PHY208 – Atoms and lasers
Lecture 3

Is laser light unique?

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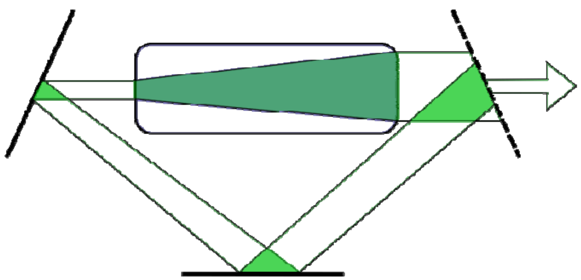
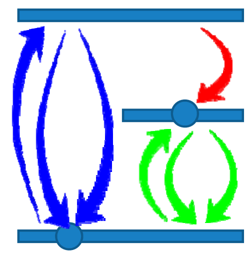
What have we seen so far? LASER

An optical cavity (« oscillator ») → condition on phase

An amplifying medium (« gain ») → condition on ampl. / intensity



Introduced a 3 level systems.



$$r_{\text{abs}} = r_{\text{stim}} = \frac{\sigma I}{h\nu} = W$$

$$r_{\text{spont}} = \Gamma$$

Need gain to compensate losses (output + parasitic)

Impossible in Lorentz model,
requires population inversion.

$$g = \sigma_{eg} \underbrace{(n_e - n_g)}_{\Delta n}$$

Basic laser properties

$$g = \frac{g_0}{1 + I/I_{\text{sat}}}$$

Laser threshold,
Gain saturation,
Steady state intensity,
Steady state population inversion

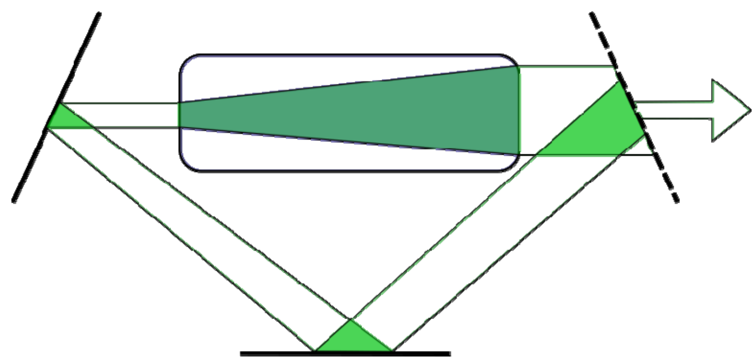
What have we seen so far?

The phenomenon LASER:

L.A.S.E.R. : Light Amplification by Stimulated Emission of Radiation

The device, laser:

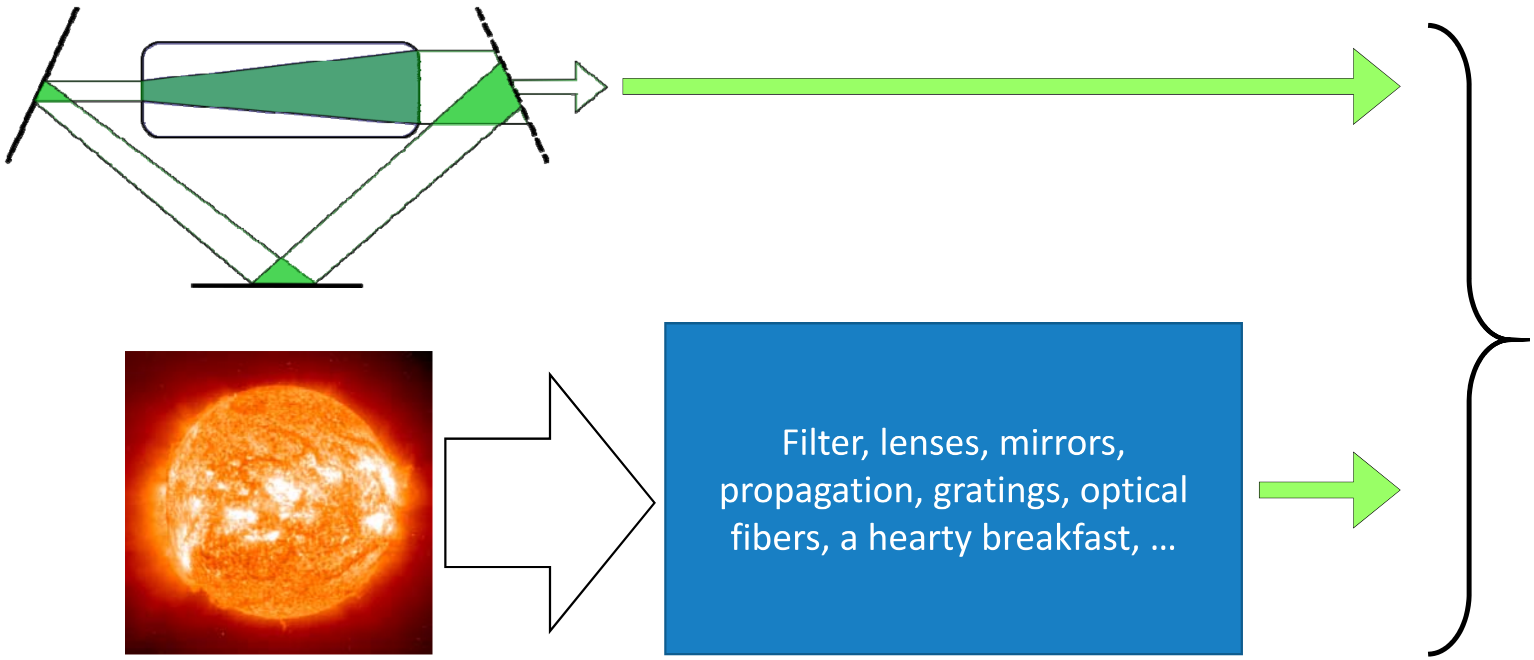
Gas laser, solid laser, semiconductor laser diode……



The outcome, a laser beam (today's subject):

« Powerful » beam
« Monochromatic » or « Coherent » beam

Today’s Question: Is laser light unique?



Can we make these beams indistinguishable?

Outline of Lecture 3

Is laser light unique?

I. Coherence
    Temporal Coherence
    Spectral Width and Wiener-Khinchin
    Spontaneous Emission Example
    Headlight Example

II. Focusing Light
    Numerical Aperture
    Optical Etendue
    Etendue and the Headlight
    Modes

III. So is laser light actually unique?

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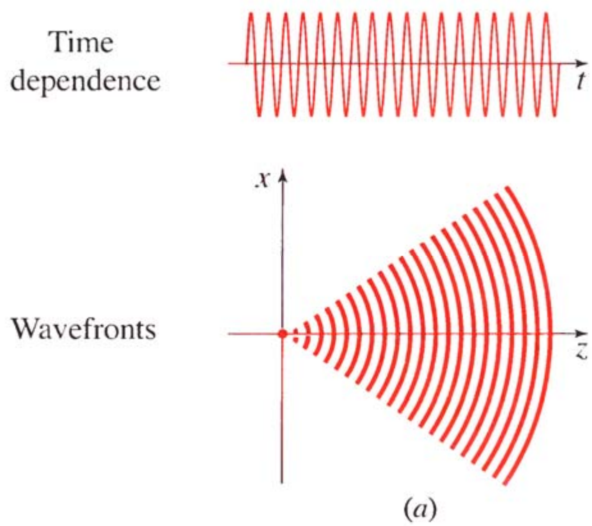
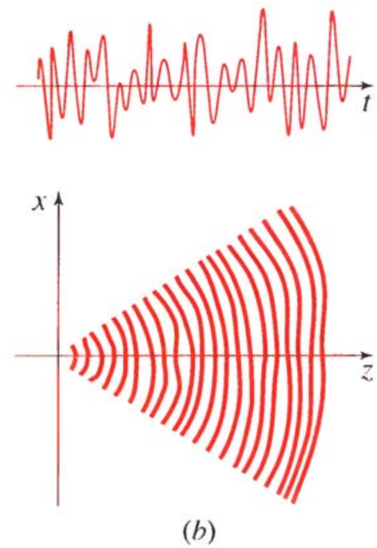
I. Deterministic vs Random Light

So far, we have treated light as a well-defined, single frequency (monochromatic) complex wavefunction.

$$U(\boldsymbol{r},t) = U(\boldsymbol{r})e^{j\omega t} \qquad \mathrm{I}(\boldsymbol{r}) = |U(\boldsymbol{r})|^2$$

In reality, the exact instantaneous value of U will fluctuate in both amplitude and phase (around some central value), and has to be described statistically.



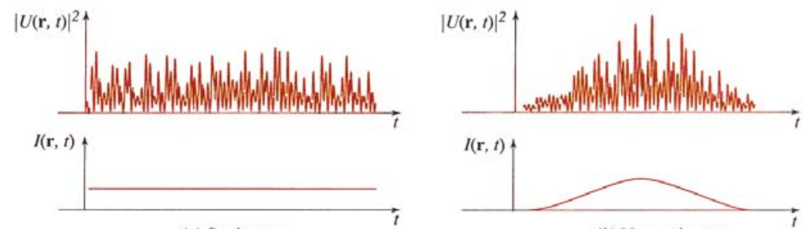


How can we answer « how coherent »?

I. Defining Temporal Coherence

We are going to consider random light, but light that is statistically stationary
(statistical descriptors like mean, skewness, etc do not change over sufficiently long time scales).

Doing this, the average intensity of the light becomes:



(a) Stationary (b) Nonstationary

$$\mathrm{I}(t) = \langle |U(t)|^2 \rangle$$

Where the $\langle - \rangle$ indicates an ensemble average.

We also need a way to quantify the “randomness” of the light. For this we use the **temporal coherence function**.

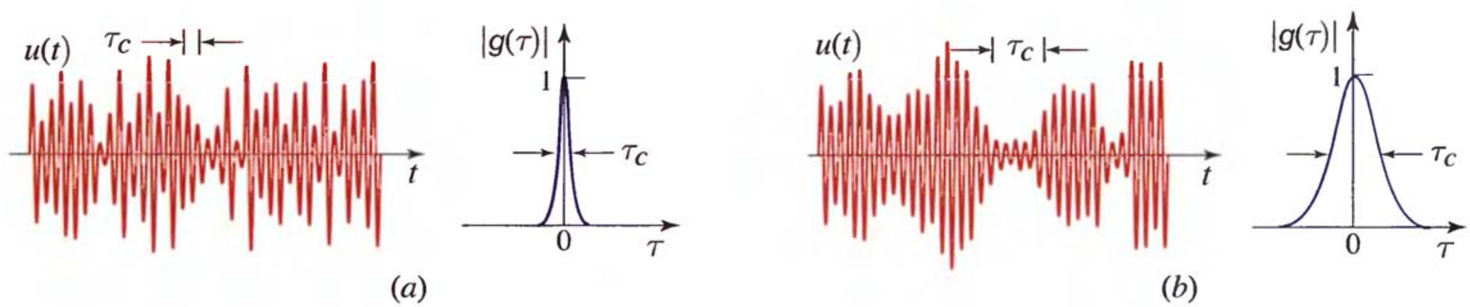
$$\mathrm{G}(\tau) = \langle U^*(t+\tau)U(t) \rangle$$

This function can be normalized (using the intensity) to get $g(\tau)$, which will vary between 0 and 1.

NB: I am using the notation of S&T, where U is the complex wavefunction. It can be replaced by E for a propagating TEM wave, but then we need a factor of 2η to get optical intensity.

I. Coherence Time

If I now plot $g(\tau)$, I can see that it decreases with τ for random waves.
What would it look like for a monochromatic, coherent wave?

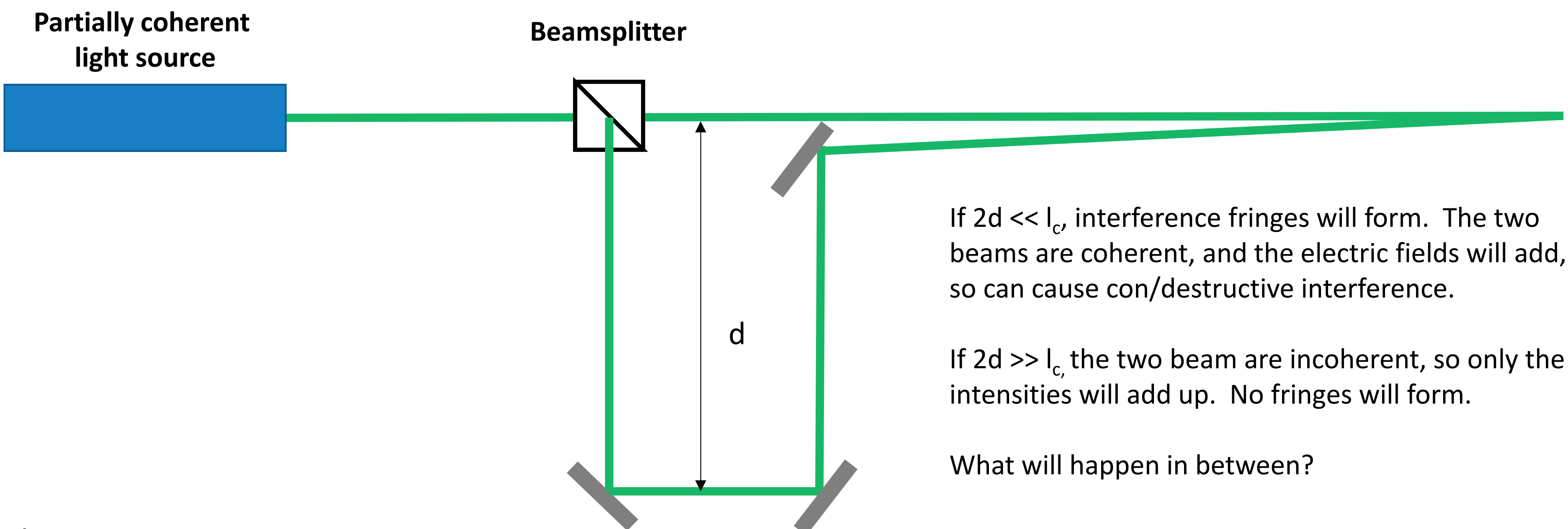


The time scale over which $g(\tau)$ falls to 0.5 is denoted as the **coherence time,** $\boldsymbol{\tau_c}$.

I. Coherence Length

In vacuum, in a coherence time $\boldsymbol{\tau_c}$, the wave will travel a distance $\boldsymbol{c\tau_c}$ (**c** is the speed of light). This is called the **coherence length, l_c**.

A good way to picture this is as the **maximum path length difference** allowed for **interference** to still occur.



If $2d \ll l_c$, interference fringes will form. The two beams are coherent, and the electric fields will add, and so can cause con/destructive interference.

If $2d \gg l_c$, the two beam are incoherent, so only the intensities will add up. No fringes will form.

What will happen in between?

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I. Wiener-Khinchin Theorem

Wiener-Khinchin Theorem

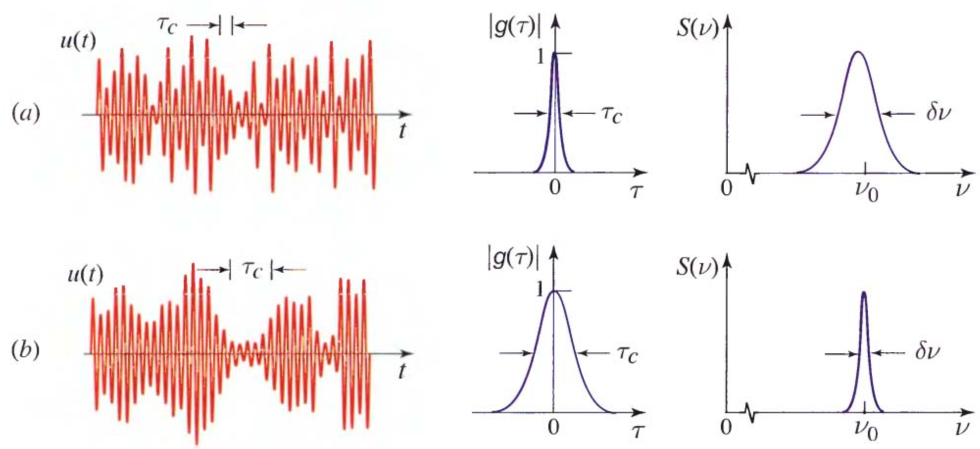
The **temporal coherence function** (or autocorrelation function) $G(\tau)$ and the **power spectral density** $S(\nu)$, (where the total average intensity, $I = \int_{-\infty}^{\infty} S(\nu)d\nu$) form a **Fourier transform pair**:

$$\mathrm{S}(\upsilon) = \int_{-\infty}^{\infty} G(\tau)\exp(-j2\pi\nu\tau)\, d\tau$$

This means that the spectral width (**linewidth**) of a source and its **coherence time** are inversely related. The exact relation will depend on the definition of linewidth that is used.

If we use: $$\Delta\nu_c = \frac{\left(\int_0^{\infty} S(\nu)d\nu\right)^2}{\int_0^{\infty} S^2(\nu)d\nu}$$ Then we get $$\Delta\nu_c = \frac{1}{\tau_c}$$

I. Examples of temporal coherence



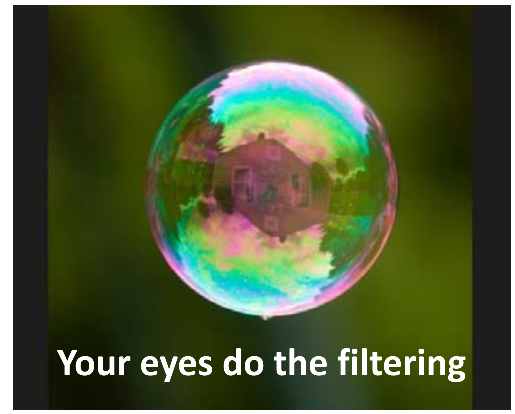


Table 11.1-2 Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

Source	$\Delta\nu_c$ (Hz)	$\tau_c = 1/\Delta\nu_c$	$l_c = c\tau_c$
Filtered sunlight ($\lambda_o = 0.4$–0.8 μm)	3.74×10^{14}	2.67 fs	800 nm
Light-emitting diode ($\lambda_o = 1$ μm, $\Delta\lambda_o = 50$ nm)	1.5×10^{13}	67 fs	20 μm
Low-pressure sodium lamp	5×10^{11}	2 ps	600 μm
Multimode He–Ne laser ($\lambda_o = 633$ nm)	1.5×10^{9}	0.67 ns	20 cm
Single-mode He–Ne laser ($\lambda_o = 633$ nm)	1×10^{6}	1 μs	300 m

In what situation are you used to seeing an interference effect for filtered sunlight?

Notice that even a single mode laser (which we have treated so far as a perfectly monochromatic source) has a finite bandwidth. Let's see why!

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I. Model for Spontaneous Emission

Wavetrain Model

We consider an ensemble of two level atoms and look at the excited state population. Spontaneously, one of the atoms can decay and release radiation at a frequency ω_0

This radiation can then cause the stimulated emission of the surrounding atoms and result in a coherent wave.

$$u_0 \, cos(\omega_0 + \varphi)$$

We will also assume that the gain is fully saturated, so the amplitude of the wave stays constant.

With each new spontaneous emission event, the phase of the radiation will change randomly. The total radiation emitted by the ensemble of atoms will therefore take the form:

$$u(t) = U(t) + U^*(t)$$

$$U(t) = U_0 e^{i(\omega_0 t + \varphi(t))}$$

Where $\varphi(t)$is a random variable that translates spontaneous emission into random jumps in phase.

I. Spont. Em. and Temporal Coherence

Probability Law for Spontaneous Emission

Let's derive the probability *p(t)* that the phase does not jump within a time *t*.

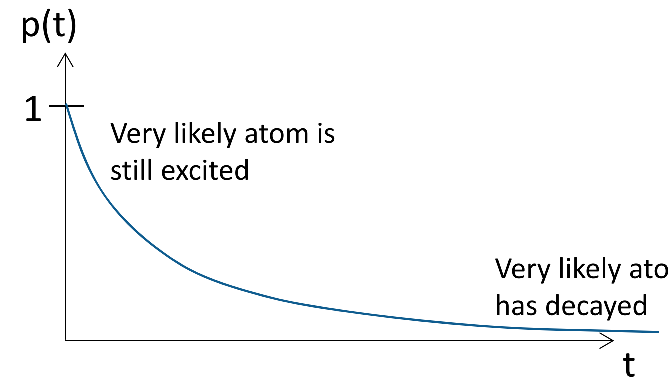
Spontaneous de-excitation is a random, memoryless process. **We can define γdt as the probability that the phase will jump between time *t* and *t+dt*. (It can be shown (but we won't do it here) that $\gamma = \Gamma/2$, where Γ is the excited state lifetime.)**

The probability p(t) must therefore satisfy

$$p(t+dt) = p(t) \times (1 - \gamma dt)$$

And if we assign p(0) = 1, we obtain

$$p(t) = e^{-\gamma t}$$



I. Spont. Em. and Temporal Coherence

Autocorrelation function for Spontaneous Emission

Let's now calculate the autocorrelation function $G(\tau)$ of the emitted radiation.
(Rather than use U and U^* to get a real value, we will leave it as imaginary until the end).

We can distinguish between two contributions :

$$G(\tau) = \langle U^*(t)U^*(t+\tau)\rangle_{\text{with phase jump}} + \langle U^*(t)U^*(t+\tau)\rangle_{\text{without phase jump}}$$

If the phase doesn't jump during the time $|\tau|$:

$$\langle U^*(t)U^*(t+\tau)\rangle = U_0^2\, e^{i\omega_0\tau}$$

This case has a probability $p(|\tau|)$ of occurring.

If the phase does jump during the time $|\tau|$:

$$\langle U^*(t)U^*(t+\tau)\rangle = U_0^2\, e^{i\omega_0\tau}\langle e^{-i\varphi(t)}\rangle = 0$$

This case has a probability $1 - p(|\tau|)$ of occurring.

I. Spont. Em. and Temporal Coherence

The full autocorrelation function $G(\tau)$ can now be calculated, using the probability of each event.

$$
\begin{aligned}
G(\tau) &= \langle U^*(t)U^*(t+\tau)\rangle \\
&= p(|\tau|) \times U_0^2 e^{i\omega_0\tau} + (1 - p(|\tau|)) \times 0 \\
&= e^{-\Gamma|\tau|} \times U_0^2 e^{i\omega_0\tau} \\
&= U_0^2\, e^{(i\omega_0)\tau - \gamma|\tau|}
\end{aligned}
$$

And now we can use the Wiener Khinchin theory to transform this into the spectrum

$$S(\omega) = \int_{-\infty}^{\infty} G(\tau) \exp(-i\omega\tau)\, d\tau$$

$$S(\omega) = \frac{I_0}{(\omega - \omega_0)^2 + \gamma^2}$$

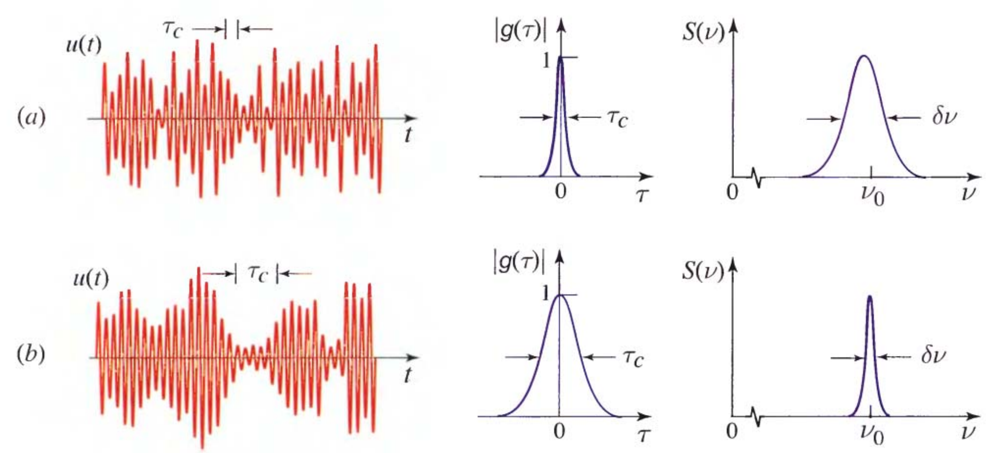
So the **randomization of phase** caused by **spontaneous emission** is a fundamental source of broadening for lasers.

I. Temporal Coherence and Bandwidth

Saying a laser has **a narrow bandwidth** and saying it is **temporally coherent** are **two ways of saying the same thing**.

A laser has a much narrower spectral width than other light sources.

However, using optical filters will decrease the bandwidth, and therefore increase the coherence length, etc.



Is this all it takes to mimic a laser?

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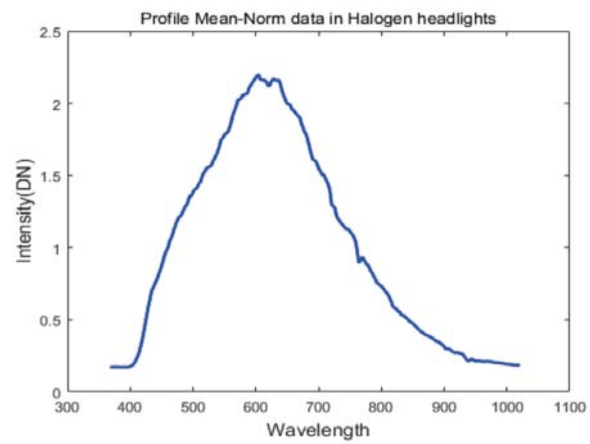
I. The Headlight Example

Could we get the same power spectral density (and total power) of a multimode HeNe laser just by filtering a car headlamp?



Headlamp (halogen): 50W emitting from 450-850 nm

HeNe: 0.5 mW with a bandwidth of 1 GHz

I. The Headlight Example

First, turn bandwidth into spectral width

$$\Delta\lambda = \frac{\lambda^2 \Delta\nu}{c} = \frac{(630 \times 10^{-9} m)^2 (10^9/s)}{3 \times 10^8 \ \frac{m}{s}}$$

$$= 1 \times 10^{-12} m$$

$$= 1 \times 10^{-3} \ nm$$

Then filter out all power outside that band:

$$50W \, \frac{1 \times 10^{-3} \ nm}{400 \ nm} = 0.125\text{mW}$$

So it is extremely wasteful, but I could get something close to the same power spectral density. However, the other special thing about lasers is being able to focus them down.

Can I focus my headlight to the size of a laser spot?

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II. Focusing/Collecting Light

What is the fundamental limit to focusing down light?

Could I focus down blackbody radiation from a warm rock to light a fire?

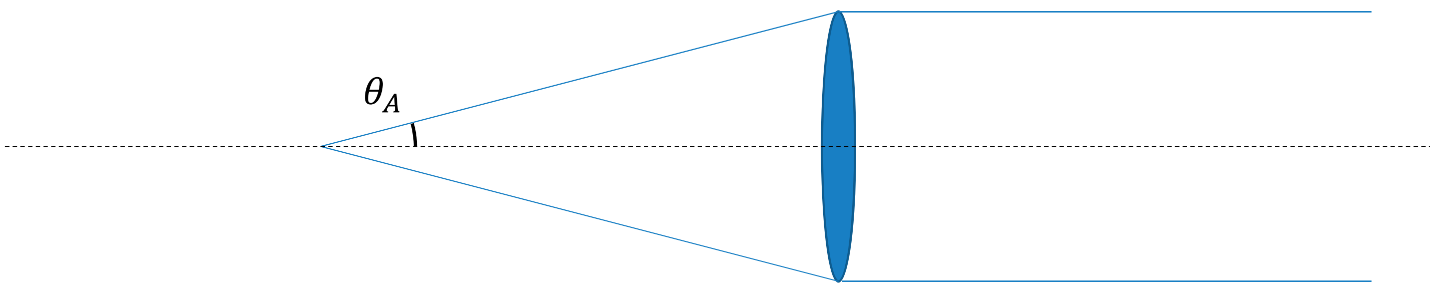
Obviously no, but how does this emerge from optics?

Let's recall some useful optical formulas to help quantify this….

II. Acceptance Angle and NA

Numerical aperture (NA) or acceptance angle of a lens/fibre describes cone of light (in 2D) that can be collimated.

$$NA = n\, sin\theta_A$$

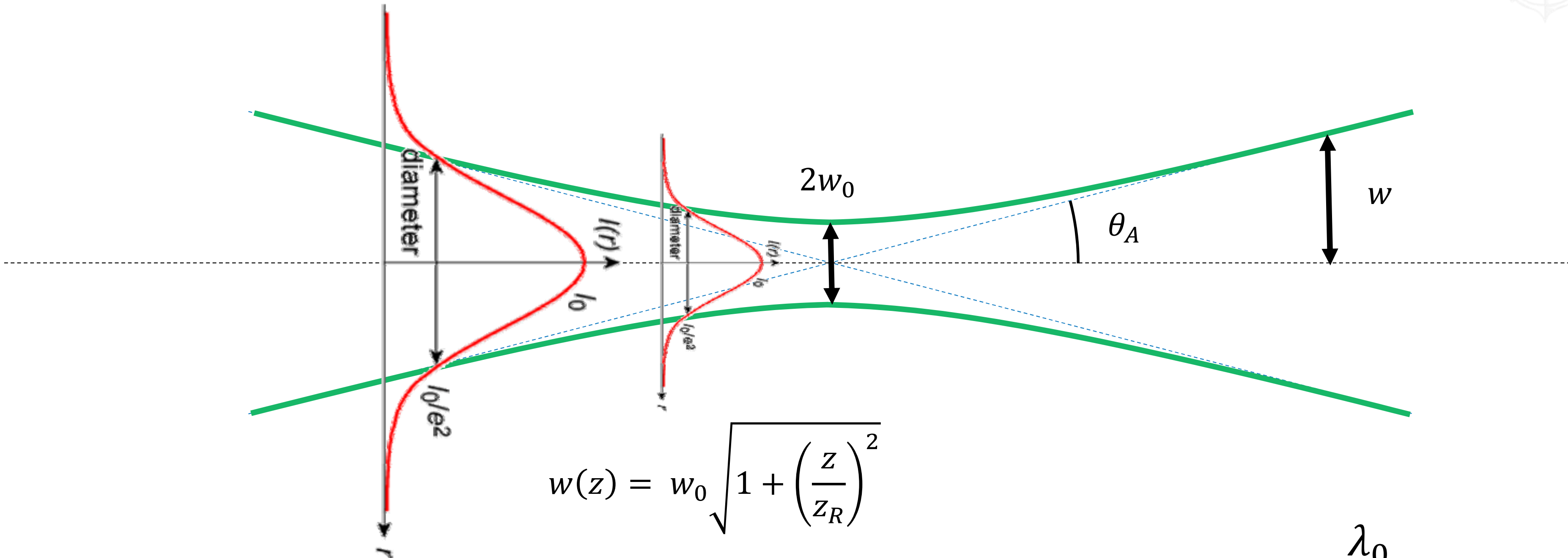


Accounts for **focal length** and **size** of lens
Treats light as a point source (and therefore assumes one can focus light to a point)
Dimensionless unit, always less than *n*
This makes it seem like infinite concentration is possible…

NB: Ray tracing model

II. NA for Gaussian Beams

For propagating Gaussian modes (such as a single mode laser beam), the entire beam profile is defined by the beam waist and the wavelength alone.



$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

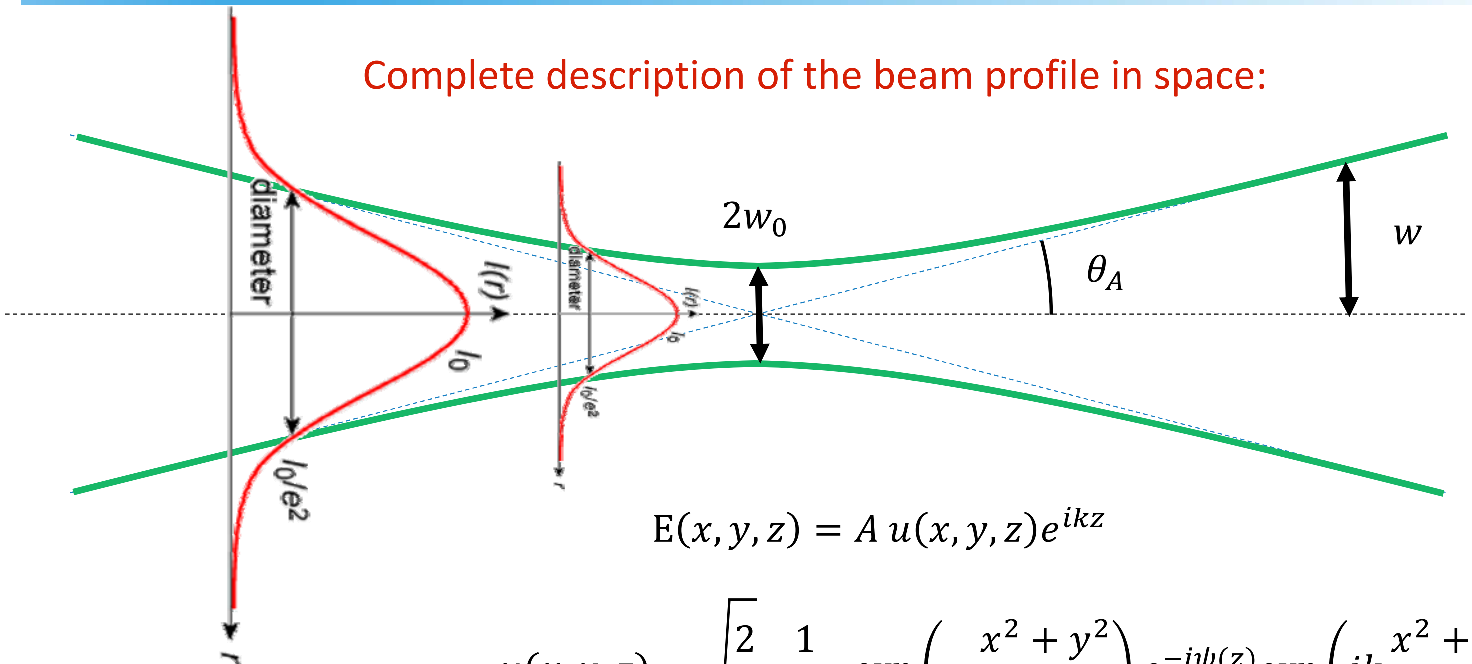
$$z_R = \frac{\pi w_0^2}{\lambda}$$ Rayleigh length

$$NA \approx \frac{\lambda_0}{\pi w_0}$$

NB: Adding in some EM

II. Gaussian Beam Profile

Complete description of the beam profile in space:



Rayleigh length

$$z_R = \frac{\pi w_0^2}{\lambda}$$

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

$$R(z) = z + \frac{z_R^2}{z}$$

$$\psi(z) = \arctan\frac{z}{z_R}$$

$$\mathrm{E}(x, y, z) = A\, u(x, y, z) e^{ikz}$$

$$u(x, y, z) = \sqrt{\frac{2}{\pi}} \frac{1}{w(z)} \exp\left(-\frac{x^2 + y^2}{w(z)^2}\right) \underbrace{e^{-i\psi(z)} \exp\left(ik\frac{x^2 + y^2}{2R(z)}\right)}$$

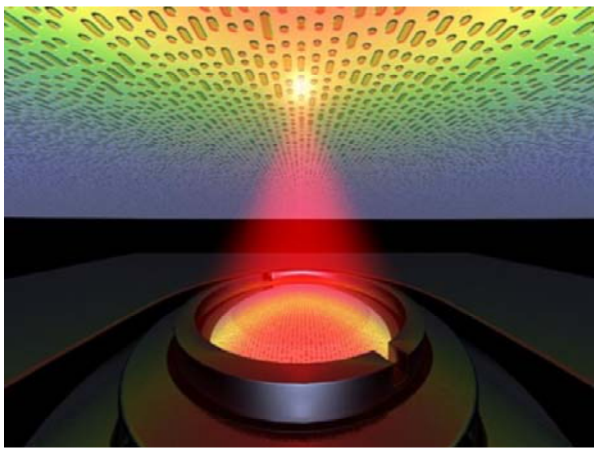
phase term, to account for wave-fronts becoming round

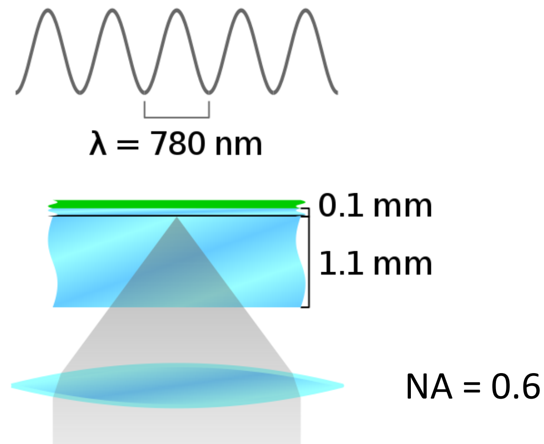
$$\rho^2 = x^2 + y^2$$

$$I(\rho, z) = \frac{2P}{\pi w_0^2} \frac{1}{\left(1 + \left(\frac{z}{z_R}\right)^2\right)} \exp\left(-2\frac{\rho^2}{w_0^2\left(1 + \left(\frac{z}{z_R}\right)^2\right)}\right)$$

II. NA for Gaussian Beams

This can tell us the spot size for a given NA (given by the lens), for example for a CD read laser.





$$w_0 \approx \frac{\lambda_0}{\pi NA}$$

This seems to put a limit on concentration, at least.
Still no basic limit – we need another quantity….

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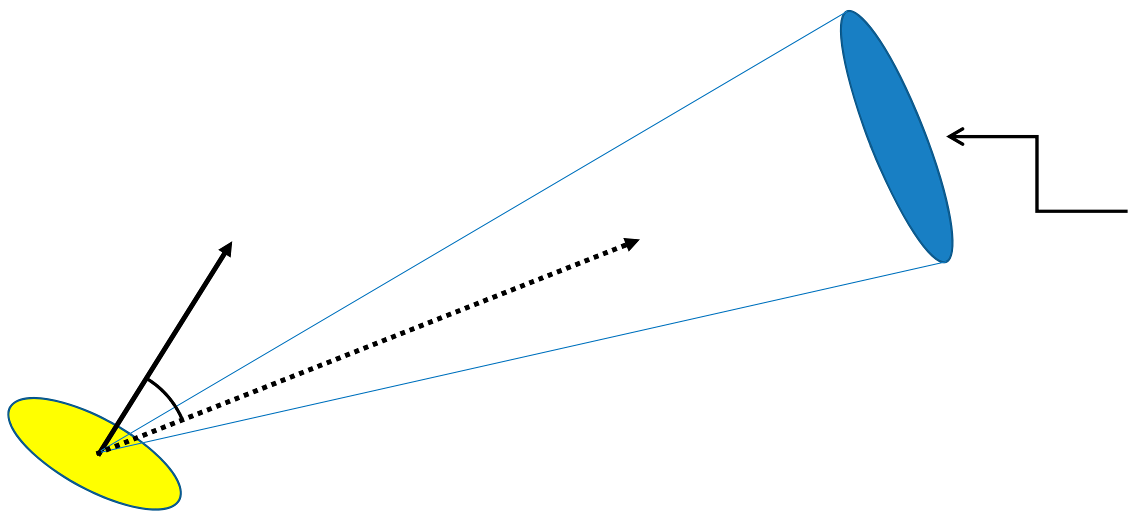
II. Extending NA to surfaces

NA was a good way to describe coupling from a point source (ray) or Gaussian beam propagation.
How do we incorporate the effect of a finite sized source?

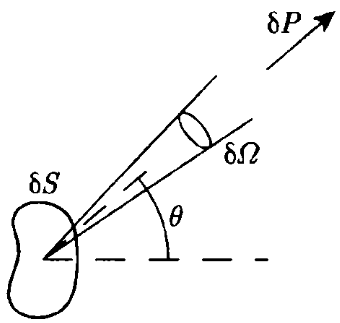


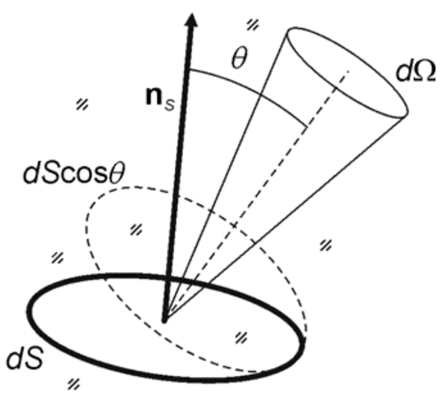
We will link power incident on this surface (or through this solid angle) to the radiance from the infinitesimal surface?

This infinitesimal surface has some radiance, B $\left[\frac{W}{str\ m^2}\right]$

NB: Back to ray tracing model

II. Optical Etendue





Consider an infinitesimal area element δS that radiates optical power.

The power δP that we can collect in a solid angle $\delta\Omega$ is proportional to $\delta\Omega$ and to $\delta S\ cos\theta$ (area of object projected along line of sight).

We define a quantity **étendue** (in a medium with refractive index n), as

$$\delta(\text{étendue}) \equiv n^2\,\delta S\,\delta\Omega\cos\theta.$$

The power δP collected is now given by:

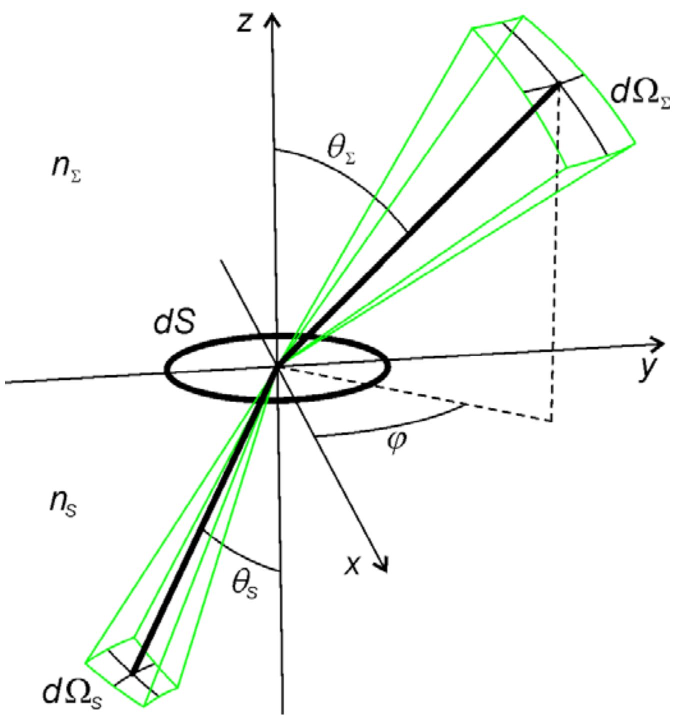
$$\delta P = B \times n^2\,\delta S\,\delta\Omega\cos\theta.$$

Where B is radiance.
Notice that we could treat either surface as the emitter and get the same étendue, so it is describing the **system**.

Etendue is a very important quantity when discussing the limits of focusing!

II. Conservation of Optical Etendue

Important fact: étendue is conserved in a perfect optical system.

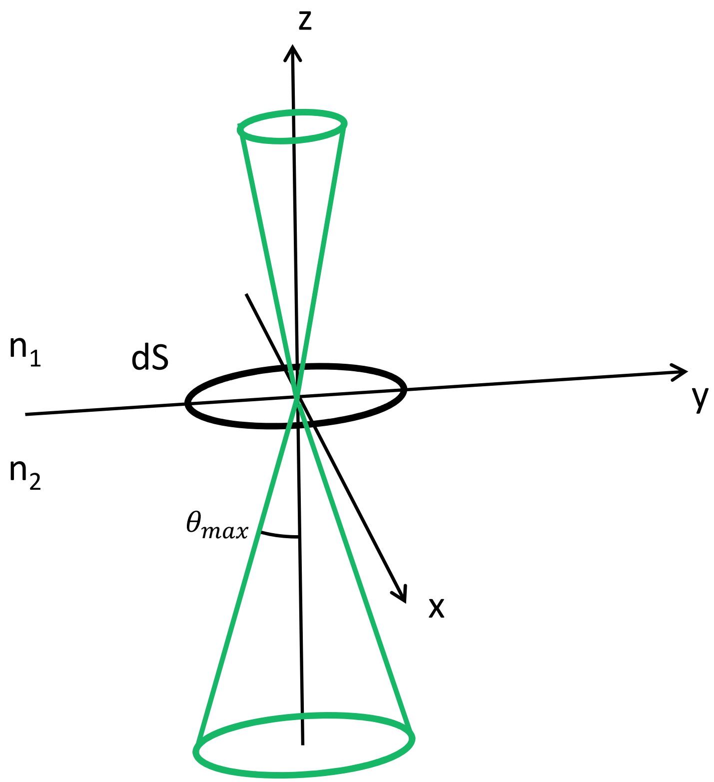


True for all perfect optical components (mirrors, lenses, …).
Example at left is for passage from one medium to another.

$$\delta(\text{étendue}) \equiv n^2\, \delta S\, \delta\Omega \cos\theta.$$

NB: Still ray tracing model

II. Optical Etendue – NA and small angles



If we gather light radiated symmetrically into a cone of semi-angle θ_{max} , then we have:

$$\text{étendue} = n^2\,\Delta S \int \cos\theta\,\mathrm{d}\Omega = \Delta S\,\pi(n\sin\theta_{\max})^2,$$

Here we can recognize the numerical aperture (NA) from classical optics, $NA = n\sin\theta_{max}$

$$étendue \;= \Delta S\;\pi NA^2$$

If we make small-angle approximations as well, we get a useful simpler version:

$$\text{small angles:} \qquad \text{étendue} \approx n^2\,\Delta S\,\Delta\Omega.$$

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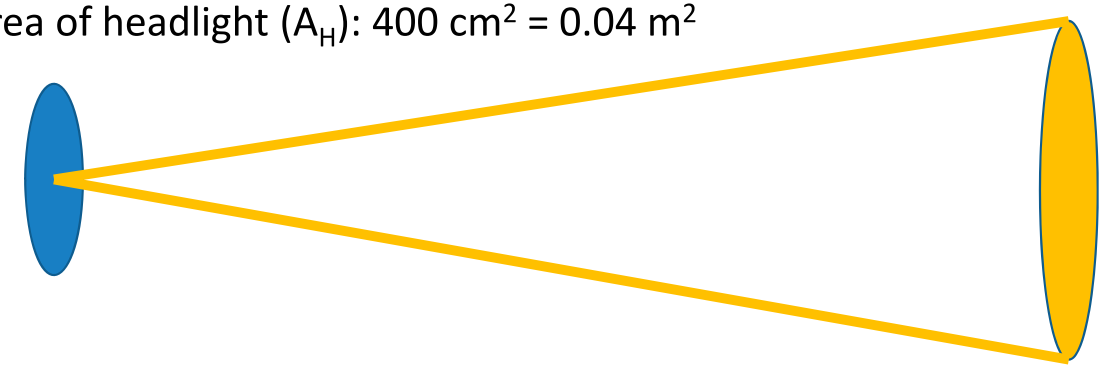
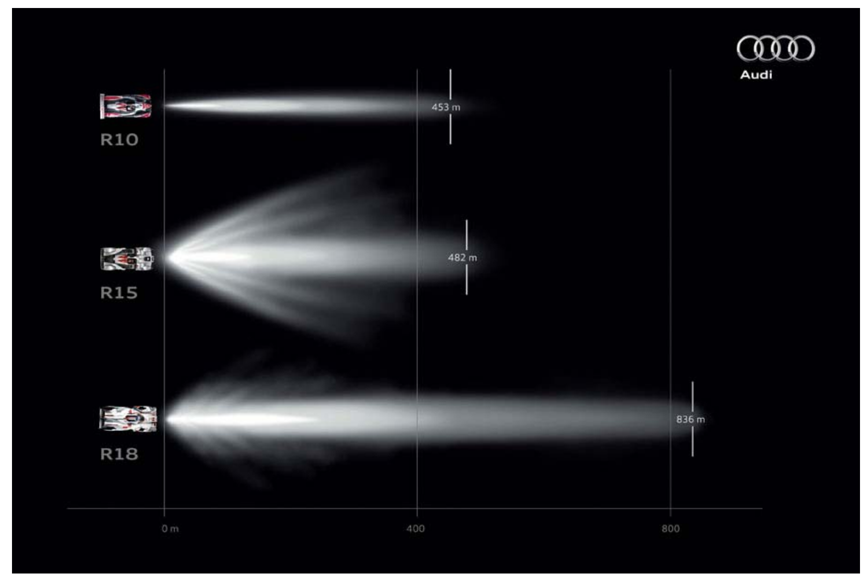
II. Optical Etendue of the Headlight

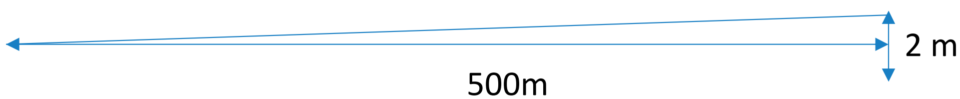
Area of headlight (A_H): 400 cm^2 = 0.04 m^2



Solid angle of radiation Ω_H



θ = arctan (2/500) = .004 rad



$$\Omega_H = 2\pi\,(1-\cos\theta) \approx \pi\theta^2$$

$$\Omega_H = 3\,(4\times10^{-3})^2 = 50\times10^{-6}$$

Canadian pi

II. Optical Etendue of the Headlight

Area of headlight (A_H) = 400 cm^2 = 0.04 m^2

$$\Omega_H = 3\,(4 \times 10^{-3})^2\,sr = 50 \times 10^{-6} sr$$

$$\text{étendue} = \Omega_H A_H$$

Like a laser, we want to focus this down to an area on the order of λ^2. ($\lambda = 630 \times 10^{-9} m$)

What is the solid angle describing the focused cone necessary to achieve this spot-size, not forgetting that **étendue (AΩ in simplified version) must be conserved**.

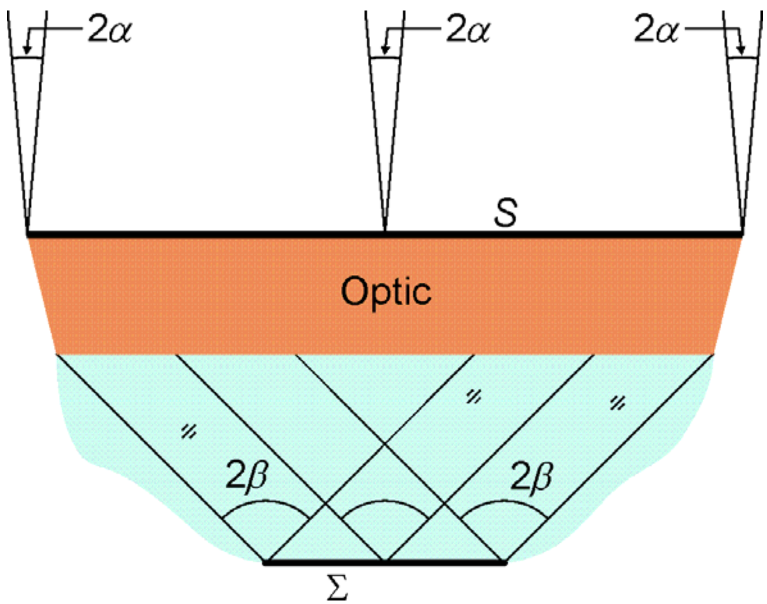
$$\Omega_{SPOT} = 50 \times 10^{-6} sr \, \frac{.04\, m^2}{(630 \times 10^{-9})^2\, m^2}$$

$$\boldsymbol{\Omega_{SPOT} = 5 \times 10^6\, sr}$$

The full unit sphere is 4 π sr !

As étendue must be conserved, we cannot focus a headlight down to the size of a laser spot (even if it has the same spectral bandwidth)

II. Application of Optical Etendue



Actually, that was kind of cheating… we used the small angle approximation for étendue, when obviously the second angle must be large.

If use the true expression, we can get the wikipedia example:

$$C = \frac{S}{\Sigma} = n^2 \frac{\sin^2 \beta}{\sin^2 \alpha},$$

$$C_{\max} = \frac{n^2}{\sin^2 \alpha}.$$

$$C_{max} = {}^{1}/_{(.004)^2} = 62500$$

A_{SPOT}= 0.04 m^2 / 62500 = 64 x 10^{-8} m^2 = (8 x 10^{-4}m$)^2$

Still, we are a long way away (1mm x 1mm) from the laser spot size (1µm x 1µm) . Why?

II. Optical Etendue and Laser Beams

We previously linked the étendue to the numerical aperture $\quad étendue \;= \Delta S\, \pi NA^2$

Using the relationship for NA for a propagating single mode Gaussian beam $\quad NA \approx \dfrac{\lambda_0}{\pi w_0}$

If we use this to express the minimum area at the beam waist, A_0 $\quad NA^2 \approx \dfrac{{\lambda_0}^2}{\pi A_0}$

We get an expression for étendue for a propagating Gaussian beam $\quad étendue \;= \left(\dfrac{\Delta S}{A_0}\right)\lambda_0^2$

This tells us that for a given propagating monochromatic beam, **étendue** informs us **how many times larger** the minimum spot size is, relative to the ideal case of a single mode.

But why would it be larger than the minimum?

NB: Using EM model

Outline of Lecture 3

Is laser light unique?

I. Coherence
   - Temporal Coherence
   - Spectral Width and Wiener-Khinchin
   - Spontaneous Emission Example
   - Headlight Example

II. Focusing Light
   - Numerical Aperture
   - Optical Etendue
   - Etendue and the Headlight
   - **Modes**

III. So is laser light actually unique?

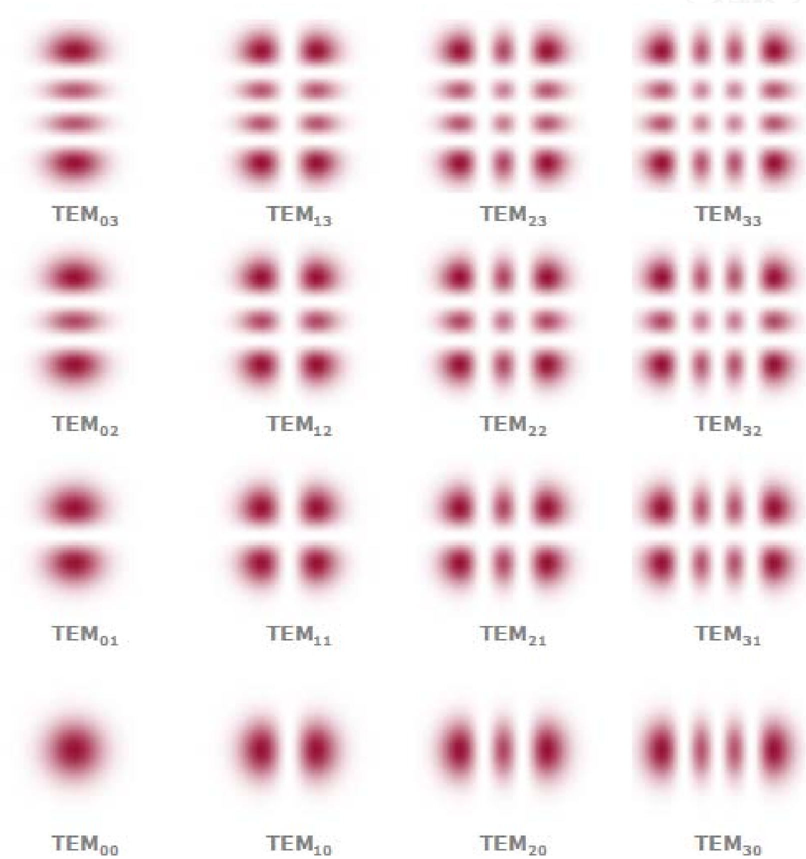
II. Transverse Optical Modes

So far, we have only been considering the simple Gaussian mode. This is just one solution for a propagating EM wave.

There exists a full basis set of Hermite-Gaussian modes (partly presented at right), and **photons may also occupy these modes**, not just the first transverse mode (TEM_{00}).

The more modes that are occupied, the larger the smallest possible spot size. The first order Gaussian mode has the smallest possible spot size.

Expressed in practice as the dimensionless Beam Parameter Product (BPP), which describes the **actual** spot size vs **minimum theoretical** size (ie for TEM_{00} alone).



NB: EM model, but now adding in photons

II. Etendue and Transverse Optical Modes

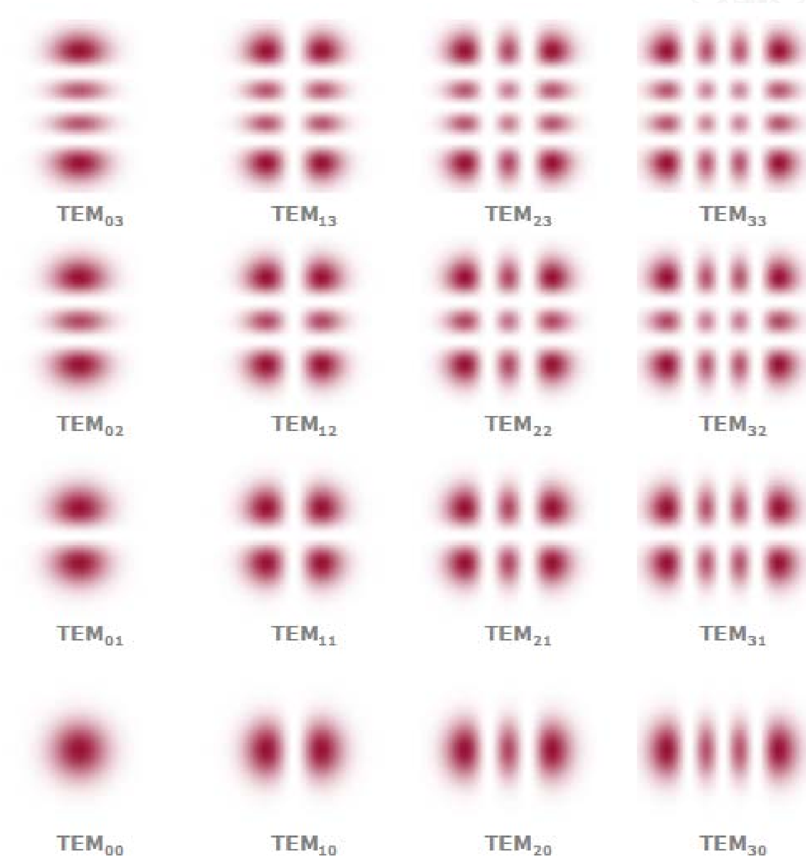
Optical étendue is another way of expressing the occupation by photons of transverse propagating modes. It can be shown that (and it feels obvious):

$$\#\, occupied\ transverse\ modes = \left(\frac{\acute{e}tendue}{\lambda_0^2}\right)$$

And so…

The minimum achievable spot size (focus area) will scale with the number of occupied transverse modes, thus also with étendue.

Key concept: We cannot « corral » photons into a mode. This would decrease entropy. Random scattering will distribute them, increasing entropy. The best we can do is filter them out.



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III. So is laser light actually unique?

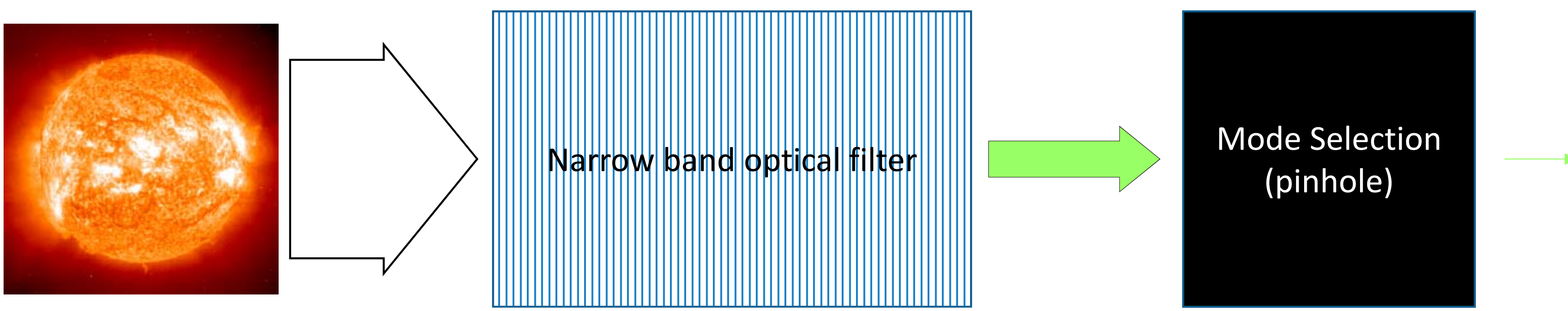
III. Is Laser light actually unique?

No, but it is special…

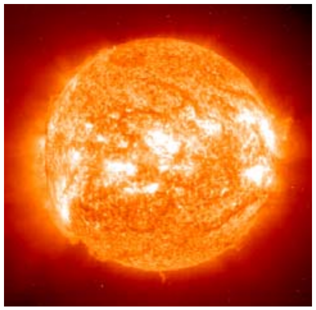
Unlike electrons (which are Fermions), photons (which are Bosons) can happily all occupy the same state (in this case, a transverse mode).

Lasers are special because the « stimulated » process can **selectively** put many, many photons into **the same transverse mode** (TEM_{00}) with the same phase, so they can then be perfectly focussed.

Although with enough filtering, we could mimic a laser beam, the losses would be spectacular.



III. Is Laser light actually unique?



Narrow band optical filter

Mode Selection (pinhole)

50W

Halogen
Headlight lamp

Etendue=
0.04 m^2 $(50 \times 10^{-6} sr)$

Bandwidth of
400 nm

Filter down to
10^{-3} nm

0.125 mW

1.25E-4 W

#occupied modes
= (étendue/λ^2)
= 2E-6 / (0.63E-6)^2
= 5E6

Assume randomly distributed, filter out all but TEM_{00}

0.25^E-10 W

25 pW

1 eV photons = 1.6E-19 J
25E-12 W/ 1.6E-19 J = 0.16 E9 /s
Photon every 8 ns
3E8 [m/s] 8E-9 [s] ≈ 2 m apart

Take home message

Laser light is almost perfectly coherent:

- Stimulated emission processes leads to photons with same frequency and phase
- Coherence time, length, and spectral width are all linked
- Spontaneous emission is a source of incoherence

$$\Delta\nu_c = \frac{1}{\tau_c}$$

$$S(\omega) = \frac{I_0}{(\omega - \omega_0)^2 + \Gamma^2}$$

Laser light can be perfectly focused:

- Minimum spot size depends on étendue, which represents the number of occupied transverse modes, and the wavelength
- Stimulated emission selectively puts photons into one transverse mode (or a few)

$$\text{étendue} = \left(\frac{\Delta S}{A_0}\right)\lambda_0^2 \qquad NA \approx \frac{\lambda_0}{\pi w_0}$$

$$\#\ occupied\ transverse\ modes = \left(\frac{\text{étendue}}{\lambda_0^2}\right)$$

References

Saleh & Teich, Fundamentals of Photonics

- Chapter 10, Statistical Optics

Brooker, Modern Classical Optics

- Chapter 11, Optical Practicalities: Etendue, interferometry, fringe localization

Some light Google/Wikipedia…