Chapter 5

Light on atoms: radiative forces

In this chapter, we will see how light can be used to exert a force on atoms - and thus to control and manipulate them. We will see two basic forces, namely the radiation pressure and the dipole force. To understand these forces, we will have to go back and fourth between the internal degress of freedom of the atoms (ie energy levels and transition rates) and the external degrees of freedom (center of mass motion).

We recall key results from previous lectures which will be used extensively in this chapter:

- An atom in the ground state has a probability per second r_{abs} of absorbing a photon and going to the ground state. An atom in the excited state has a probability per second r_{stim} of emitting a photon by stimulated emission and Γ of emitting a photon by spontaenous emission, with

$$r_{\text{abs}} = r_{\text{stim}} = \frac{\sigma(\omega) I}{h\nu} = \frac{\Gamma}{2} \frac{1}{1 + \left(\frac{\omega - \omega_0}{\Gamma/2}\right)^2} \frac{I}{I_{\text{sat}}} = \frac{\Gamma}{2} s \tag{5.0.1}$$

  where we have introduced the following quantities

$$\sigma(\omega) = \frac{\sigma_0}{1 + \left(\frac{\omega - \omega_0}{\Gamma/2}\right)^2} \qquad I_{\text{sat}} = \frac{h\nu\Gamma}{2\sigma_0} \qquad s = \frac{I}{I_{\text{sat}}} \frac{1}{1 + \left(\frac{\omega - \omega_0}{\Gamma/2}\right)^2} \tag{5.0.2}$$

- In steady state, a 2 levels system addressed by a light beam follows the detailed balance

$$n_e r_{\text{stim}} + n_e r_{\text{spt}} = n_g r_{\text{abs}} \tag{5.0.3}$$

$$n_e + n_g = 1 \tag{5.0.4}$$

  leading to

$$n_g = \frac{1 + s/2}{1 + s} \qquad n_e = \frac{s/2}{1 + s} \tag{5.0.5}$$

- Adiabatic elimination : consider that fast variables (typically, internal variables) have time relaxed towards steady state while the slow variable (typically, external degrees of freedom) are still changing.

- Perturbation theory : starting from a well known equilibrium state, expand the perturbation as a series, and consider together terms of same order.

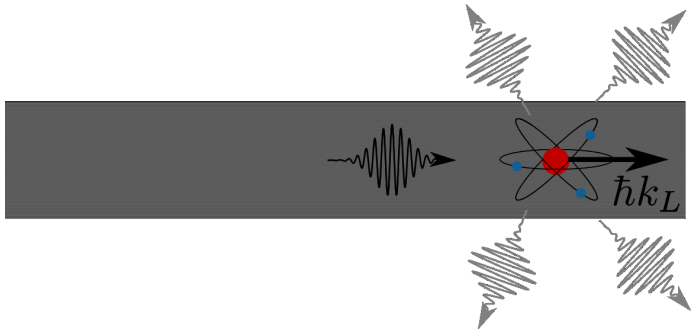


Figure 5.1: Radiation pressure. Photons from the laser beam (in black) always push the atom in the same direction when they are absorbed, while spontaneously emitted photons (in gray) can go all directions.

5.1 Radiation pressure

5.1.1 A very simple approach

Just to get the orders of magnitudes, let us consider a beam of photons being absorbed on a material and perform a momentum balance (same as problem 1).

$$\underbrace{p(t+dt)}_{0} = \underbrace{p(t)}_{n\,c dt\,S\hbar k} + F_{\text{wall}\to\text{photon}}dt \tag{5.1.1}$$

$$F_{\text{wall}\to\text{photon}} = -\frac{I}{\hbar\omega}\,S\,\hbar k \tag{5.1.2}$$

$$F_{\text{photon}\to\text{wall}} = \frac{I}{c}\,S \tag{5.1.3}$$

Order of magnitude : for sunlight reaching the Earth, $I \simeq 1\,\text{kW.m}^{-2}$ so $p \simeq 3\,10^{-6}\,\text{Pa}$. Can be sufficient to act on dust particles in vacuum, but certainly not to slow a river flow !

This very simple model has several flaws. Notably, we consider that all light is being absorbed - but we know that a material which absorbs light is also able to emit some. From a semi-quantum perspective, once an atom has absorbed a photon and jumped to the excited state, it needs to emit a photon before being able to absorb again. With increasing intensity, atoms should be stimulated back to the ground state. How to account for all these considerations?

5.1.2 Semi-quantum approach

Consider a two levels atom in a laser beam. Every time the atom absorbs a photon from the beam, it gets a recoil momentum $\hbar\mathbf{k_L}$. Every time the atom emits a photon from stimulated emission, it gets a recoil $-\hbar\mathbf{k_L}$ (stimulated emission is the exact opposite of absorption). Every time the atom emits a photon from spontaneous emission, it gets a recoil $-\hbar\mathbf{k}_{\text{spt}}$ which fixed modulus $k_{\text{spt}} = \omega_{eg}/c$ but a random direction (see Fig. 5.1). Over many cycles, the resulting force from absorption pushes the atom along the direction of the beam, while the average recoil from spontaenous emission is zero.

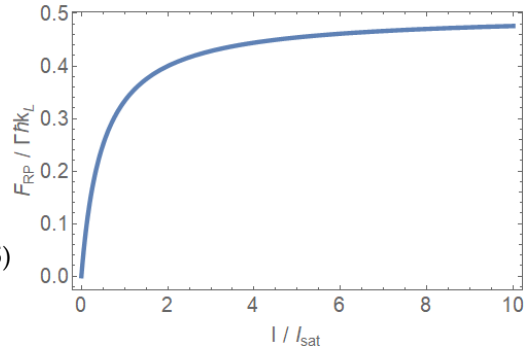
Let's now treat this problem quantitatively. We assume the internal degrees of freedom relax fast enough to consider they are always in steady state (adiabatic elimination). The momentum

balance reads

$$\frac{d}{dt}\mathbf{p}_{\text{atom}} = \mathbf{F}_{RP} = -n_e r_{\text{spt}} \hbar \mathbf{k}_{\text{spt}}(t) - n_e r_{\text{stim}} \hbar \mathbf{k}_L + n_g r_{\text{abs}} \hbar \mathbf{k}_L \tag{5.1.4}$$

where $\mathbf{k}_{\text{spt}}(t)$ is the wave vector of photons emitted by spontaneous emission. Those photons are emitted randomly in all directions, so the average value of their contribution is zero and the average force applied is

$$\begin{aligned} \langle \mathbf{F}_{RP} \rangle &= \left(n_g r_{\text{abs}} - n_e r_{\text{stim}} \right) \hbar \mathbf{k}_L \\ &= \left(\frac{1 + s/2}{1+s} \frac{\Gamma}{2} s - \frac{s/2}{1+s} \frac{\Gamma}{2} s \right) \hbar \mathbf{k}_L \\ &= \frac{s}{1+s} \frac{\Gamma}{2} \hbar \mathbf{k}_L \end{aligned} \tag{5.1.5}$$



Comments:

- At low intensity ($I \ll I_{\text{sat}}$),

$$F_{PR} \underset{s \ll 1}{\longrightarrow} \frac{I}{I_{\text{sat}}} \frac{\Gamma}{2} \hbar k_L = \frac{I}{c} \frac{\sigma_0}{1 + 4\delta^2/\Gamma^2} \tag{5.1.6}$$

  and we recover the previous result, where the size of the atom is the cross section. This effective size is maximal at resonance, and decreases with the detuning of the laser.

- At high intensity limit ($I \gg I_{\text{sat}}$),

$$F_{PR} \underset{s \gg 1}{\longrightarrow} \frac{\Gamma}{2} \hbar k_L \tag{5.1.7}$$

  Remarkably, this result is independant of the laser intensity. The transition is completly saturated: there so many photons that as soon as the atom is back to the ground state, he is readily brought again to the excited state.

- Expression (5.1.5) offers a very clear way of estimating the *scattering rate* of an atom, ie the number of absorption - spontaneous emission cycles per second. Considering that each of these cycles provides a momentum $\hbar k_L$ to the atom, we read:

$$\Gamma_{\text{scat}} = \frac{s}{1+s} \frac{\Gamma}{2} \tag{5.1.8}$$

  So if you shoot an atom with a continuous laser beam, this is the number of photons scattered by the atom.

5.1.3 A word about the classical approach (optional)

Note that it is also possible to express the radiation pressure with a purely classical model. Considering an electric field $\mathbf{E}(\mathbf{r}, t) = \mathcal{R}\left(\boldsymbol{\mathcal{E}}(\mathbf{r}) e^{i(\varphi(\mathbf{r}) - \omega t)} \right)$ in a dielectic medium, the volumic radiation

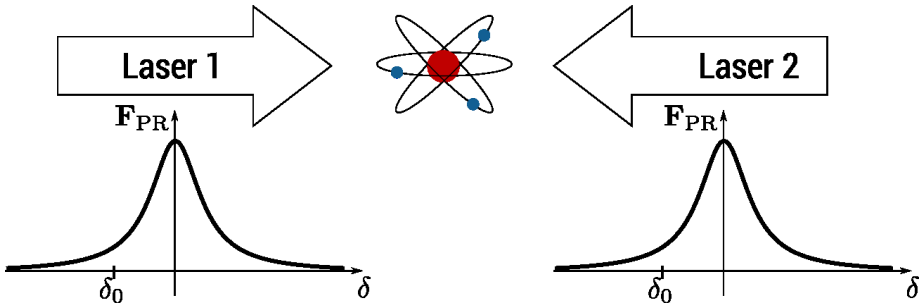


Figure 5.2: An optical molasses results from the radiation pressure of two red-detuned counter propagating beams.

pressure[1] appears as

$$\mathbf{f}_{\mathrm{RP}} = \frac{1}{2c}\chi'' I(\mathbf{r})\boldsymbol{\nabla}\left(\varphi(\mathbf{r})\right) \tag{5.1.9}$$

and for a plane wave, $\boldsymbol{\nabla}\left(\varphi(\mathbf{r})\right) = \mathbf{k}_L$. The radiation pressure is related to the imaginary part of the susceptibility, ie to the absorption coefficient of the medium (remember section 2.1.1.).

5.2 Application: cooling atoms with lasers

It possible possible to reach extremely low temperatures by shooting atoms with lasers !

Let's take an atom, and shoot it with two counter propagating laser beams. If the atom is at rest, $\langle \mathbf{F}_{\mathrm{PR},1}\rangle = -\langle \mathbf{F}_{\mathrm{PR},2}\rangle$. If the atom moves it will experience a Doppler shift: the laser coming towards the atom will look bluer, the laser going in the same direction as the atom will look redder. If both beams are initially red-detuned with a detuning $\omega_L - \omega_0 = \delta_0 < 0$, the atom will interact more with the contra-propagating beam, and thus experience a force opposed to its motion.

We can use the previous expression of the radiation pressure to calculate explicitly this force. For simplicity, we will work in the low and use the expression (5.1.6), leading go

$$\langle \mathbf{F}_{\mathrm{tot}}\rangle = \langle \mathbf{F}_{\mathrm{PR},1}\rangle + \langle \mathbf{F}_{\mathrm{PR},2}\rangle$$

$$\text{with } \langle \mathbf{F}_{\mathrm{PR},1}\rangle = \frac{I}{c}\frac{\sigma_0}{1+4\frac{(\delta_0-\mathbf{k}_L.\mathbf{v})^2}{\Gamma^2}}\mathbf{u}, \qquad \langle \mathbf{F}_{\mathrm{PR},2}\rangle = -\frac{I}{c}\frac{\sigma_0}{1+4\frac{(\delta_0+\mathbf{k}_L.\mathbf{v})^2}{\Gamma^2}}\mathbf{u}$$

We can use this expression to calculate the force applied on the atom as a function of the atom velocity (see Fig. 5.3). As expected, we see that for negative velocities (atoms going to the left), the force is positive (pushing atoms towards the right) and vice versa. For small velocities, the force scales linearly with v, like a frictious force. We can estimate the value of the corresponding friction coefficient.

To do so, we will Taylor expand the expression of each force around $v \sim 0$, using

$$\frac{1}{1+4\frac{(\delta_0-\mathbf{k}_L.\mathbf{v})^2}{\Gamma^2}} \simeq \frac{1}{1+4\frac{\delta_0^2}{\Gamma^2}} + 8\frac{\delta_0}{\Gamma^2}\frac{\mathbf{k}_L.\mathbf{v}}{\left(1+4\frac{\delta_0^2}{\Gamma^2}\right)^2}$$

[1] ie the force applied onto a small volume dV is $\mathbf{f}_{\mathrm{RP}}dV$

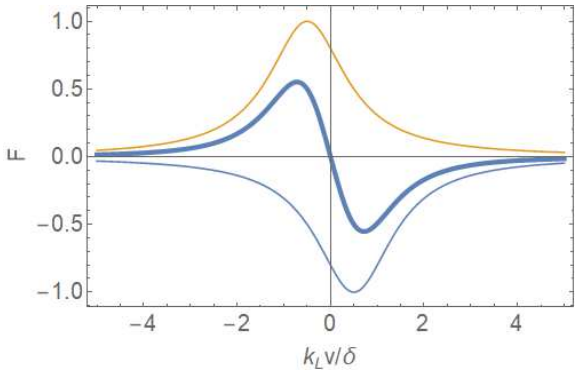


Figure 5.3: Force applied by right-propagating laser (thin yellow), the left-propagating laser (thin blue) and the sum of both (thick blue).

and we can write the total force to highlight its friction-like behavior

$$\langle \mathbf{F}_{\text{tot}} \rangle \simeq 16 \frac{I}{c} \sigma_0 \frac{\delta_0/\Gamma^2}{\left(1 + 4\delta_0^2/\Gamma^2\right)^2} k_L \, v = -m\alpha v \tag{5.2.1}$$

This force slows down the atom if $\delta_0 < 0$, as intuited in Fig 5.2. Considering that temperature is related to the average velocity of the atoms as

$$\frac{3}{2} k_B T = \frac{1}{2} m \left\langle v^2 \right\rangle , \tag{5.2.2}$$

slowing down the atoms actually meaning cooling down the ensemble. Using this kind of method, it is possible to cool atomic samples down to a fraction of mK ! There is however a lower bound for this cooling technique: if the atom is perfectly at rest, we would like it to stop interacting with light. However, because of the finite linewidth of the atomic transition, it can still interact with the detuned lasers, and start jiggling due to absorption / emission cycles. The proper way to account for these fluctuations is to consider a diffusion model, which leads to the following result (not proven here):

$$k_B T_{\text{min}} = \frac{\hbar \Gamma}{2}$$

5.3 Dipole force

Radiation pressure corresponds to a force *pushing* atoms along the direction of the beam. This force is clearly unable to explain experiments like optical tweezers, where atoms are *attracted* towards the center of the beam. There is indeed a second force at stake, the so called *optical dipole force*, which we will derive and interpret in this section.

5.3.1 Failure of the semi quantum model

While radiation pressure corresponds to absorption / spontaneous emission cycles, the optical dipole force corresponds to absorption / *stimulated emission* cycles. Now, absorbing a photon from one beam, then emitting a similar photon under stimulated emission has zero net impact onto the atom, as the two recoils compensate perfectly each other. For such cycles to result in a net effect onto the atoms, we need to consider at least two differents modes. In Fig. 5.4, the atom absorbs a

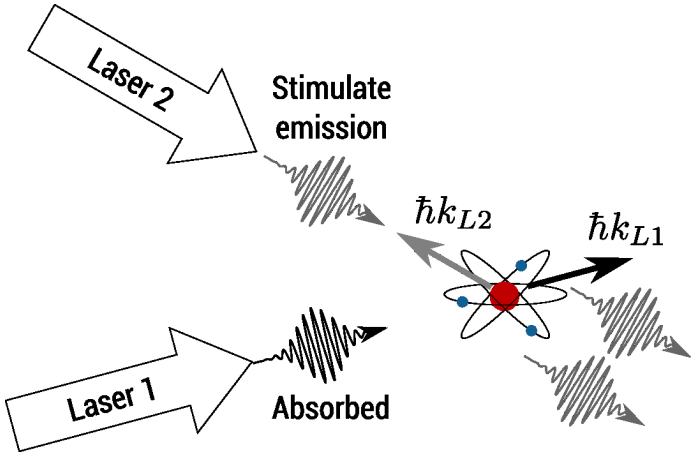


Figure 5.4: An optical molasses results from the radiation pressure of two red-detuned counter propagating beams.

photon from laser 1, then decay back to the ground state by emitting a photon stimulated by laser 2.

Yet, this simple process cannot be descirbed with our semi-quantum model... If we try to write a momentum balance corresponding to this situation:

$$\frac{d\mathbf{p}}{dt} = n_g \frac{\sigma I}{h\nu}\hbar\mathbf{k_1} + n_g \frac{\sigma I}{h\nu}\hbar\mathbf{k_2} - n_e \frac{\sigma I}{h\nu}\mathbf{k_1} - n_e \frac{\sigma I}{h\nu}\hbar\mathbf{k_2} - n_e\Gamma\hbar\mathbf{k}_{spt} \tag{5.3.1}$$

$$= n_g \frac{\sigma I}{h\nu}\hbar\left(\mathbf{k_1} + \mathbf{k_2}\right) - n_e \frac{\sigma I}{h\nu}\hbar\left(\mathbf{k_1} + \mathbf{k_2}\right) - n_e\Gamma\hbar\mathbf{k}_{spt} \tag{5.3.2}$$

we obtain the average radiation pressure along $\mathbf{k}_1 + \mathbf{k}_2$, but nothing more than what we discussed in the previous section... The reason for this limitation is that the dipole force relies on coherent processes - somehow, the excited state keeps memory of the phase of the laser it absorbed. Phases are not included in our toy model so this effect can't be captured.

5.3.2 Results from a semi-classical model

To account for the optical dipole force, we thus need to consider a model which includes the notion of phase. The most proper way of doing this would be a classical model, where the atom is treated in a quantum way while light is described as an electromagnetic field. However, the accurate treatment requires some technical tools beyond the scope of this lecture[2]. We simply give the result here without deriving it: unlike radiation pressure, the optical dipole force is conservative, and derives from a potential energy as given by

$$\mathbf{F}_{\text{Dip}} = -\frac{\hbar\delta}{2}\frac{\boldsymbol{\nabla} s}{1+s} = -\boldsymbol{\nabla} U_{\text{Dip}} \tag{5.3.3}$$

$$U_{\text{Dip}} = \frac{\hbar\delta}{2}\log\left(1+s\right) \underset{s\ll 1}{\simeq} \frac{\hbar}{8}\frac{\Gamma^2}{I_{\text{sat}}}\frac{I_L}{\delta} \tag{5.3.4}$$

Furthermore, the full quantum treatment shows that the resonant cross-section is given by $\sigma_0 = \frac{3\lambda_0^2}{2\pi}$ (admitted here, not proven). Plugging in this expression in I_{sat} in the previous expression, we

[2]Remember the discussion on spontaneous emission in section **??**.

get

$$U_{\text{Dip}}(\mathbf{r}) = \frac{3\pi}{2}\frac{c^2}{\omega_0^3}\frac{\Gamma}{\delta}I_L(\mathbf{r}) \tag{5.3.5}$$

which is truly a beautiful result, depending only on basic atomic quantities.

What this expression tells us, is that an atom immersed in a light field experiences a potential proportional to the light intensity, which can be attractive or repulsive depending on whether the laser frequency is red-detuned or blue-detuned with respect to the atomic transition. It is the "light-shift" mentioned in the previous lecture (section 4.3.4).

5.3.3 Analysis with a simple classical model

To get a basic understanding of these results in the low intensity regime, we can use a fully classical model. Let us consider a single atom as described by the Lorentz model. When immersed in a electric field $\mathbf{E}(\mathbf{r},t) = \mathcal{R}\left(\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t}\right)$, the atom forms an electric dipole

$$\mathbf{p}(t) = -e\mathbf{r}(t) = \mathcal{R}\left(\alpha\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t}\right) \tag{5.3.6}$$

where α is the atomic polarizatibilty (don't confuse it with the absorption coefficient !)

$$\alpha = \frac{\epsilon_0\chi_{\text{Lorentz}}}{n} = \frac{e^2}{m}\frac{\omega_0^2-\omega^2}{\left(\omega_0^2-\omega^2\right)^2+\Gamma^2\omega^2} + i\frac{e^2}{m}\frac{\omega\Gamma}{\left(\omega_0^2-\omega^2\right)^2+\Gamma^2\omega^2} \tag{5.3.7}$$

The potential energy corresponding to this dipole moment in the field is[3]

$$U_{\text{Atom}} = \int_0^E p\,dE = -\frac{1}{2}p.E = -\frac{1}{2}\frac{\alpha\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t}+\alpha^*\boldsymbol{\mathcal{E}}^*(\mathbf{r})e^{+i\omega t}}{2}\frac{\boldsymbol{\mathcal{E}}(\mathbf{r})e^{-i\omega t}+\boldsymbol{\mathcal{E}}^*(\mathbf{r})e^{+i\omega t}}{2} \tag{5.3.8}$$

Keeping only the slowly varying terms, we get

$$U_{\text{Atom}} = -\frac{1}{2}\alpha'\left|\boldsymbol{\mathcal{E}}(\mathbf{r})\right|^2 = -\frac{\alpha'}{2\epsilon_0 c}I(\mathbf{r}) \tag{5.3.9}$$

where we have used $I = |\mathbf{E}\times\mathbf{B}/\mu_0|$. From there, the volumic optical dipole force is

$$\mathbf{f}_{\text{Dip}}(\mathbf{r}) = -\boldsymbol{\nabla}nU_{\text{Atom}} = \frac{\chi'}{2c}\boldsymbol{\nabla}I(\mathbf{r}) \tag{5.3.10}$$

This expression already contains everything about the optical dipole force. However, we can rewrite to introduce the quantities we have been discussing during the last chapters. Considering the relation at resonance between susceptibility, absorption coefficient a and interaction cross-section (where all atoms are assumed to be in the ground state)

$$a(\omega_0) = 2k'' = \frac{2\pi}{\lambda_0}\chi''_{\text{Lorentz}}(\omega_0) = \sigma_0 n \tag{5.3.11}$$

[3]Be careful: when multiplying quantities, we can't use complex notations !

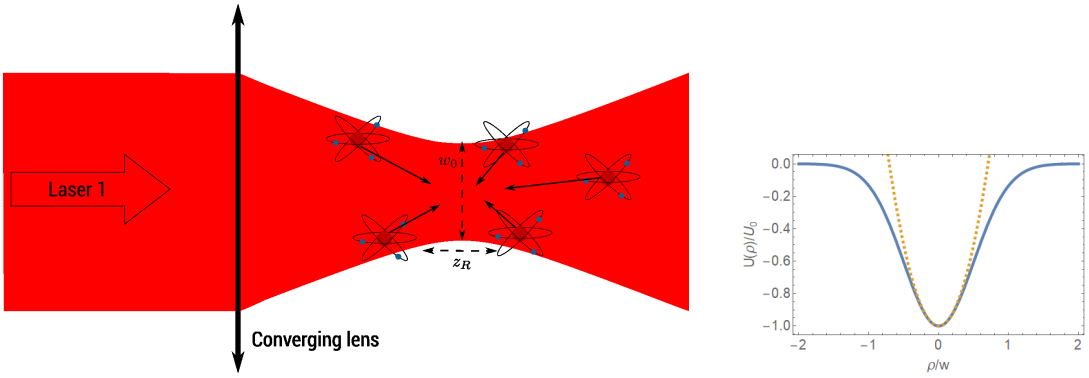


Figure 5.5: Transverse traping potential of an optical dipole trap.

we can reach

$$U_{\text{Dip}} = -\frac{\sigma_0\Gamma}{2}\frac{\omega_0^2-\omega^2}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}I(\mathbf{r}) = -\frac{\hbar\omega_0\Gamma^2}{4}\frac{\omega_0^2-\omega^2}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{I(\mathbf{r})}{I_{\text{sat}}} \tag{5.3.12}$$

where σ_0 is the interaction cross section at resonance (ie for $\omega = \omega_0$). Finally, if the laser frequency is far from resonance, such that

$$\Gamma \ll \delta \ll \omega_0 \tag{5.3.13}$$

this expression can be simplified to recover eq. (5.3.4)

$$U_{\text{Dip}}(\mathbf{r}) \simeq \frac{\hbar\omega_0\Gamma^2}{4}\frac{1}{\omega^2-\omega_0^2}\frac{I(\mathbf{r})}{I_{\text{sat}}} = \frac{\hbar\Gamma^2}{8\delta}\frac{I(\mathbf{r})}{I_{\text{sat}}} \tag{5.3.14}$$

5.4 Application: optical traps

We highlight a physical consequence of eq. 5.3.4: atoms will be attracted towards high-intensity region if the laser is red-detuned with respect to the atomic transition. Based on this notion, if we focus a red-detuned gaussian laser beam into a cloud of atoms, atoms will be trapped around the beam focal point. This technique can be used to trap and manipulate cold atoms.

Quantitatively, the intensity profile of a gaussian beam with power P is given by

$$I(\boldsymbol{\rho},z) = \frac{2P}{\pi w_0^2}\frac{1}{1+\left(\frac{z}{z_R}\right)^2}\exp\left(-2\frac{\rho^2}{w_0^2\left(1+\left(\frac{z}{z_R}\right)^2\right)}\right) \tag{5.4.1}$$

so the atoms experience a gaussian traping potential $U(\boldsymbol{\rho},z) = \frac{3\pi}{2}\frac{c^2}{\omega_0^3}\frac{\Gamma}{\delta}I(\boldsymbol{\rho},z)$ (see Fig. 5.5). The depth of this trap is

$$U_0 = \frac{3c^2}{\omega_0^3}\frac{\Gamma}{|\delta|}\frac{P}{w_0^2} \tag{5.4.2}$$

so a 1064 nm laser of $P \simeq 10\,\text{W}$ focused into a $w_0 = 40\,\mu\text{m}$ waist can trap Potassium atoms ($\lambda_0 = 767\,\text{nm}$, $\Gamma = 2\pi \times 6\,\text{MHz}$) if their typical kinetic energy is lower than U_0 - ie at temperatures lower than

$$T_0 = \frac{U_0}{k_B} \simeq 470\,\mu\text{K} \tag{5.4.3}$$

Around the bottom of this trap, the trap can be approximated by a harmonic potential (see Fig. 5.5) as

$$U(\rho \ll w_0) \simeq -U_0 + \underbrace{\frac{6c^2}{\omega_0^3}\frac{\Gamma}{|\delta|}\frac{P}{w_0^4}\rho^2}_{\frac{1}{2}m\omega_\perp^2\rho^2} + \underbrace{\frac{6c^2}{\omega_0^3}\frac{\Gamma}{\delta}\frac{P}{w_0^2 z_R^2}z^2}_{\frac{1}{2}m\omega_\parallel z^2} \tag{5.4.4}$$

and typical values for theses frequencies in cold atoms experiments are around hundreds of Hertz.

As we are trying to trap the atoms around the focal point with the dipole force, radiation pressure operates as a detrimental effect, pushing the atoms away in the direction of the beam. What saves us is that the influence of radiation pressure decreases with detuning *squared*, while the dipole force is scales simply with the inverse of the detuning

$$F_{PR} \propto \frac{I_L(\mathbf{r})}{\delta^2} \qquad F_{\mathrm{Dip},z} \propto \frac{I_L(\mathbf{r})}{\delta} \tag{5.4.5}$$

so we can work far enough from the resonance so that the radiation pressure can be neglected, but close enough so that the dipole force is still significant.

Remark: Optical tweezers

A similar trapping effect can be observed not only on atoms, but also on small but macroscopic objects. This is the idea behind the so-called *optical tweezers*, rewarded with a Nobel prize to Arthur Ashkin in 2018. The usual approach is to describe the target (typically a small bead) as a dielectric medium with a known susceptibility, and to consider the classical expression derived above $\mathbf{F}_{\mathrm{Dip}} = \frac{1}{2c}\chi' \boldsymbol{\nabla}\left(I(\mathbf{r})\right)$. For pratical application, it is useful to consider the Lorentz–Lorenz equation which gives the susceptibility for a small spherical bead of radius a and optical index n immersed in a medium of optical index n_0:

$$\chi'_{\mathrm{at}} = 4\pi a^3 \frac{m^2-1}{m^2+2} \Rightarrow \mathbf{F}_{\mathrm{Dip}} = \frac{2\pi a^3}{c}\frac{m^2-1}{m^2+2}\boldsymbol{\nabla}\left(I(\mathbf{r})\right) \tag{5.4.6}$$

where $m = n/n_0$. The tyipcal traping force corresponds to a stiffness of some pN/nm per Watt of incident light. This technique has notably allow the optical manipulation of biological systems.

5.5 Take home message

Force	Radiation pressure	Dipole force
Cycles	Abs / spt emission	Abs / stim emission
Nature	Dissipative	Conservative
Quantum	$\mathbf{F}_{PR} = \frac{s}{1+s}\frac{\Gamma}{2}\hbar\mathbf{k}_L$ $\Gamma_{\mathrm{scat}} = \frac{s}{1+s}\frac{\Gamma}{2}$	$\mathbf{F}_{\mathrm{Dip}} = -\boldsymbol{\nabla}U_{\mathrm{Dip}}$ $U_{\mathrm{Dip}} = \frac{\hbar\delta}{2}\log(1+s)$ $\underset{s\ll 1}{\simeq} \frac{\hbar}{8}\frac{\Gamma^2}{I_{\mathrm{sat}}}\frac{I_L}{\delta} = \frac{3\pi}{2}\frac{c^2}{\omega^3}\frac{\Gamma}{\delta}I_L(\mathbf{r})$
Classical	$\mathbf{f}_{\mathrm{RP}} = \frac{1}{2c}\chi'' I(\mathbf{r})\boldsymbol{\nabla}\left(\varphi(\mathbf{r})\right)$	$\mathbf{f}_{\mathrm{Dip}} = \frac{1}{2c}\chi'\boldsymbol{\nabla}\left(I(\mathbf{r})\right)$
Application	Laser cooling	Optical traps