PHY208 – atoms and lasers
Lecture 5

Radiative forces

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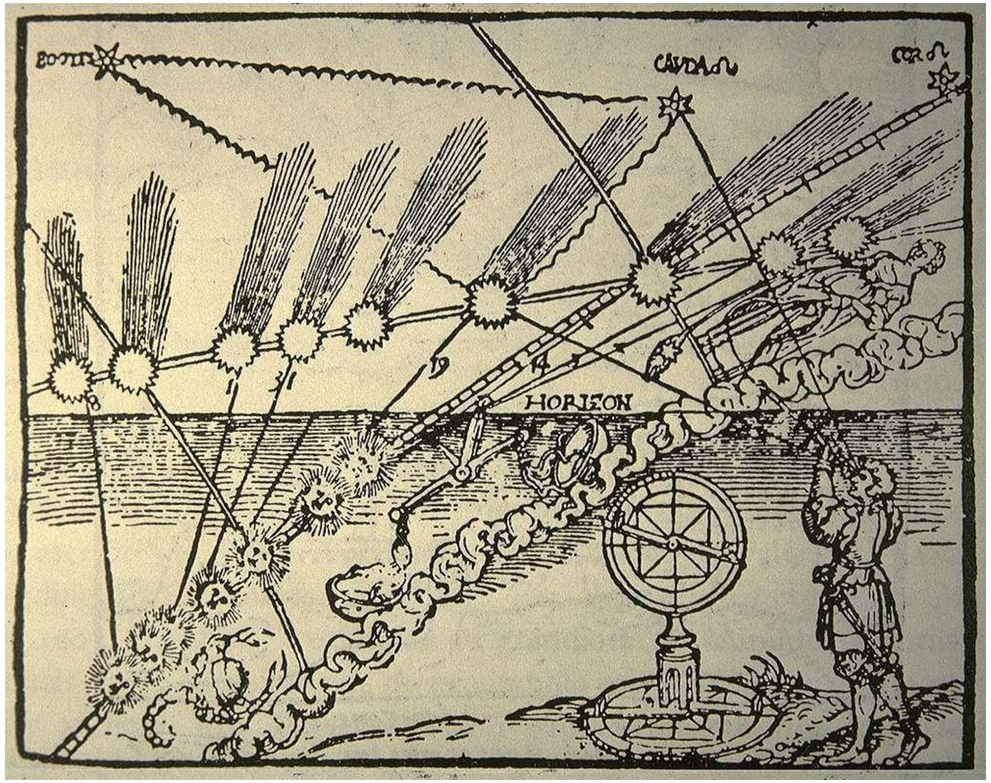
Radiative forces – trial and errors

A comet's tail is formed by matter that the Sun's rays chase through their impulses outside the comet's body

Johannes Kepler, 1619

Commuters are adamant that the Danube is far slower in the morning when the Sun's rays counter its course than in the afternoon when they aid it

Nicolas Hartsoeker, 1696



Outline of lecture 4 – radiative forces

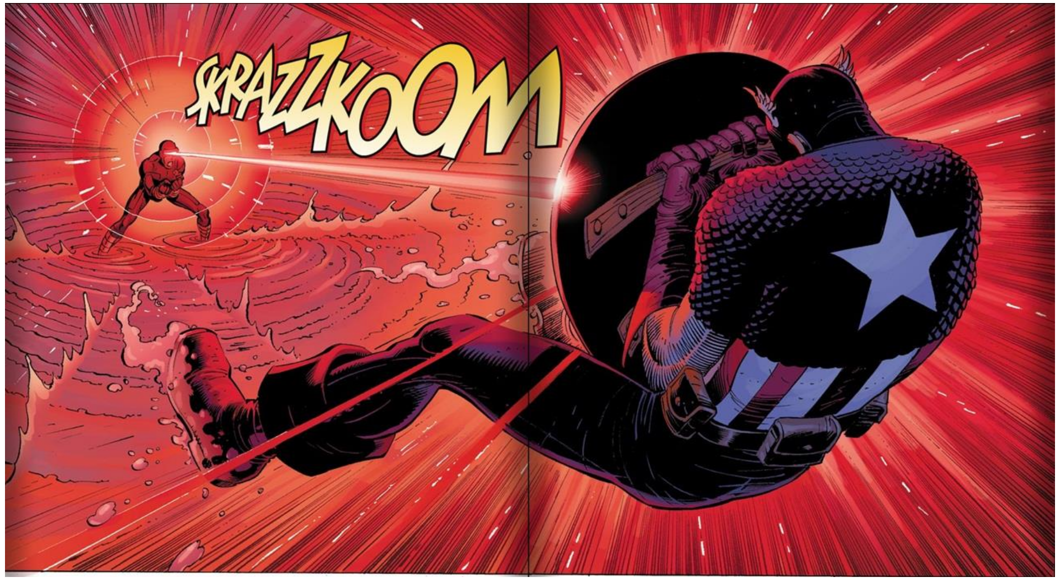
I. Reminder from previous lectures

II. Radiation pressure

III. Application to atoms cooling

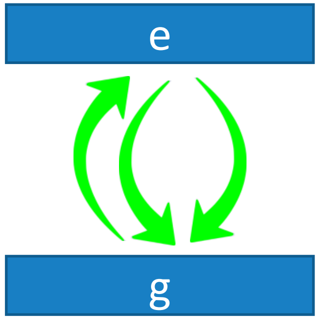
IV. Dipole force

V. Application to optical tweezers



Reminder of previous lectures - rates

Lecture 1



$$r_{\text{spt}} = \Gamma$$

$$r_{\text{abs}} = r_{\text{stim}} = \frac{\sigma(\omega) I}{h\nu}$$

$$= \frac{\Gamma}{2} \frac{1}{1+\left(\frac{\omega-\omega_0}{\Gamma/2}\right)^2} \frac{I}{I_{\text{sat}}} = \frac{\Gamma}{2} s$$

Lecture 3&4

Intrinsic line shape

$$\sigma(\omega) = \frac{\sigma_0}{1+\left(\frac{\omega-\omega_0}{\Gamma/2}\right)^2}$$

Lecture 2

Saturation intensity

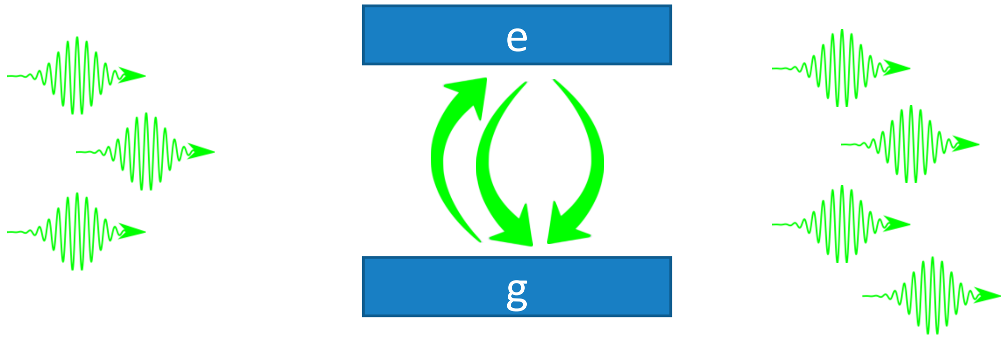
$$I_{\text{sat}} = \frac{h\nu\Gamma}{2\sigma_0}$$

Saturation parameter

$$s = \frac{I}{I_{\text{sat}}} \frac{1}{1+\left(\frac{\omega-\omega_0}{\Gamma/2}\right)^2}$$

Reminder of previous lectures - populations

Lecture 2



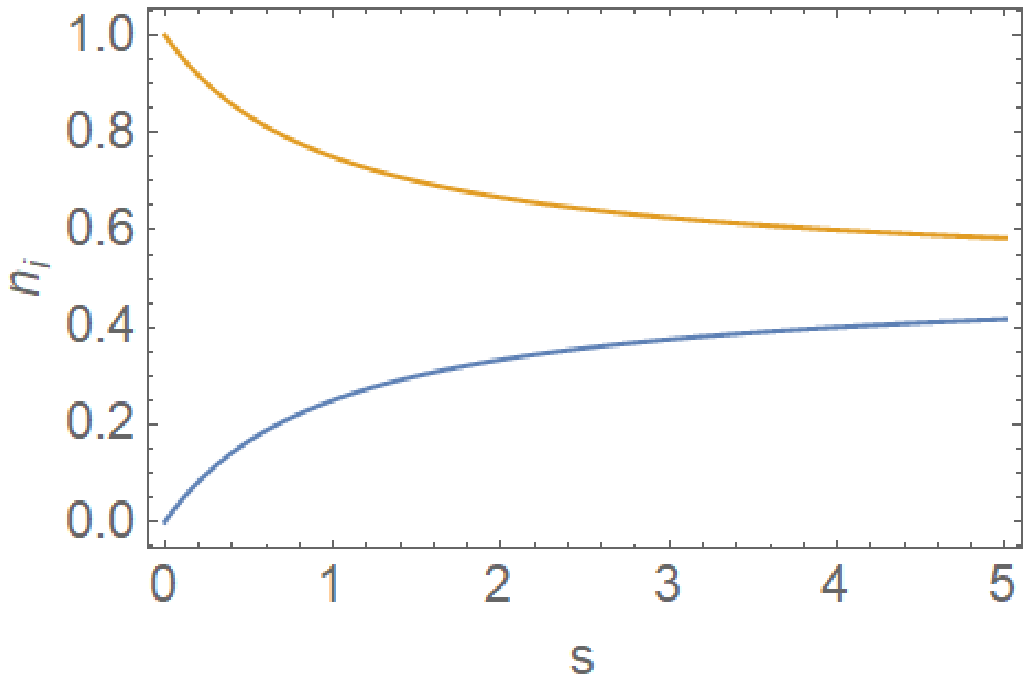
$$r_{\rm spt} = \Gamma$$

$$r_{\rm abs} = r_{\rm stim} = \frac{\Gamma}{2} s$$

$$n_e r_{\rm stim} + n_e r_{\rm spt} = n_g r_{\rm abs}$$

$$n_e + n_g = 1$$

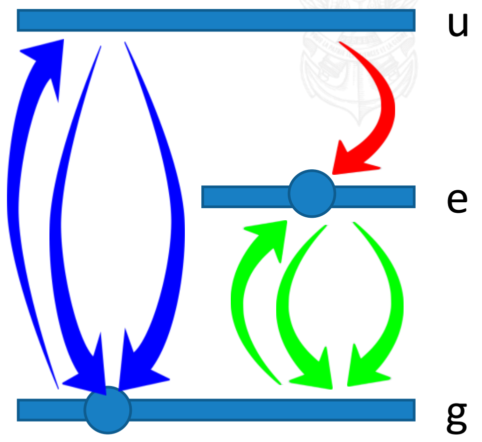
$$n_e = \frac{s/2}{1+s} \qquad n_g = \frac{1+s/2}{1+s}$$



Reminder of previous lectures - methods

Adiabatic elimination

$$\frac{d}{dt}n_u \simeq 0$$



If one dynamic is much faster than all the others,

consider the corresponding degree of freedom is always at steady state

Perturbation method

$$\Delta E_n = \langle \psi_n | \hat{V} | \psi_n \rangle$$

Starting from equilibrium, we add a small perturbation.

the response of the system can be estimated from the steady state properties.

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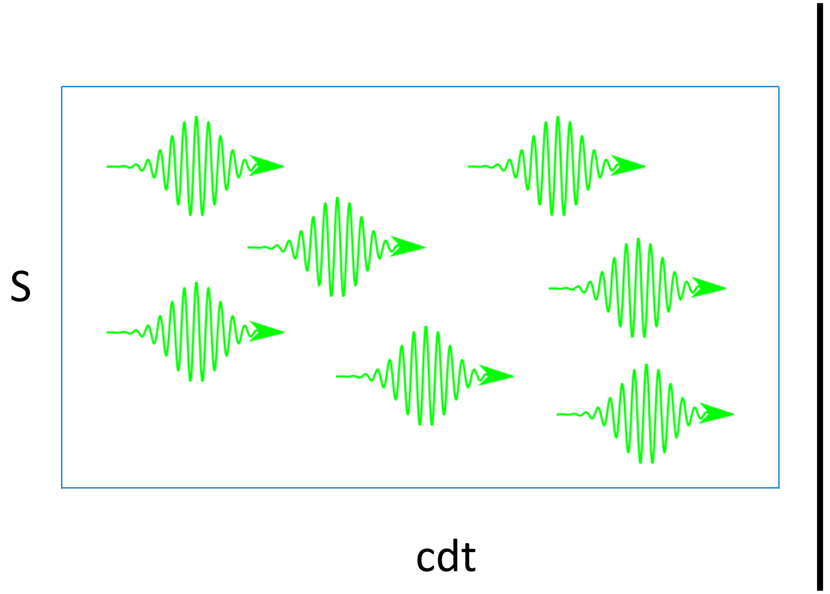
Radiation pressure – a naive approach

Momentum balance on photon reaching the well between t and t+dt



$$\frac{p(t+dt)-p(t)}{dt} = F_{\text{wall}\to\gamma} = -F_{\gamma\to\text{wall}}$$

$$F_{\gamma\to\text{wall}} = \frac{I}{h\nu c} \times Sc \times \frac{h\nu}{c} = \frac{I}{c} \times S$$

$$p(t) = n \times S\,cdt \times \hbar k = \frac{I}{h\nu c} \times S\,cdt \times \hbar k$$

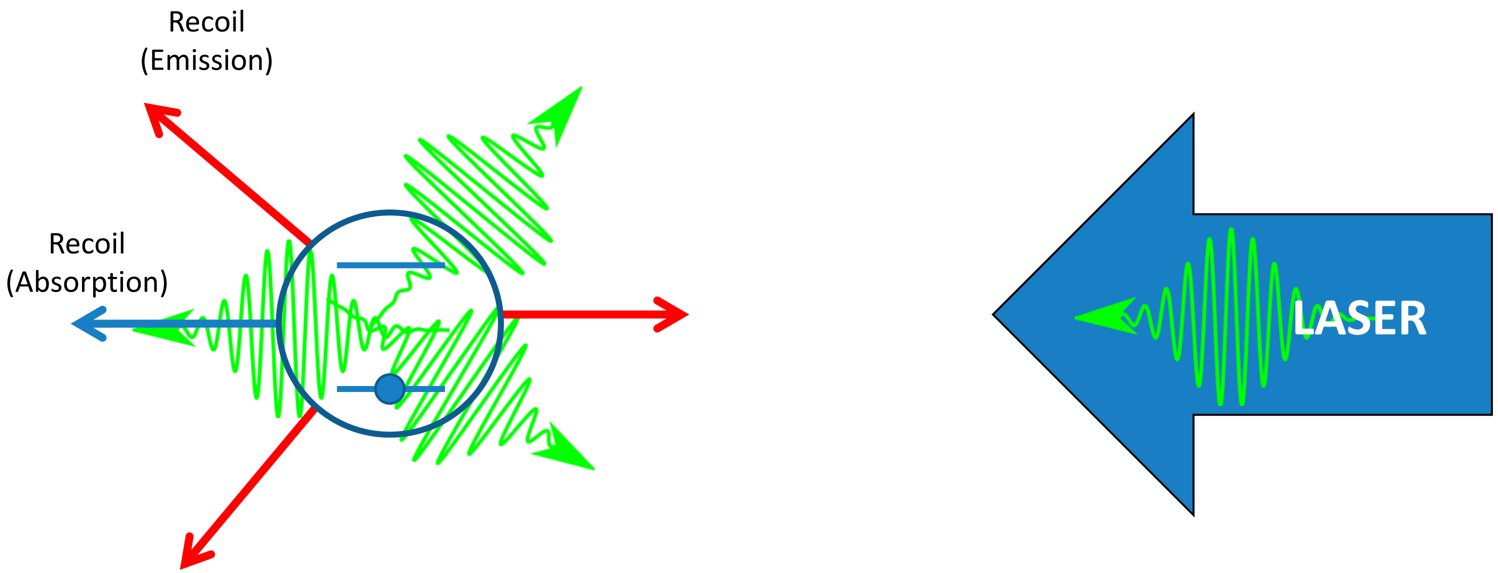
$$dE = n \times S\,cdt \times h\nu = ISdt$$

$$p(t+dt) = 0$$

Is the wall always able to absorb a photon ?

Absorption <-> Emission. What about the emitted photons ?

Radiation pressure – semi-quantum model



Momentum balance

$$\frac{d}{dt}\mathbf{p}_{\mathrm{atom}} = \mathbf{F}_{RP} = -n_e r_{\mathrm{spt}} \hbar \mathbf{k}_{\mathrm{spt}}(t) - n_e r_{\mathrm{stim}} \hbar \mathbf{k}_L + n_g r_{\mathrm{abs}} \hbar \mathbf{k}_L$$

Average force over many cycles :

$$\langle \mathbf{F}_{RP} \rangle = (n_g r_{\mathrm{abs}} - n_e r_{\mathrm{stim}})\, \hbar \mathbf{k}_L \quad = \frac{s}{1+s}\frac{\Gamma}{2}\, \hbar \mathbf{k}_L$$

Radiation pressure

Radiation pressure

$$\langle \mathbf{F}_{RP} \rangle = \frac{s}{1+s}\frac{\Gamma}{2}\,\hbar\mathbf{k}_L$$

Low intensity regime

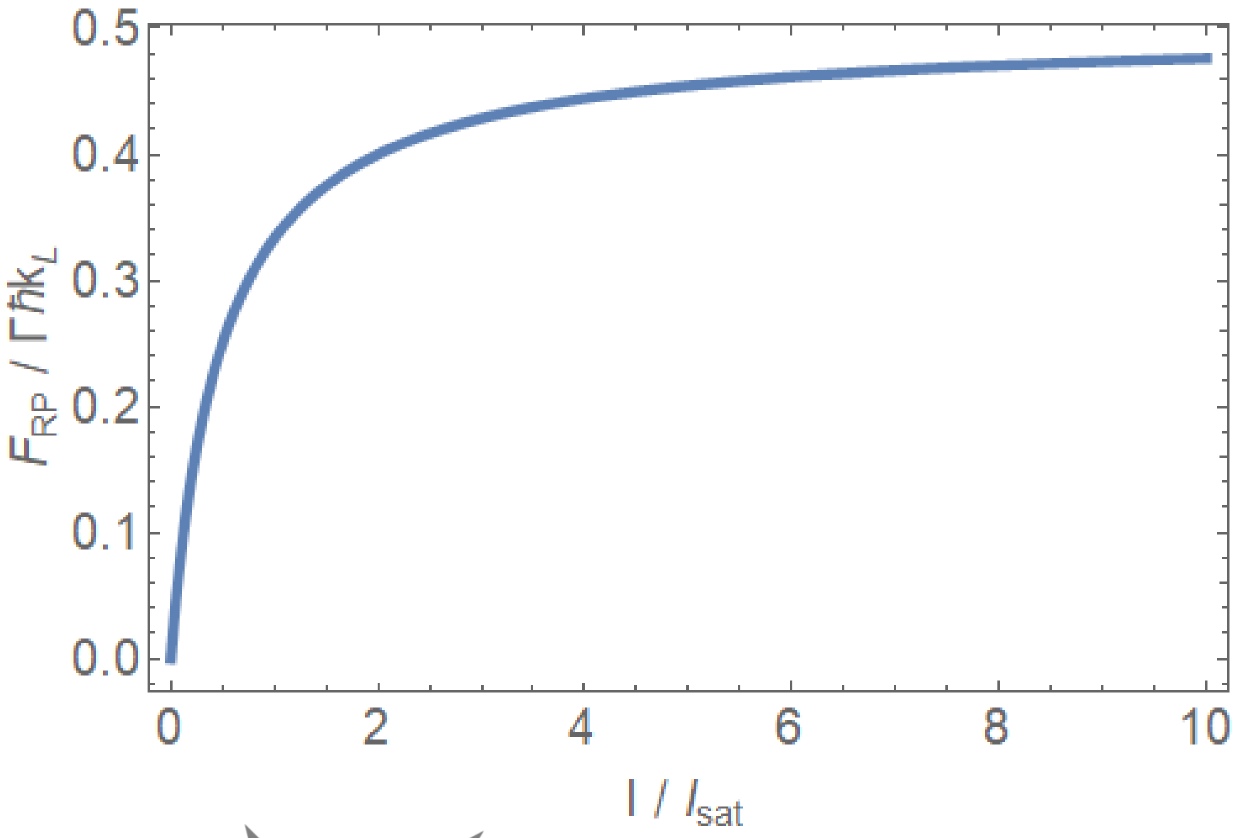
$$F_{PR} \underset{s\ll 1}{\longrightarrow} s\frac{\hbar\Gamma k_L}{2} = \frac{I}{c}\,\sigma$$

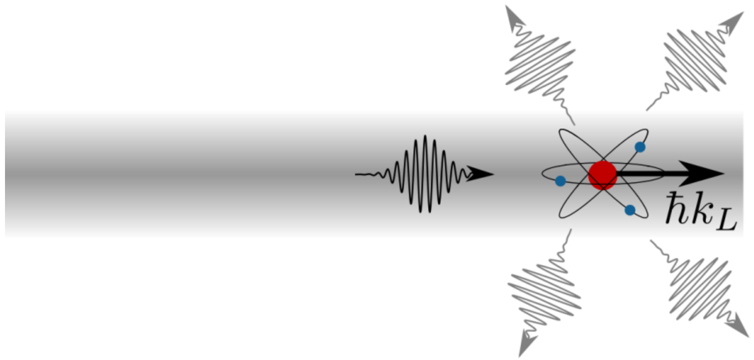
High intensity regime

$$F_{PR} \underset{s\gg 1}{\longrightarrow} \frac{\Gamma}{2}\hbar k_L$$

Scattering rate

$$\Gamma_{\text{scat}} = \frac{s}{1+s}\frac{\Gamma}{2}$$





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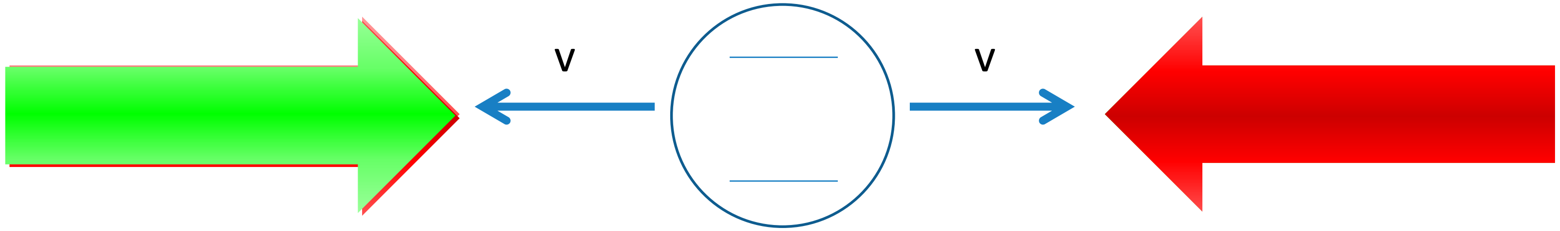
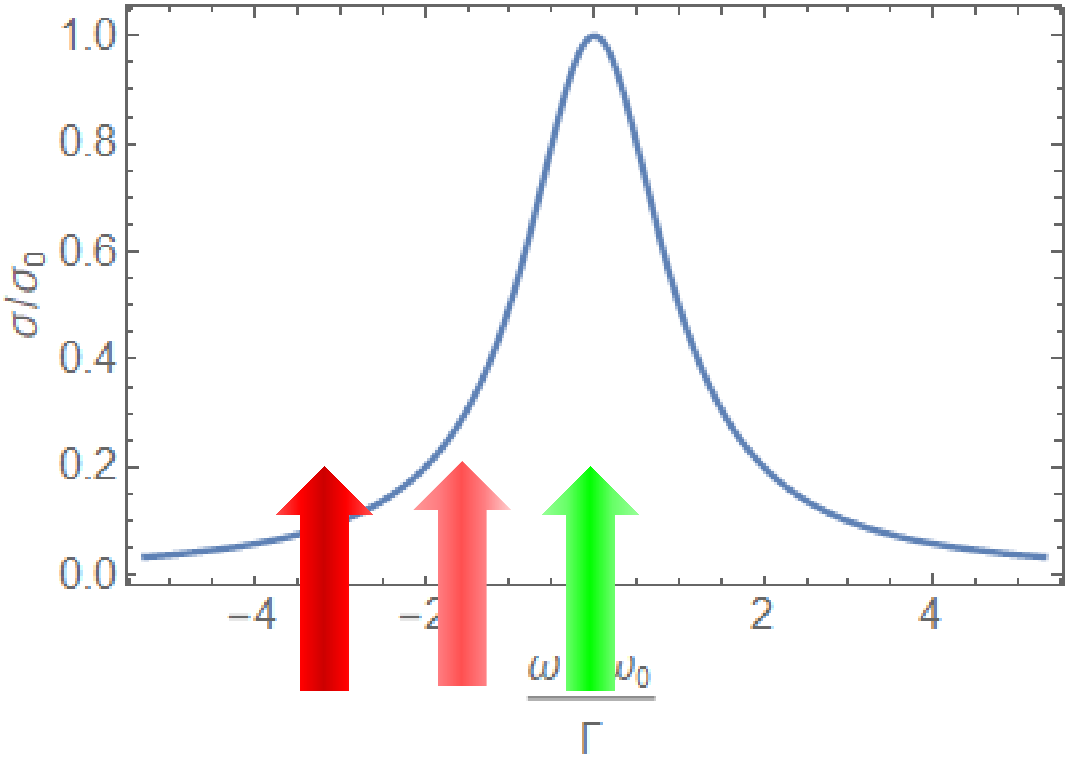
Application to laser cooling





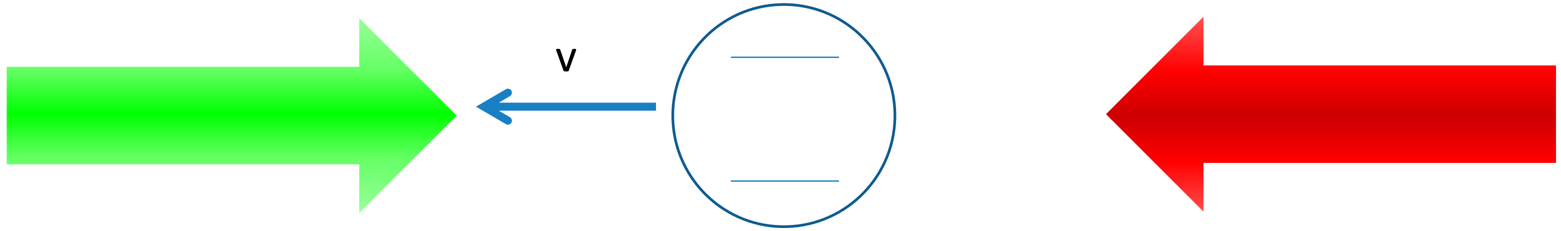
Low intensity limit : $F_{PR} \xrightarrow[s \ll 1]{} s\frac{\hbar \Gamma k_L}{2} = \frac{I}{c}\sigma$

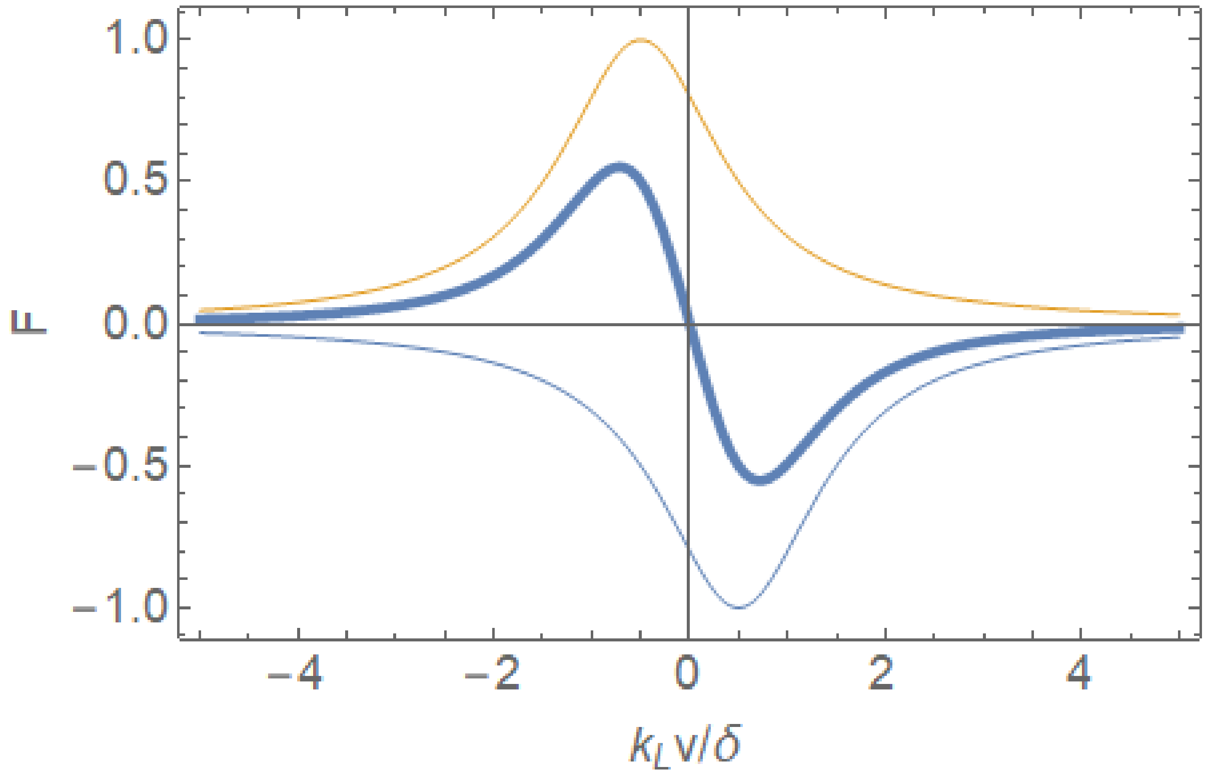
$$\langle \mathbf{F}_{\mathrm{PR},1} \rangle = \frac{I}{c} \frac{\sigma_0}{1+4\frac{(\delta_0 - \mathbf{k}_L \cdot \mathbf{v})^2}{\Gamma^2}} \mathbf{u}$$

$$\langle \mathbf{F}_{\mathrm{PR},2} \rangle = -\frac{I}{c} \frac{\sigma_0}{1+4\frac{(\delta_0 + \mathbf{k}_L \cdot \mathbf{v})^2}{\Gamma^2}} \mathbf{u}$$

$$\langle \mathbf{F}_{\mathrm{tot}} \rangle = \langle \mathbf{F}_{\mathrm{PR},1} \rangle + \langle \mathbf{F}_{\mathrm{PR},2} \rangle$$

Application to laser cooling

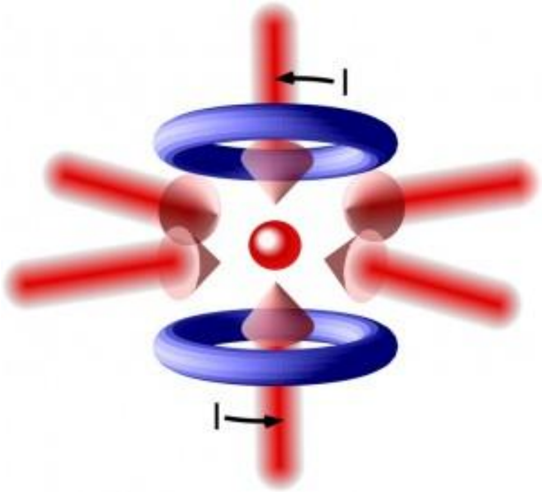


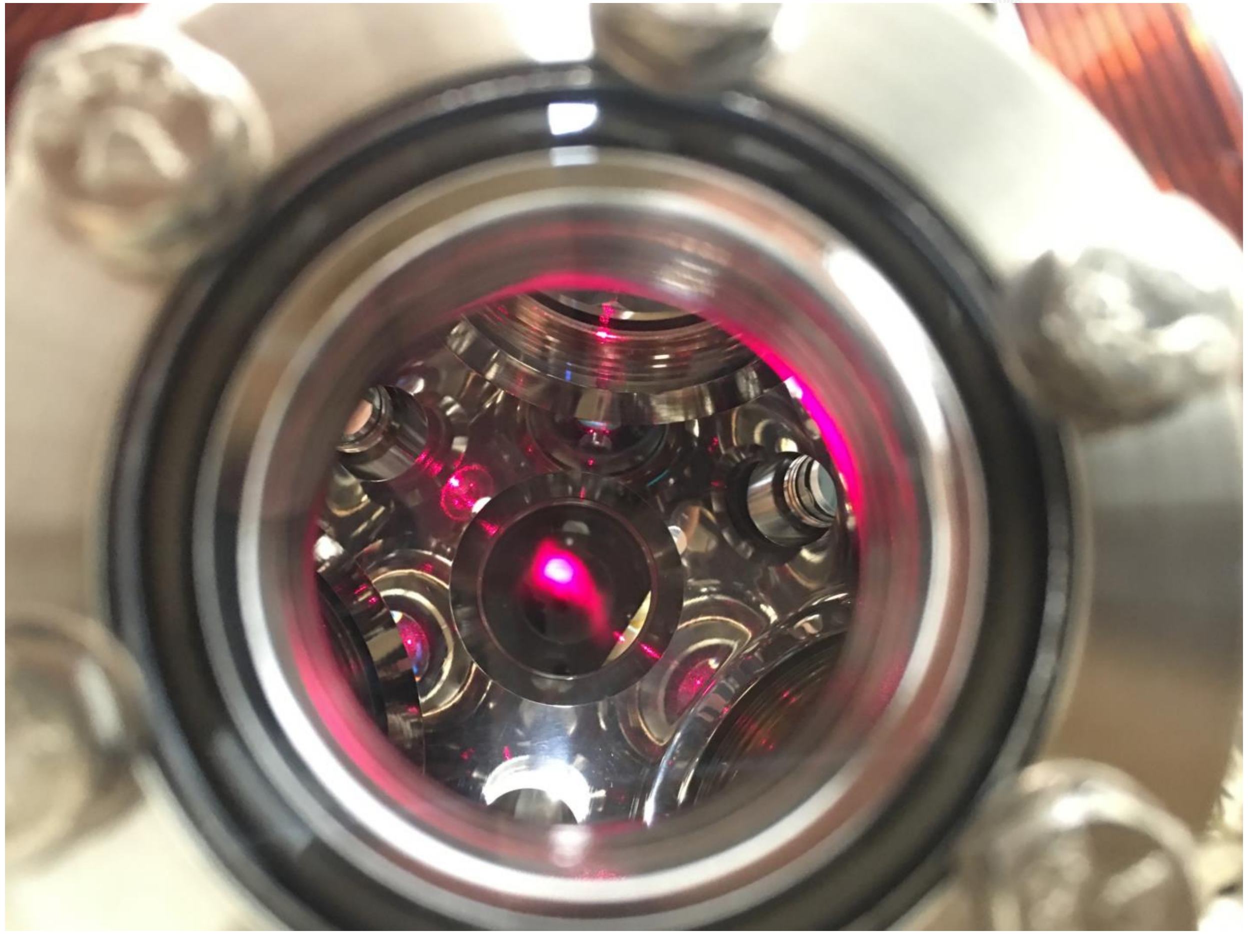


$$\langle \mathbf{F}_{\mathrm{tot}} \rangle = \langle \mathbf{F}_{\mathrm{PR},1} \rangle + \langle \mathbf{F}_{\mathrm{PR},2} \rangle$$

$$\simeq 16 \frac{I}{c} \sigma_0 \frac{\delta_0/\Gamma^2}{\left(1+4\delta_0^2/\Gamma^2\right)^2} k_L\, v = -m\alpha v$$

Optical molasses

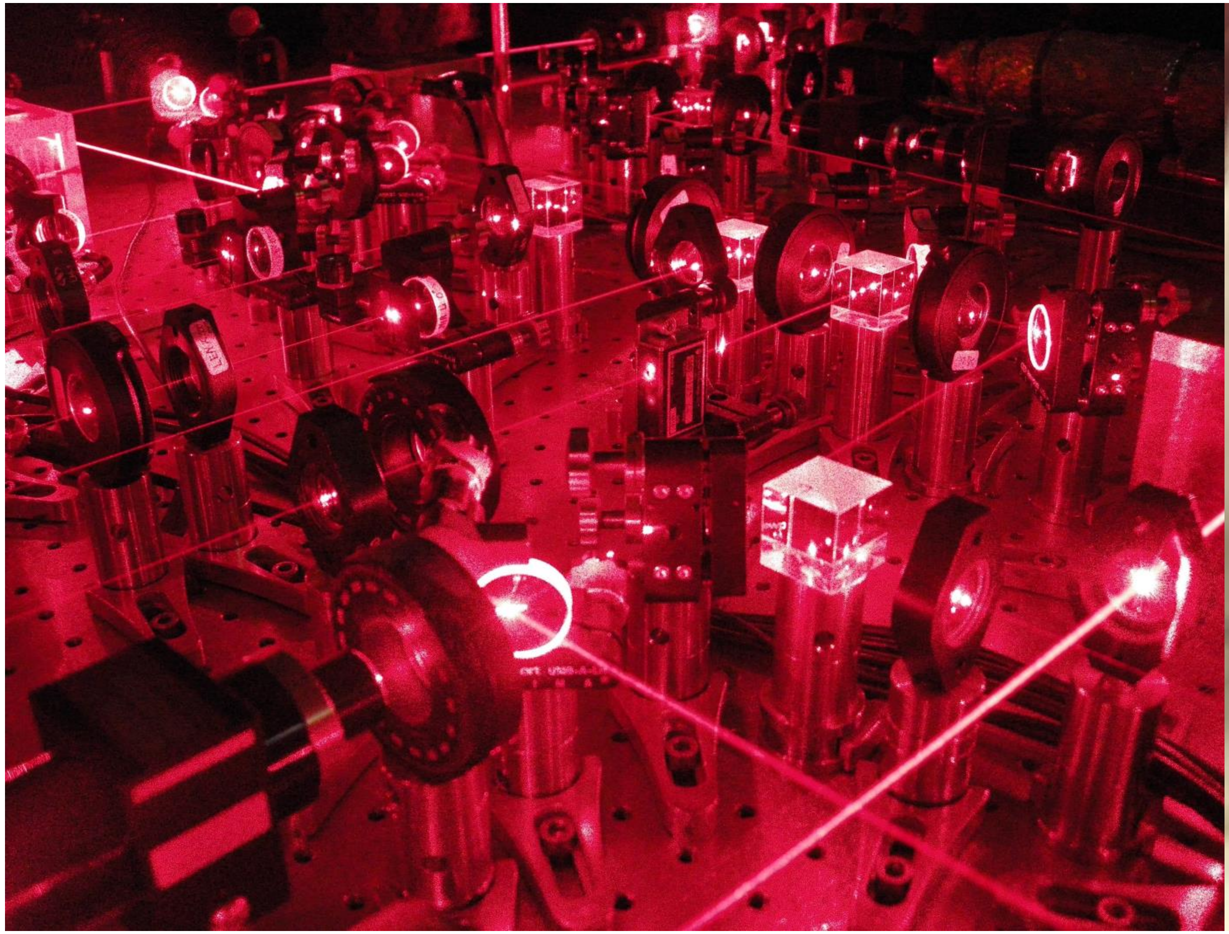


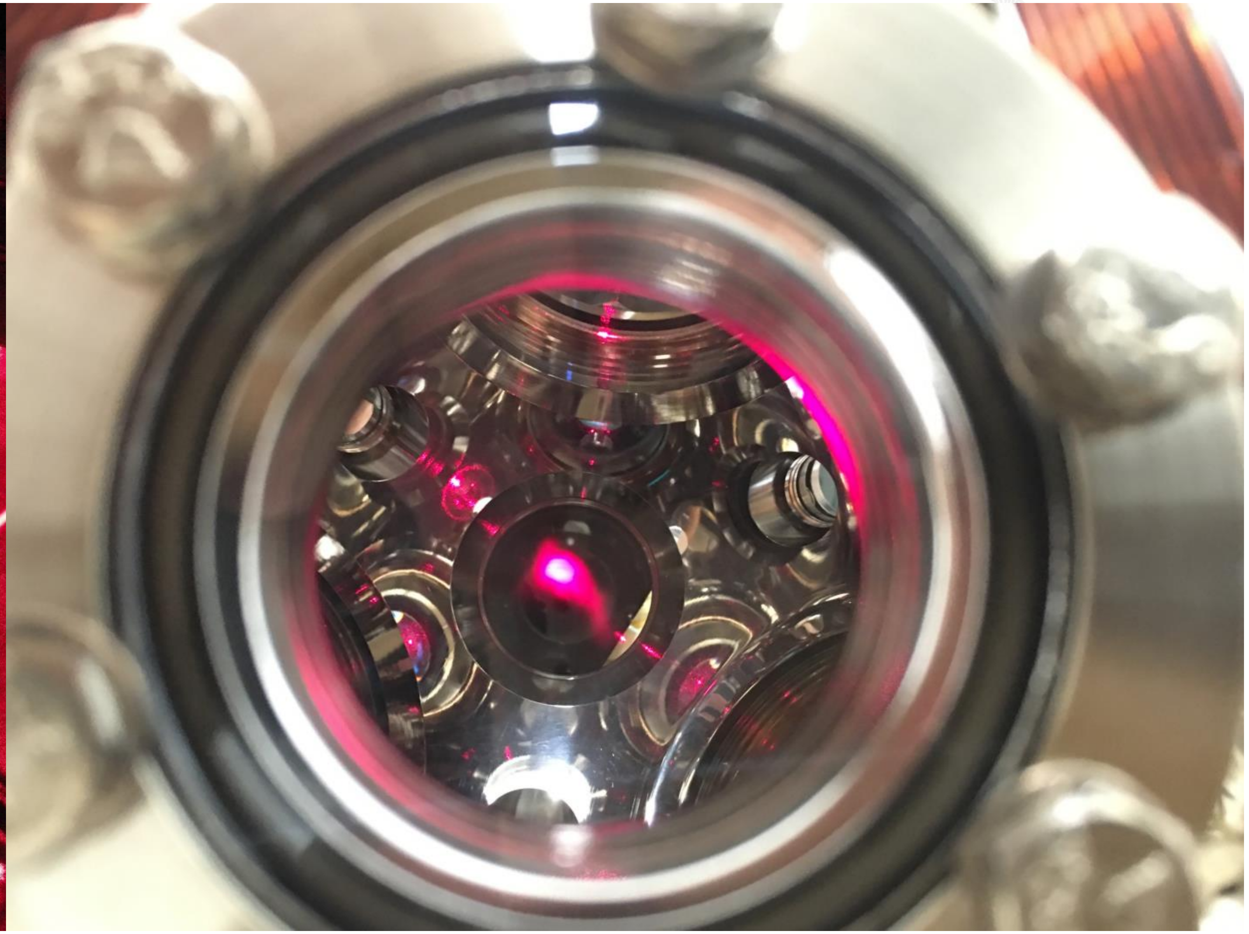


Steve Chu, Claude Cohen-Tannoudji, Bill Phillips

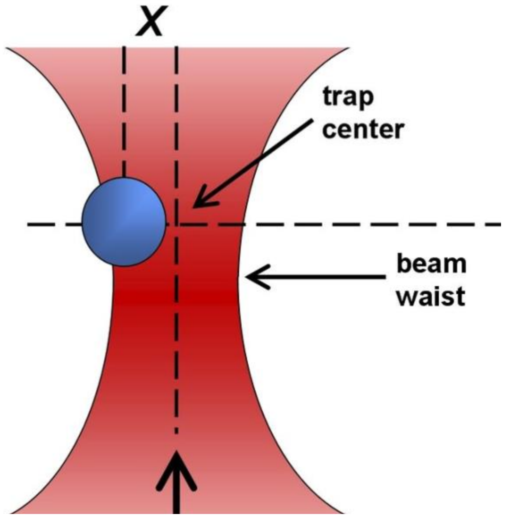
1997 « for development of methods to cool and trap atoms with laser light. »

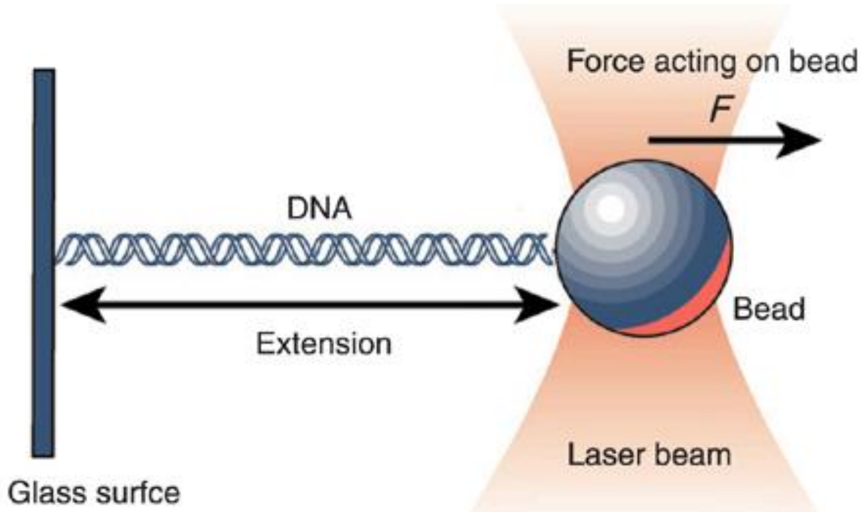
Optical molasses

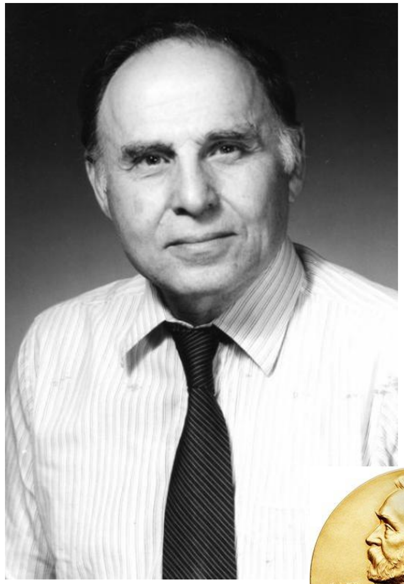


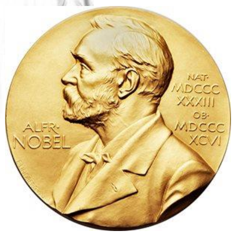


Radiation pressure is not enough !









Arthur Ashkin

2018 « in particular "for the optical tweezers and their application to biological systems" »

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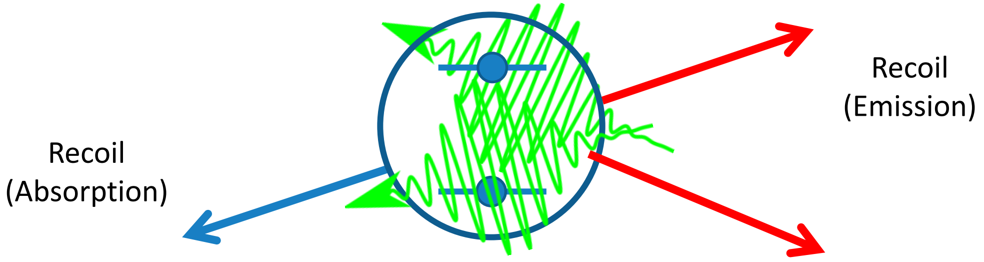
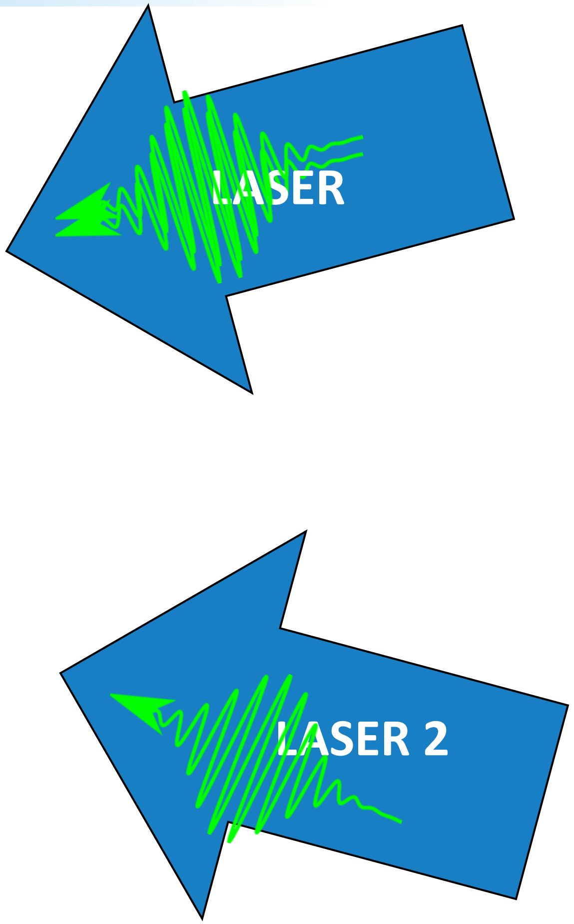
Semi-quantum model





Momentum balance

$$\frac{d\mathbf{p}}{dt} = n_g \frac{\sigma I}{h\nu}\hbar \mathbf{k_1} + n_g \frac{\sigma I}{h\nu}\hbar \mathbf{k_2} - n_e \frac{\sigma I}{h\nu}\mathbf{k_1} - n_e \frac{\sigma I}{h\nu}\hbar \mathbf{k_2} - n_e \Gamma \hbar \mathbf{k}_{spt}$$

$$= n_g \frac{\sigma I}{h\nu}\hbar \left(\mathbf{k_1} + \mathbf{k_2}\right) - n_e \frac{\sigma I}{h\nu}\hbar \left(\mathbf{k_1} + \mathbf{k_2}\right) - n_e \Gamma \hbar \mathbf{k}_{spt}$$

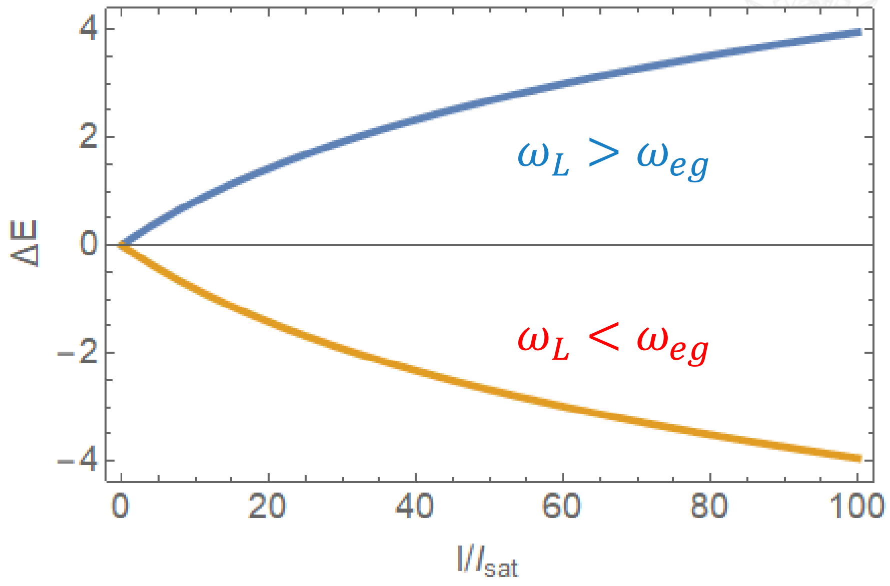
Results from a semi-quantum model

Admitted, not proven :

$$\mathbf{F}_{\mathrm{Dip}} = -\frac{\hbar\delta}{2}\frac{\boldsymbol{\nabla} s}{1+s} = -\boldsymbol{\nabla}\underbrace{\frac{\hbar\delta}{2}\log(1+s)}_{U_{\mathrm{Dip}}}$$

Optical dipole potential:

$$U_{\mathrm{Dip}} = \frac{\hbar\delta}{2}\log(1+s) \underset{s\ll 1}{\simeq} \frac{\hbar}{8}\frac{\Gamma^2}{I_{\mathrm{sat}}}\frac{I_L}{\delta}$$

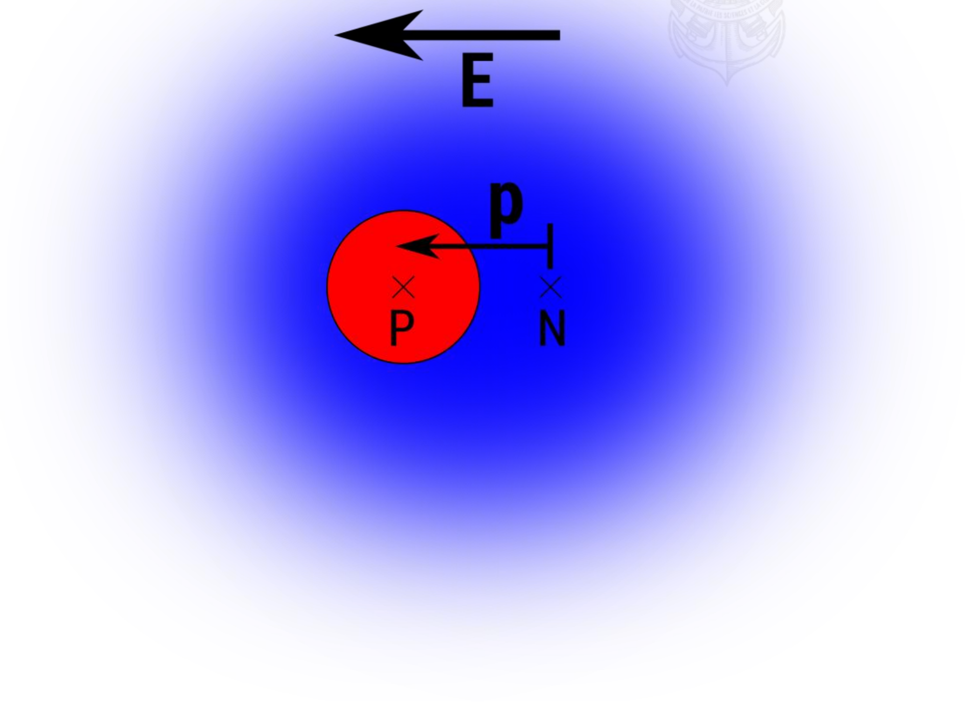


Lamp (or light) shift

A semi-classical approach

$$U_{\text{Dip}} = \frac{\hbar\delta}{2}\log(1+s) \underset{s\ll 1}{\simeq} \frac{\hbar}{8}\frac{\Gamma^2}{I_{\text{sat}}}\frac{I_L}{\delta}$$

$$U_{\text{Dip}} = \int_0^E p\,dE = -\frac{1}{2}pE$$



Electric field $\mathbf{E}(\mathbf{r},t) = \mathcal{R}\left(\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t}\right)$

Atomic dipole $\mathbf{p}(t) = -e\mathbf{r}(t) = \mathcal{R}\left(\alpha\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t}\right)$

Dipole potential

$$U_{\text{Dip}} = -\frac{1}{2}\frac{\alpha\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t} + \alpha^*\boldsymbol{\mathcal{E}}^*(\mathbf{r})e^{+i\omega t}}{2}\frac{\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t} + \boldsymbol{\mathcal{E}}^*(\mathbf{r})e^{+i\omega t}}{2}$$

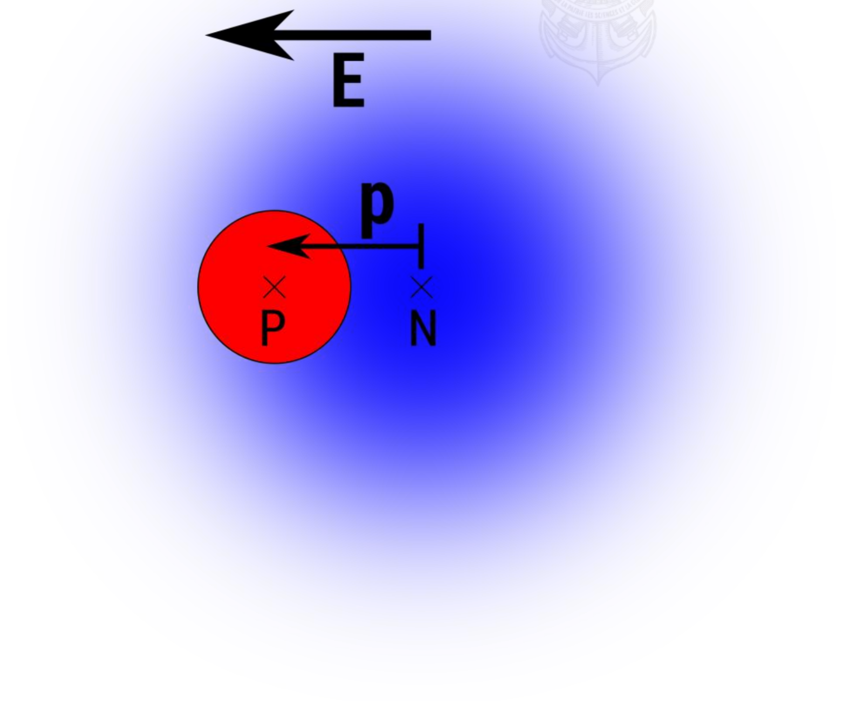
$$\simeq -\frac{1}{2}\left(\frac{\alpha+\alpha^*}{2}\right)\frac{|\boldsymbol{\mathcal{E}}(\mathbf{r})|^2}{2}$$

$$U_{\text{Dip}} = -\frac{\alpha'}{2\epsilon_0 c}I(\mathbf{r})$$

A semi-classical approach (cont.)

$$U_{\text{Dip}} = \frac{\hbar\delta}{2}\log(1+s) \underset{s\ll 1}{\simeq} \frac{\hbar}{8}\frac{\Gamma^2}{I_{\text{sat}}}\frac{I_L}{\delta}$$

$$U_{\text{Dip}} = -\frac{\alpha'}{2\epsilon_0 c} I(\mathbf{r})$$



Atomic polarizability

$$\alpha = \frac{\epsilon_0 \chi_{\text{Lorentz}}}{n} = \frac{e^2}{m}\frac{\omega_0^2-\omega^2}{\left(\omega_0^2-\omega^2\right)^2+\Gamma^2\omega^2} + i\,\frac{e^2}{m}\frac{\omega\Gamma}{\left(\omega_0^2-\omega^2\right)^2+\Gamma^2\omega^2}$$

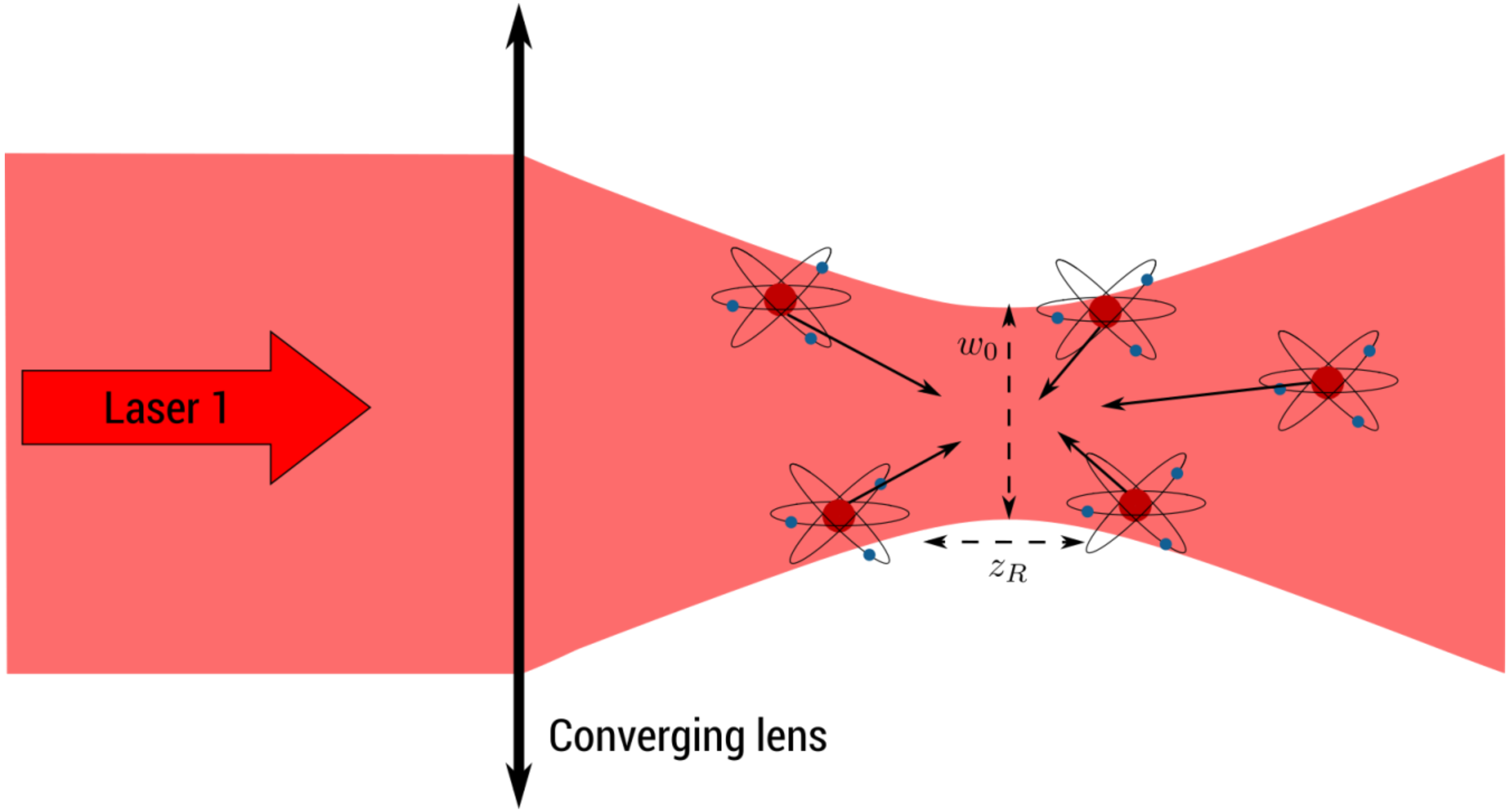
$$U_{\text{Dip}} = -\frac{\sigma_0\Gamma}{2}\frac{\omega_0^2-\omega^2}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2} I(\mathbf{r})$$

$$= \frac{\hbar\omega_0\Gamma^2}{4}\frac{\omega_0^2-\omega^2}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{I(\mathbf{r})}{I_{\text{sat}}} \simeq \frac{\hbar\omega_0\Gamma^2}{4}\frac{1}{\omega^2-\omega_0^2}\frac{I(\mathbf{r})}{I_{\text{sat}}}$$

$$= \frac{\hbar\Gamma^2}{8\delta}\frac{I(\mathbf{r})}{I_{\text{sat}}}$$

Outline of lecture 4

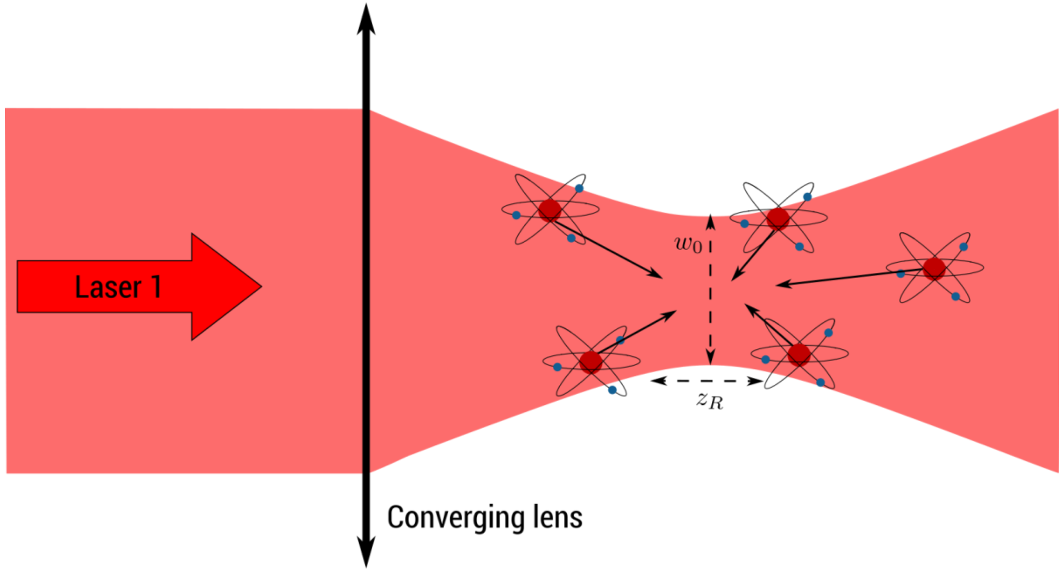
Optical dipole trap - concept

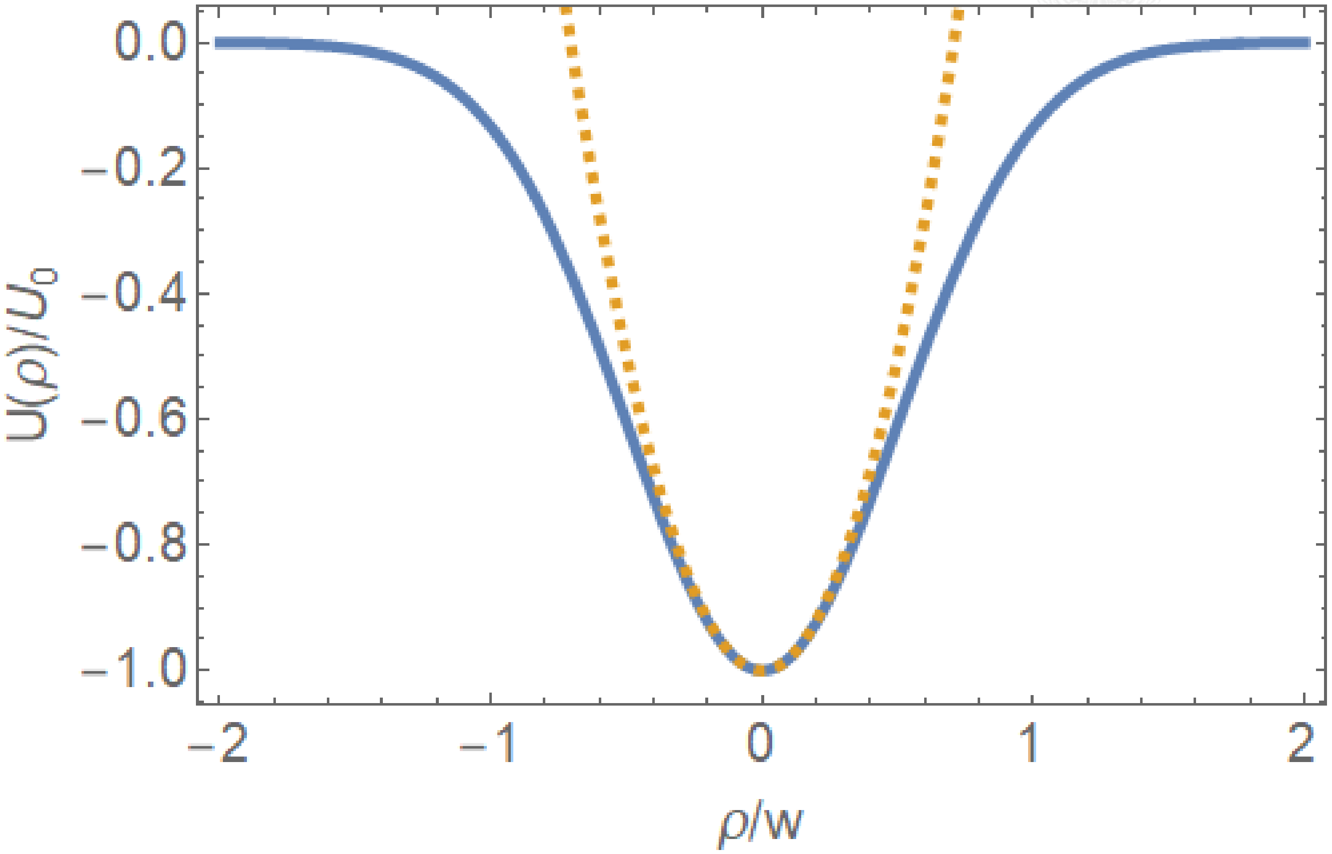


$$I = I_0 \exp\left(-2\frac{r^2}{w_0^2}\right)$$

$$U_{\mathrm{Dip}}(\mathbf{r}) = \frac{\hbar\Gamma^2}{8\delta}\frac{I(\mathbf{r})}{I_{\mathrm{sat}}}$$

Optical dipole trap – harmonic approx.



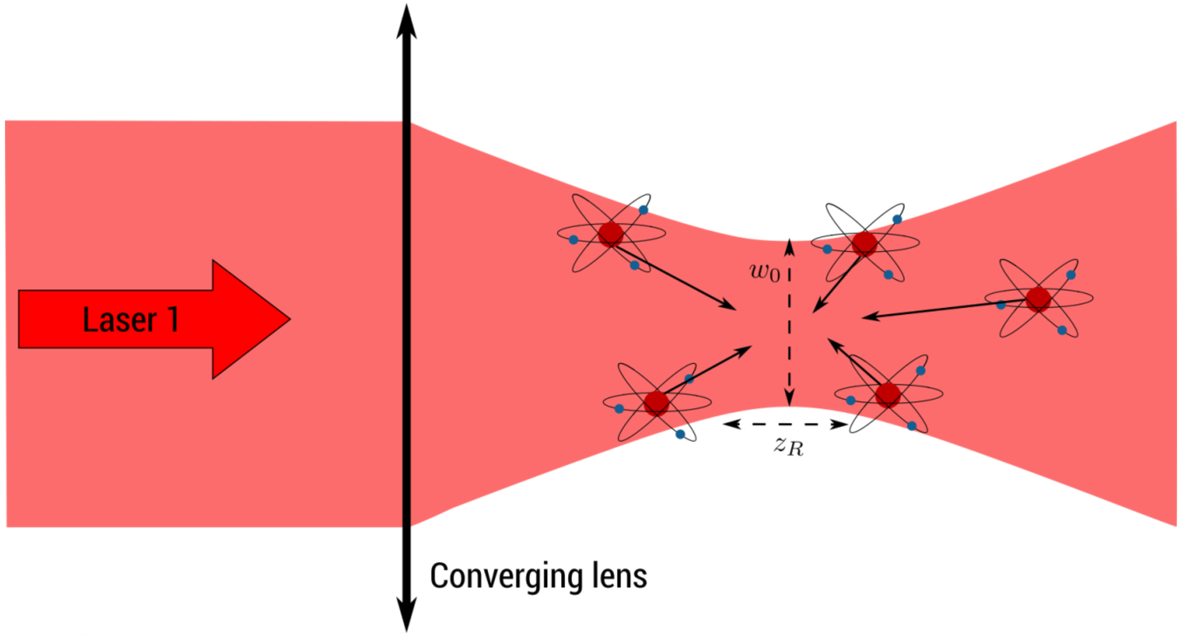


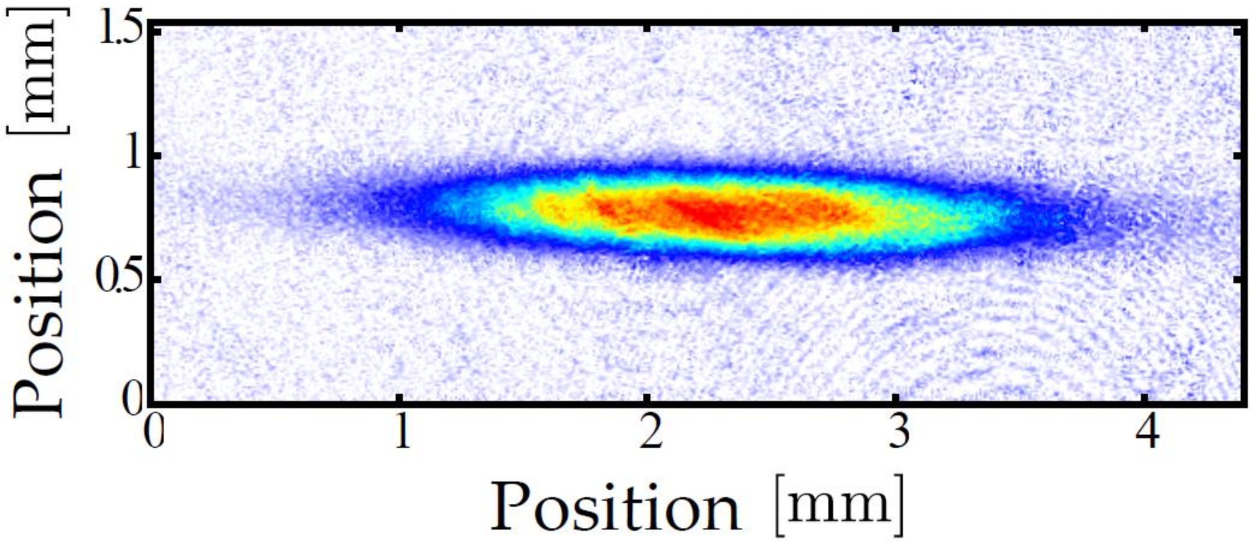
$$I = I_0 \exp\left(-2\frac{r^2}{w_0^2}\right) \simeq I_0 - 2I_0\frac{r^2}{w_0^2}$$

$$U_{\mathrm{Dip}}(\mathbf{r}) = \frac{\hbar\Gamma^2}{8\delta}\frac{I(\mathbf{r})}{I_{\mathrm{sat}}}$$

$$U \simeq -U_0 + \frac{1}{2}m\omega_\perp^2\rho^2 + \frac{1}{2}m\omega_\parallel z^2$$

Optical dipole trap – competing effects





$$F_{PR} \propto \frac{I_L(\mathbf{r})}{\delta^2}$$

$$F_{\mathrm{Dip,z}} \propto \frac{I_L(\mathbf{r})}{\delta}$$

7 10^6 ^{40}K atoms at 2µK.

Take home message

Atom in a laser beam => 2 forces

Radiative pressure

$$\langle \mathbf{F}_{RP} \rangle = \frac{s}{1+s}\frac{\Gamma}{2}\hbar\mathbf{k}_L$$

$$\Gamma_{\text{scat.}} = \frac{s}{1+s}\frac{\Gamma}{2}$$

Dissipative force, χ''

$$\boldsymbol{\nabla}\varphi$$

Dipole force

$$\mathbf{F}_{\text{Dip}} = -\frac{\hbar\delta}{2}\frac{\boldsymbol{\nabla}s}{1+s} = -\boldsymbol{\nabla}U_{\text{Dip}}$$

$$U_{\text{Dip}}(\mathbf{r}) = \frac{\hbar}{8}\frac{\Gamma^2}{I_{\text{sat}}}\frac{I_L}{\delta}$$

Reactive force, χ'

$$\boldsymbol{\nabla}|E|^2$$

Application :
Optical molasses

Application :
Optical tweezer