Geometrical optics

Short wavelength limit

Focus, spherical mirrors, thin lenses, image formation and magnification, aberrations and resolving power

Feynman Vol. I Chapters 26-27

Reminder from last lecture

Monochromatic EM wave: $\vec{E}(\vec{r},t) = \underbrace{\vec{E}_0(\vec{r})}_{\text{amplitude}} e^{i(\overbrace{\phi(\vec{r})}^{\text{phase}} - \omega t)}$ *single color*

Wavefronts (or phase fronts) = equiphase surfaces: $\phi(\vec{r}) = \text{const}$

Short wavelength limit = geometrical optics approximation: $L \gg \lambda$ (L: *typical dimension of system*; λ: *wavelength*)

Local wavenumber: $\vec{k}_{\text{local}}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) \qquad \|\vec{k}_{\text{local}}\| = n\omega/c$

EM energy propagates in the direction of local wavenumber

Optical rays = field lines of Poynting vector / local wavenumber

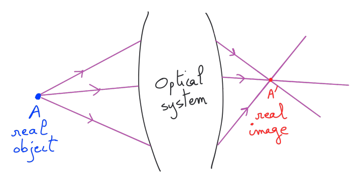
rays perpendicular to wavefronts

Fermat's principle: light takes path requiring the least time.

Snell-Descartes law: $\theta_r = -\theta_i$ (*reflection*) $\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$ (*refraction*)

Image formation:

$A \longrightarrow A'$ (*optical system*)



Flat mirror: $A' = \text{sym}_{\Pi}\, A$

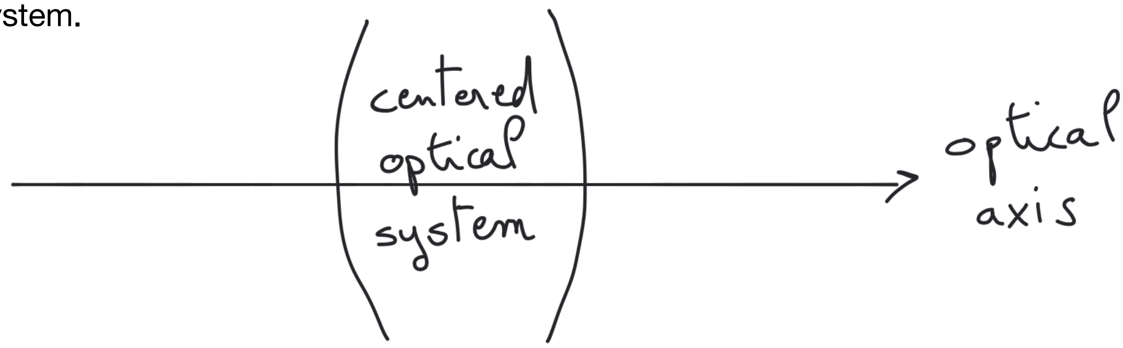
Dielectric interface: $HA' \simeq \dfrac{n_2}{n_1} HA$

Optical axis and focal points

1. Optical axis and focal points

Centered optical systems:

An optical system is said to be centered if it has an axis of symmetry, called the optical axis. An incident ray on the optical axis is not deviated by the optical system.



Gauss conditions or paraxial approximation:

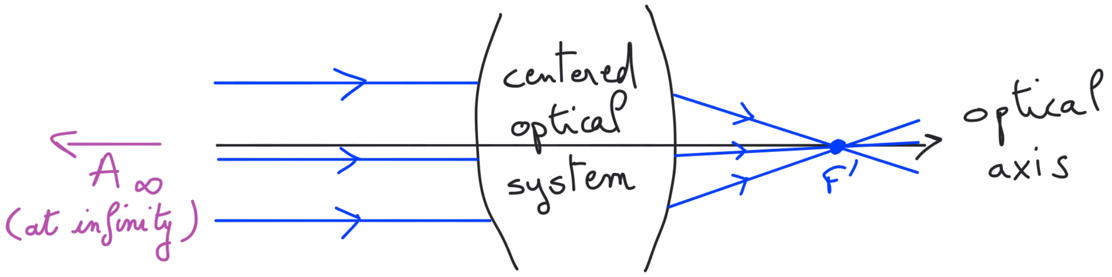
We only consider paraxial rays that:

- form a small angle with respect to the optical axis
- are close the optical axis

1. Optical axis and focal points

Image focal point:

A point at infinity and on axis, A_∞, sends rays parallel to the optical axis towards the optical system. The rays emerging from the optical system intersects at the image focal point F′.



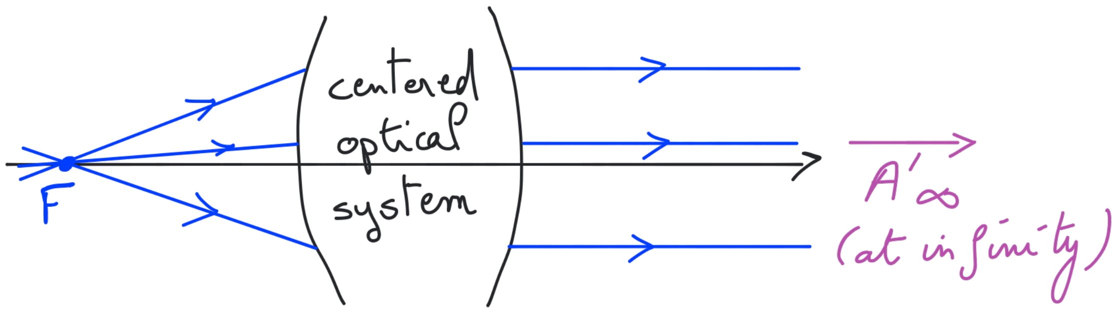
A_∞ (on axis) $\xrightarrow[\text{system}]{\text{optical}}$ F'

The image focal point F′ is the image of an object point located on axis at infinity.

1. Optical axis and focal points

Object focal point:

All rays passing through the object focal point F emerge from the optical system parallel to the optical axis.



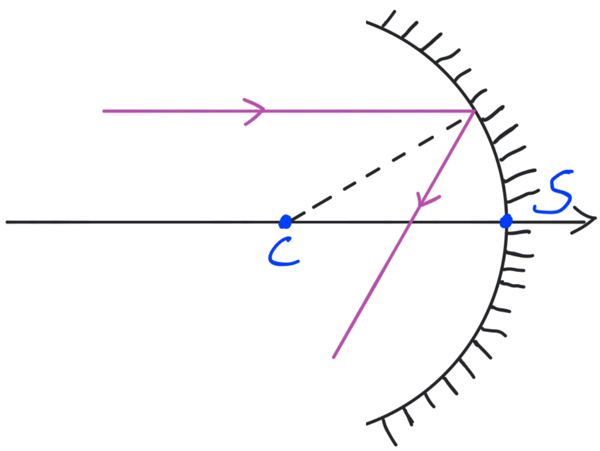
F $\xrightarrow[\text{system}]{\text{optical}}$ A'_∞ (on axis)

The object focal point F is imaged to a point located on axis at infinity.

Spherical mirrors

2. Spherical mirrors

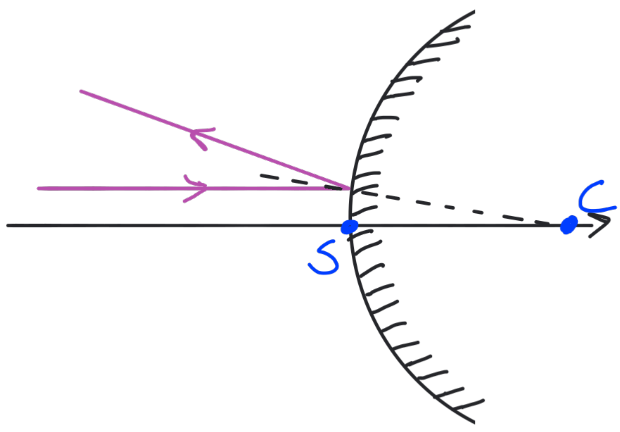
<u>Spherical mirrors:</u>



Concave mirror

$$\overline{SC} < 0$$

(negative algebraic distance means C is on the left of S)



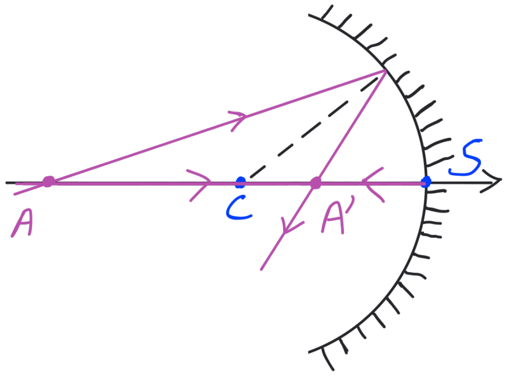
Convex mirror

$$\overline{SC} > 0$$

(positive algebraic distance means C is on the right of S)

2. Spherical mirrors

Image formation by a spherical mirror:



$A \xrightarrow[\text{mirror}]{\text{spherical}} A'$ (approximate stigmatism)

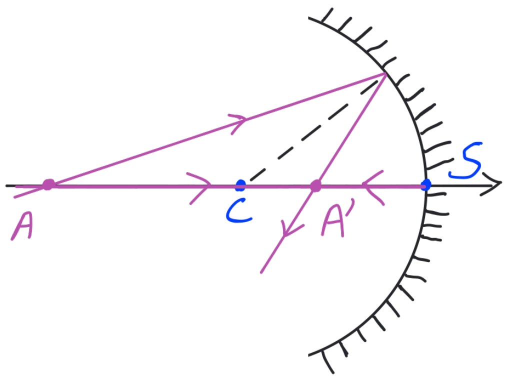
$C \xrightarrow[\text{mirror}]{\text{spherical}} C$ (perfect stigmatism)

Object and image focal points: $\overline{SF'} = \overline{SF} = \overline{SC}/2$

→ The focal length (object focal length $f = \overline{SF}$ or image focal length $f' = \overline{SF'}$) of the spherical mirror is half its radius of curvature.

2. Spherical mirrors

Image formation by a spherical mirror:



$A \xrightarrow[\text{mirror}]{\text{spherical}} A'$ (approximate stigmatism)

$C \xrightarrow[\text{mirror}]{\text{spherical}} C$ (perfect stigmatism)

Object and image focal points: $\overline{SF'} = \overline{SF} = \overline{SC}/2$

Mirror equation: $\frac{1}{\overline{SA}} + \frac{1}{\overline{SA'}} = \frac{1}{\overline{SF'}}$

Magnification: $\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$

2. Spherical mirrors

4 particular rays for the spherical mirror (important for ray tracing):

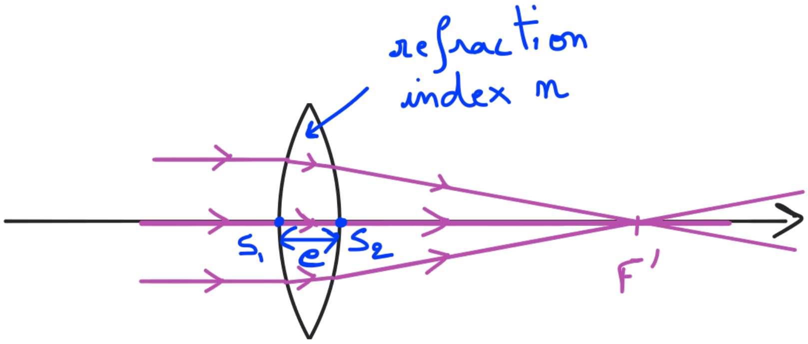
- If the incident ray is parallel to the optical axis, the reflected ray passes through F′ (=F)
- If the incident ray is passing through F (=F’), the reflected ray is parallel to the optical axis
- If the incident ray is passing through C, the ray is reflected on itself
- If the incident ray is passing through S, the reflected ray is the symmetric of the incident ray with respect to the optical axis

Thin lenses

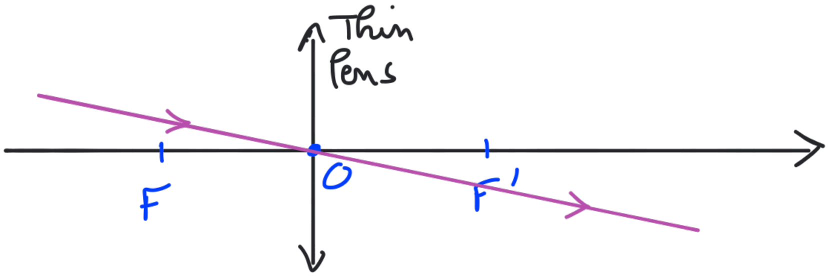
3. Thin lenses

Converging lens

Spherical lens: consists in two spherical dielectric interfaces



Thin lens: the thickness of the lens e is negligible compared to the radii of curvature of the dielectric interfaces. Modeled by:



focal length:

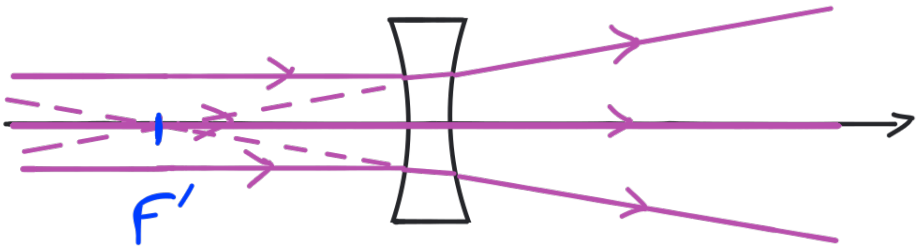
$$f = \overline{OF} < 0$$

$$f' = \overline{OF'} > 0$$

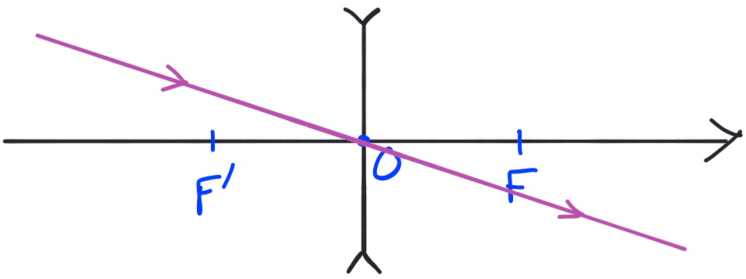
$$f' = -f$$

3. Thin lenses

Diverging lens



Diverging thin lens modeled by:



focal length:

$$f = \overline{OF} > 0$$

$$f' = \overline{OF'} < 0$$

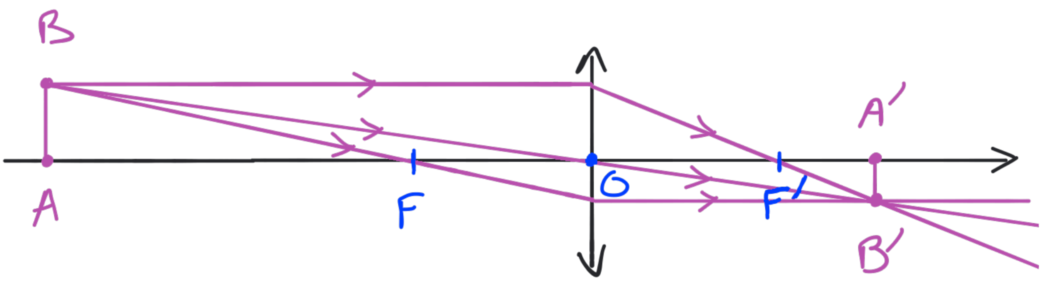
$$f' = -f$$

Vergence of the lens: $V = \dfrac{1}{f'}$ in units of dioptres (equivalent to m^{-1})

3. Thin lenses

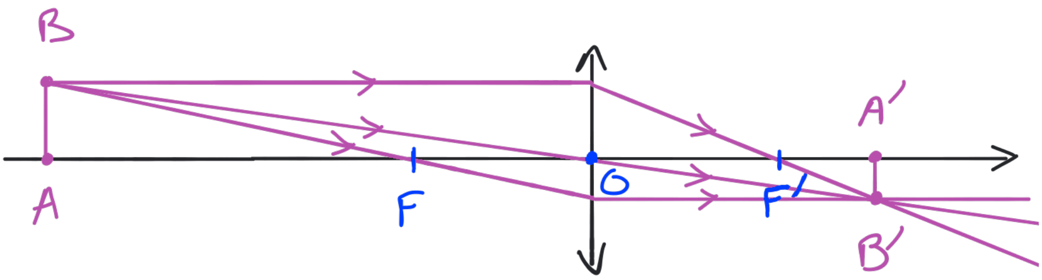
<u>3 particular rays for the thin lens (important for ray tracing):</u>

1. If the incident ray is parallel to the optical axis, the transmitted ray passes through F′
2. If the incident ray is passing through F, the transmitted ray is parallel to the optical axis
3. If the incident ray is passing through O, the ray is not deflected



3. Thin lenses

<u>Image formation by a thin lens:</u>



$A \xrightarrow[\text{lens}]{\text{thin}} A'$ $\qquad$ $B \xrightarrow[\text{lens}]{\text{thin}} B'$

Thin lens equation: $\dfrac{1}{\overline{OA'}} - \dfrac{1}{\overline{OA}} = \dfrac{1}{f'}$

Magnification: $\gamma = \dfrac{\overline{A'B'}}{\overline{AB}} = \dfrac{\overline{OA'}}{\overline{OA}}$

3. Thin lenses

Association of two thin lenses:

- In general: no simple result, have to apply thin lens equations successively.
- If the two lenses are next to each other: equivalent to a single lens and the vergences add, $V = 1/f' = V_1 + V_2 = 1/f_1' + 1/f_2'$.

$$A \xrightarrow{L_1} A' \xrightarrow{L_2} A''$$

$$O_1 = O_2 = O : \quad \frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f_1'} \quad \text{and} \quad \frac{1}{\overline{OA''}} - \frac{1}{\overline{OA'}} = \frac{1}{f_2'}$$

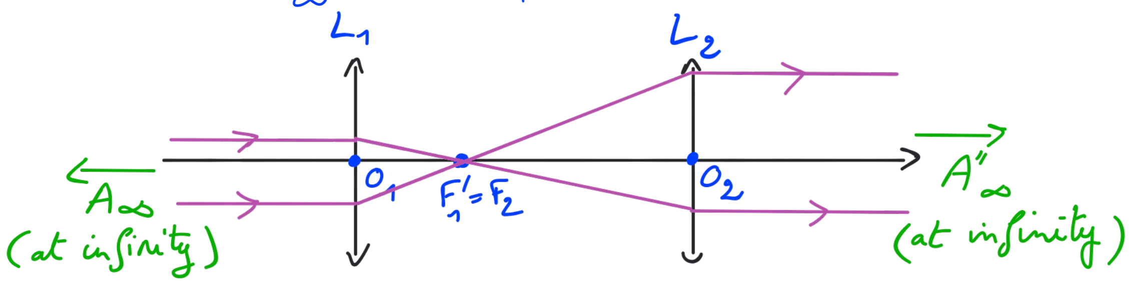
$$\Longrightarrow \quad \frac{1}{\overline{OA''}} - \frac{1}{\overline{OA}} = \frac{1}{f_1'} + \frac{1}{f_2'} = \frac{1}{f'}$$

3. Thin lenses

Association of two thin lenses:

- In general: no simple result, have to apply thin lens equation successively.
- If $F_1'=F_2$, we have an afocal system: the object and image focal points of the optical system are at infinity.

$$A_\infty \xrightarrow{L_1} F_1' = F_2 \longrightarrow A''_\infty$$

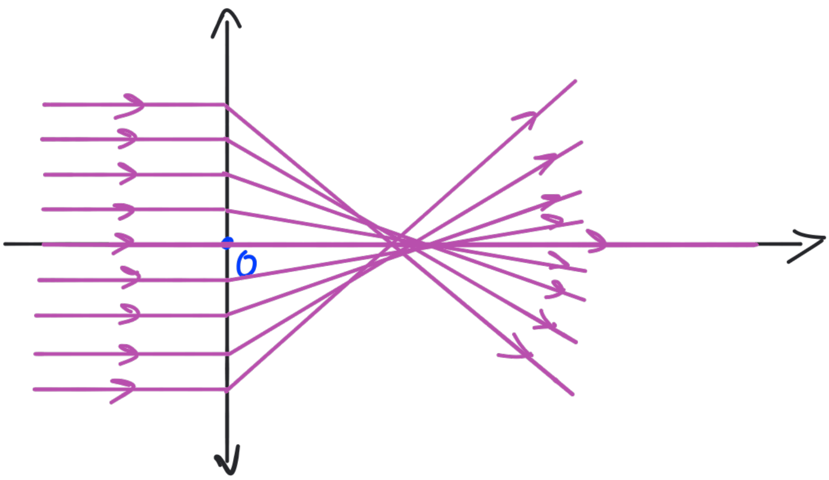


Aberrations and resolution power

4. Aberrations and resolving power

Aberrations:

- Approximate stigmatism: incident rays parallel to optical axis with different radii do not cross at the same point: spherical aberrations

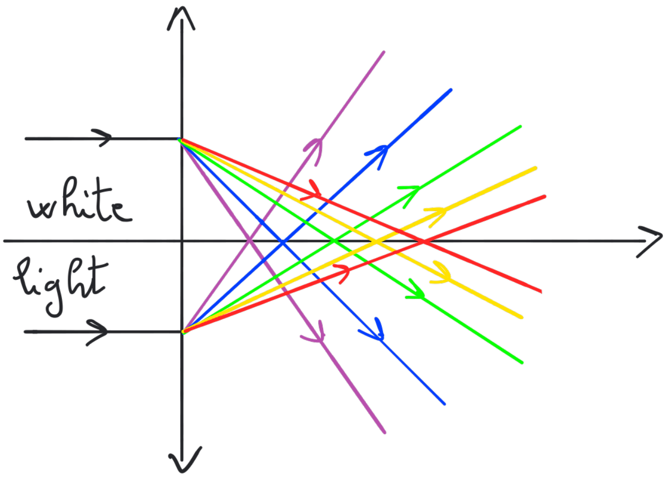


- Example of other common geometrical aberrations: astigmatism (set of horizontally-spread rays and set of vertically-spread rays are not focused to the same point, if lens is tilted for example), coma (incident parallel rays with different transverse position focus to different points), trefoil, etc.

4. Aberrations and resolving power

Aberrations:

- Chromatic aberrations: the index of refraction varies with the wavelength, and therefore the focal length too: different wavelengths are not focused to the same points.

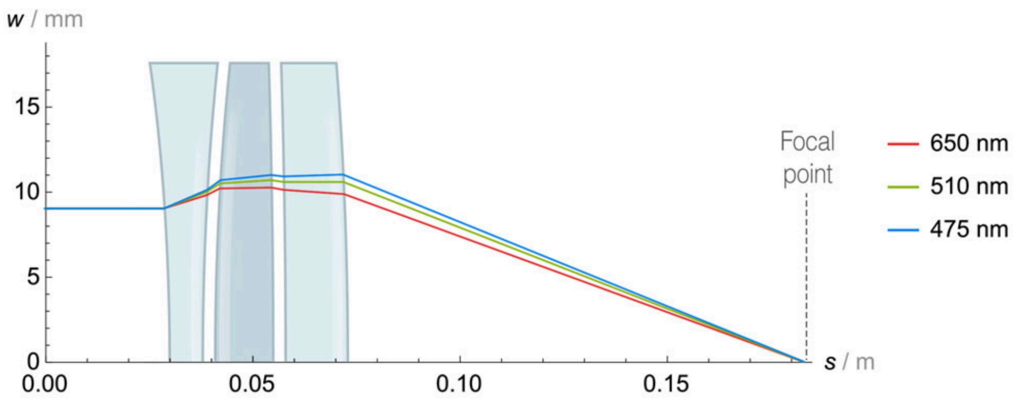


4. Aberrations and resolving power

Aberrations:

- Objectives for photography and cameras: set of up to 10 lenses to correct for chromatic (and geometrical) aberrations over the full spectrum of visible light.

- Example of 3-color apochromatic optical system:

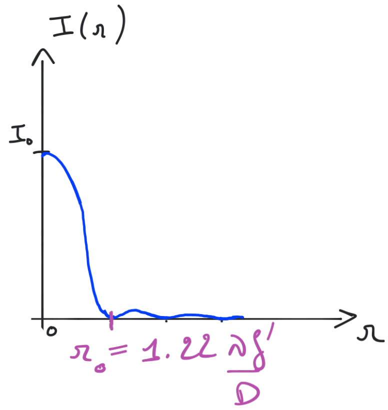


4. Aberrations and resolving power

Resolving power and Airy disk:

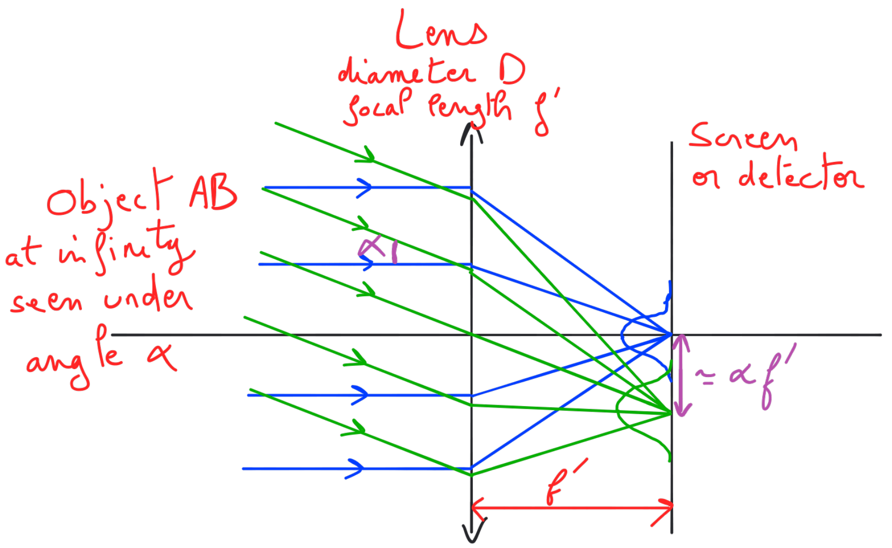
- Even if the optical system is free of aberrations, when parallel rays are focused to a point, the geometrical optics approximation breaks close to the focus.
- Geometrical optics approximation do not hold because the transverse extent of the light becomes comparable to the wavelength close to the focus.
- We have to use wave equations, which predict diffraction. If the lens has a diameter D, focal length f', and is uniformly illuminated by parallel rays, the light intensity profile in the focal plane takes the form of an Airy disk with a first zero (dark ring) at:

$$r_0 = 1.22\,\frac{\lambda f'}{D}$$



4. Aberrations and resolving power

Resolving power and Airy disk:



- Looking at an object AB far away (at infinity), the two Airy disks need to be separated by more than r_0 in order to resolve it, which writes:

$$\alpha f' \geq r_0$$

- The best angular resolution we can achieve is therefore:

$$\boxed{\alpha_{\min} = 1.22\,\frac{\lambda}{D}}$$

Summary

Centered optical system: optical axis = axis of symmetry

Object and image focal points: $A_\infty \longrightarrow F'$ (*object at infinity* → *image focus*) $F \longrightarrow A'_\infty$ (*object focus* → *image at infinity*)

Spherical mirror:

$$\overline{SF'} = \overline{SF} = \overline{SC}/2 \qquad \gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}}$$

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{\overline{SF'}}$$

Thin lens:

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{f'} \qquad \gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{OA'}}{\overline{OA}}$$

Ray tracing for lens:

- Ray parallel to optical axis goes through F′
- Ray that goes through F emerges parallel to optical axis
- Ray that goes through O is not deviated

Resolving power limited by aberrations and diffraction. Diffraction leads to an Airy disk which gives the smallest angle we can resolve:

$$\alpha_{\min} = 1.22\,\frac{\lambda}{D}$$

Remarks on PHY104 end of semester

- Due date for homework #4: today
- Due date for homework #5 (distributed today): June 7 = next week
- Due date for special relativity project: June 14 = in two weeks
- Week before final exam (June 21): revision in tutorial sessions, but no new content:

no lecture on June 21

- What to expect for final exam on June 28? Electromagnetic induction, EM waves, geometrical optics and circuit electricity.