far too large value. Must consider that we are receiving heat from environment. Considering that a human will be comfortable without clothing in an environment of 28°C, the difference between received and emitted will be the energy expended (about 100W).

   Typical work (steps): difference in potential energy of person at top and bottom of stairs (mgh), divided by time to climb.

   Power when cycling: Power = Force x Velocity. To get force exerted by air molecules, consider their change in momentum when struck by rider (with a surface area S).

   $$\Delta p = \rho \times v \times S \times (2 \times v) = 2\rho \times v^2 \times S$$

   We can make this slightly more exact by including lift and drag (factor of ½), and a penetration coefficient. Using $F = {}^{\Delta p}/_{\Delta t}$, we get the formula for power, $P = \frac{1}{2}\rho v^3 S$. The efficiency would be this value divided by the power expended (taking into account all the correct units), and is about 20%. Remaining energy must be dissipated by evaporating water. For this, we need to use the latent heat of evaporation of water (~2260 kJ/kg).

2. First estimate is that muscle power scales with mass. Horse weighs 7 times human, gives 700W (not far from DIN horsepower of 735W).

   For car, use same calculation as for cyclist. For petrol consumption, use 1L gas = 40 MJ. This neglects efficiency of the thermal engine.

   For the semi-trailer, use difference in kinetic energy (${}^1/_2\, mv^2$) divided by time (about 1 min). Should give horsepower of 250, which is reasonable.

   For fuel consumption, note that dependence on velocity is to power of 3.

2 – Energy Sources

1. Same calculation as for cyclist. Additional factor – cannot completely stop wind, which leads to Betz limit (coefficient of 16/27) for maximum power. Gives 4.4 MW. Current turbines are between 2 and 5 MWe peak.
2. Load factor: 24% . The installed cost of a wind turbine is around at 1.5 €/kWe. What about the entirety of French electricity production? French installed electricity production is about 65 GW.
3. Energy flow is river flow multiplied by gravitational potential energy change (don't need width for this). Payback time will give about one month.
4. Photon @ 0.5 μm will have 2.48 eV of energy. 60 photons to make one glucose (2871 kJ/mol). Efficiency of photosynthesis = 22%. Does not take into account self-use and respiration. Real yield of 1%. Calculation should reveal area 3xFrance needed for primary energy consumption of France.

3 – Consumption

1. For consumption with growth rate, $C = C_0 e^{\alpha t}$. Must integrate to get cumulative consumption. Match this to reserves and solve for t. Changing growth rate to 5% dramatically changes the timeline.

4 – Astrophysics

1. Use Stefan Boltzmann again and surface of sun. For power, estimate 1 eV for a single chemical reaction (so per molecule), and use volume of sun (assuming sun is mostly protons). Should give around 15000 years.

   For gravitational binding energy, $\mathrm{d}U = \frac{-G\,m(r) \times 4\pi r^2 \rho}{r}\mathrm{d}r \quad \text{with} \quad m(r) = \frac{4}{3}\pi\rho r^3$

   And integrate. Assume that light emission P compensates for the change in this binding energy.

   $$P = \frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\mathrm{d}U}{\mathrm{d}R} \times \frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{U}{R} \times \frac{\mathrm{d}R}{\mathrm{d}t} \quad \Rightarrow \quad \frac{1}{R} \times \frac{\mathrm{d}R}{\mathrm{d}t} = \frac{P}{U}$$

   Gives a lifetime of about 20 million years. To get real lifetime of 10 billion, magnitude of energy involved per reaction needs to be ~1 MeV.