PHY208 – atoms and lasers
Lecture 2

Light Amplification by Stimulated Emission of Radiation

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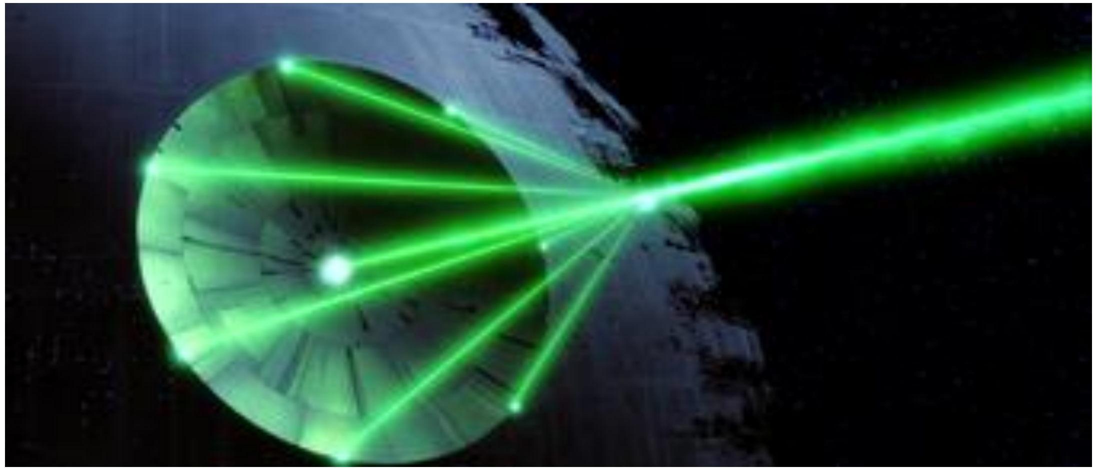
Outline of lecture 2

How does a (continuous monomode) laser actually work ?

I. A classical model for lasers: gain and phase conditions

II. Optical gain, population inversion
and saturation intensity



III. Laser operation (steady state)

IV. Laser operation (mode competition)

I. What is a LASER ?

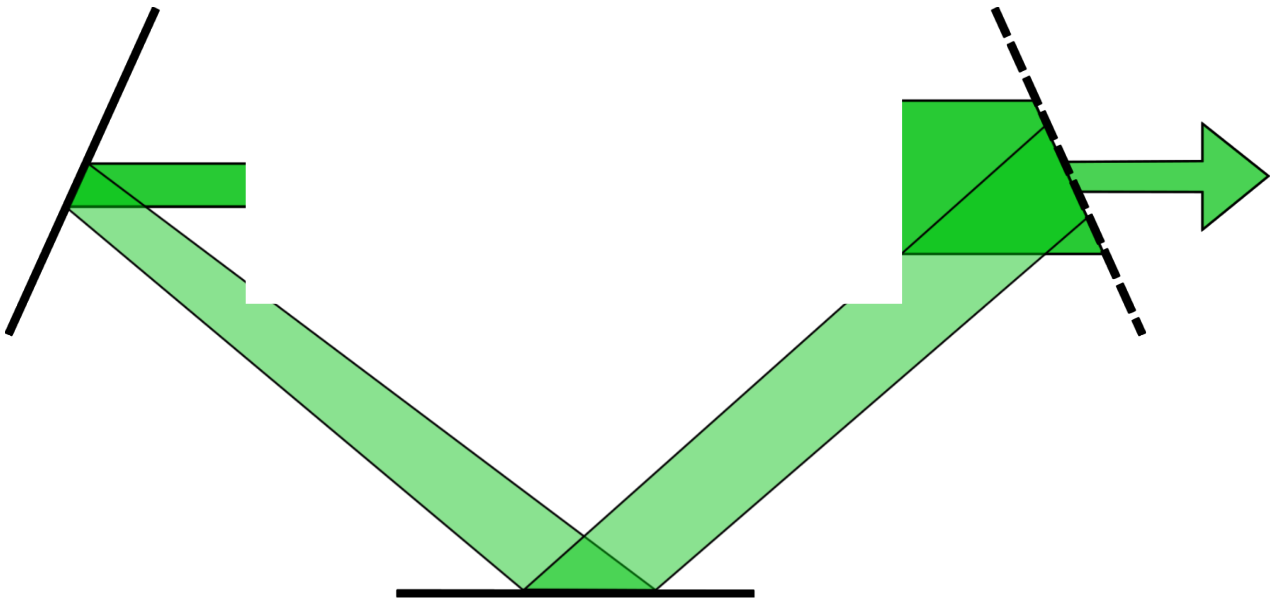
L.A.S.E.R. : Light Amplification by Stimulated Emission of Radiation

Two main ingredients :

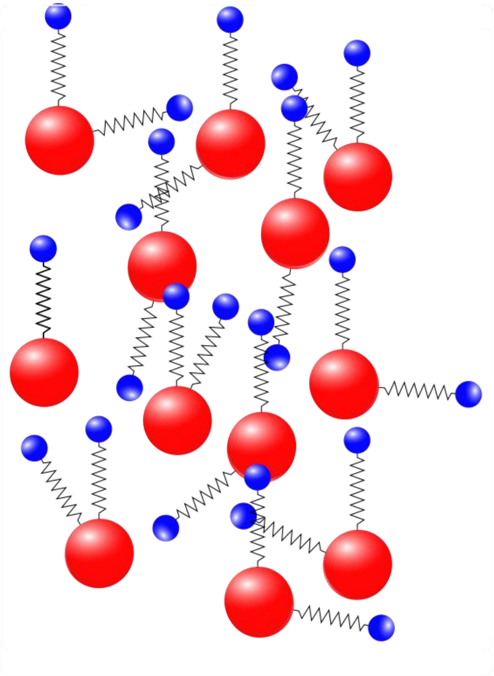
An optical cavity (« oscillator »)

An amplifying medium (« gain »)

(an output mirror)



I. Reminder on light propagation (1/2)



Classical model

Lorentz model : $$m_e \frac{d^2}{dt^2}\mathbf{r} = -m_e\omega_0^2\mathbf{r} - m\Gamma\frac{d}{dt}\mathbf{r} - q\mathbf{E}$$

Monochromatic excitation : $$\mathbf{r} = \mathbf{r}_0 e^{-i\omega t}$$

Susceptibility definition : $$\mathbf{P} = -nq\mathbf{r_0} = \epsilon_0\chi\mathbf{E}$$

Lorentz susceptibility : $$\chi = -\frac{ne^2}{m\epsilon_0}\frac{1}{\omega^2-\omega_0^2+i\omega\Gamma} = \underbrace{\frac{ne^2}{m\epsilon_0}\frac{\omega_0^2-\omega^2}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}}_{\chi'} + i\underbrace{\frac{ne^2}{m\epsilon_0}\frac{\omega\Gamma}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}}_{\chi''}$$

I. Reminder on light propagation (2/2)

Dispersion relation :
Wave vecteur ↔ frequency

$$k^2 = \frac{\omega^2 n^2}{c^2}$$

Linear homogeneous isotropic medium :
Optical index ↔ susceptibility

$$n^2 = 1 + \chi$$

Small susceptibility limit:
Wave vector ↔ susceptibility

$$k \simeq \underbrace{\left(1 + \frac{\chi'}{2}\right)\frac{\omega}{c}}_{k'} + i\underbrace{\frac{\chi''}{2}\frac{\omega}{c}}_{k''}$$

Propagation through this medium :

$$\mathcal{E}(x) = e^{ikx}\mathcal{E}(0) = e^{-k''x}e^{ik'x}\mathcal{E}(0)$$

Notion of gain :

Intensity:

$$I = \frac{1}{2\mu_0 c}\left|\mathcal{E}\right|^2$$

Beer Lambert :

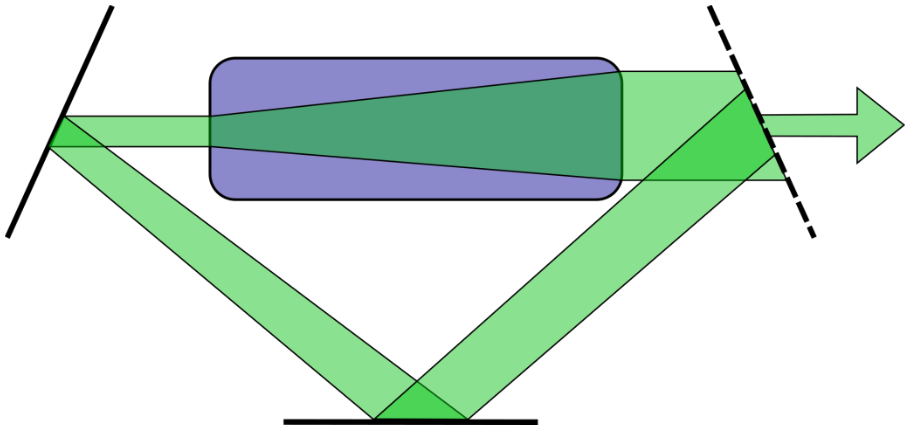
$$I(x) = I(0)e^{-2k''x}$$

$$-2k'' = -\alpha = g$$

Wave number Abs. Coeff. Gain

Units : [(c)m^{-1}] sometimes [dB.m^{-1}]

I. Classical model



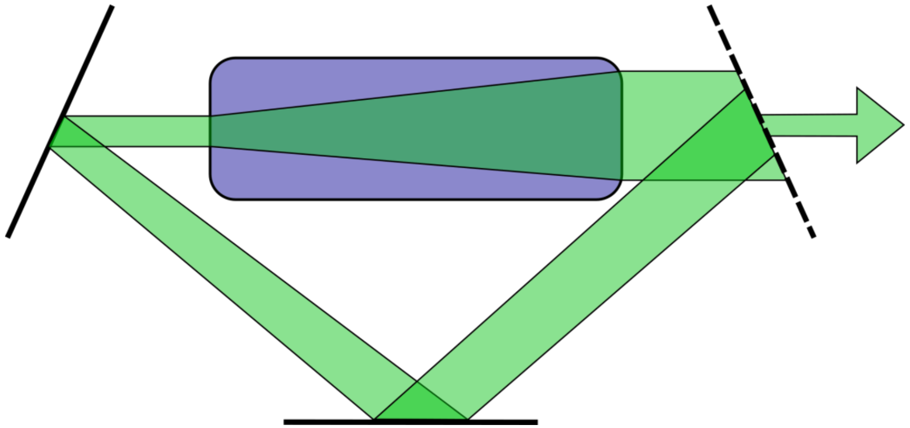
Consider the field propagation in a cavity round trip

$$\mathcal{E}(0)$$

- « Amplifying medium » of tickness d
- Reflexion on the output mirror
- Propagation in the empty cavity
- (Two additionnal reflexions)
- Parasitic losses

$$\mathcal{E}(L) = \mathcal{E}(0) \times \underbrace{r e^{-k''d} e^{-\frac{\alpha_0 L}{2}}}_{\text{amplitude}} \times \underbrace{e^{ik'd} e^{ik_0(L-d)} e^{i\theta}}_{\text{phase}}$$

I. Lasing conditions



$$\mathcal{E}(L) = \mathcal{E}(0) \times \underbrace{re^{-\frac{\alpha_0 L}{2}} e^{-k''d}}_{\text{amplitude}} \times \underbrace{e^{ik'd} e^{ik_0(L-d)} e^{i\theta}}_{\text{phase}}$$

Steady state : $E(L) = E(0)$

Two conditions are necessary to allow such a steady-state :

→ Condition on amplitude :

$$r \exp\left(-k''d\right) \exp\left(-\frac{\alpha_0 L}{2}\right) = 1$$

Amplification compensates losses

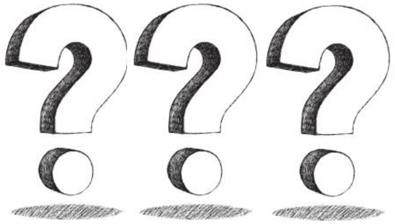
$$\Rightarrow \quad \chi'' < 0$$

$$\Rightarrow \quad \frac{ne^2}{m\epsilon_0} \frac{\omega\Gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\Gamma^2} < 0$$

→ Condition on phase :

$$k'd + k_0\left(L - d\right) + \theta = 2p\pi$$

Constructive interference



Outline of lecture 2

How does a (continuous monomode) laser actually work ?

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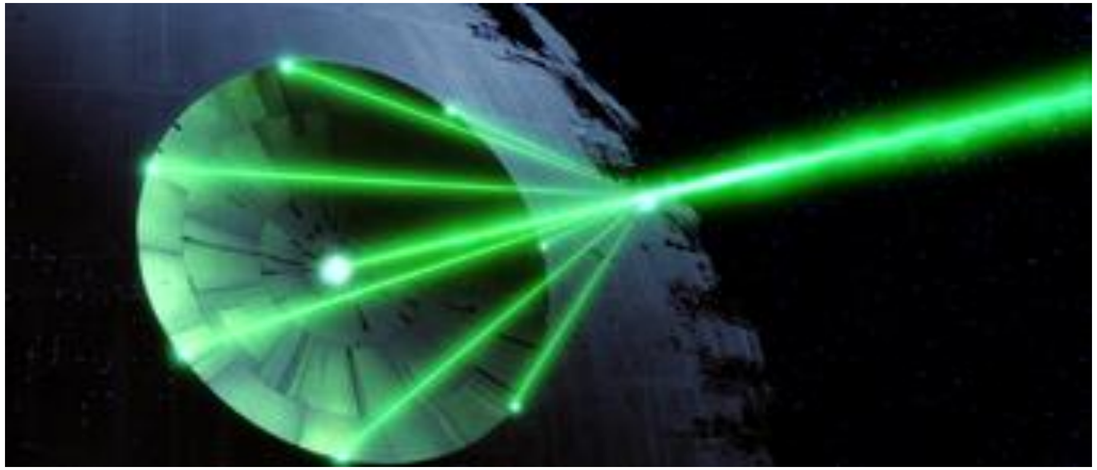
II. Optical gain, population inversion and saturation intensity

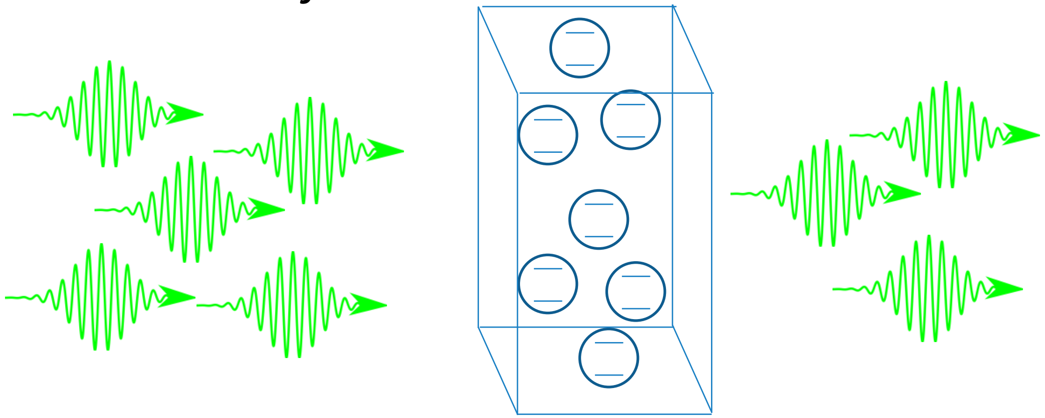


III. Laser operation (steady state)

IV. Laser operation (mode competition)

II. Population inversion

Reminder from lecture 1 :



Semi classical

Energy balance on the radiation traveling through a slab :

Interaction cross section

$$r_{\text{abs}} = r_{\text{stim}} = \frac{\sigma\, I}{h\nu}$$

Beer lambert law

$$\frac{d}{dx} I = \sigma\ (n_e - n_g)\, I$$

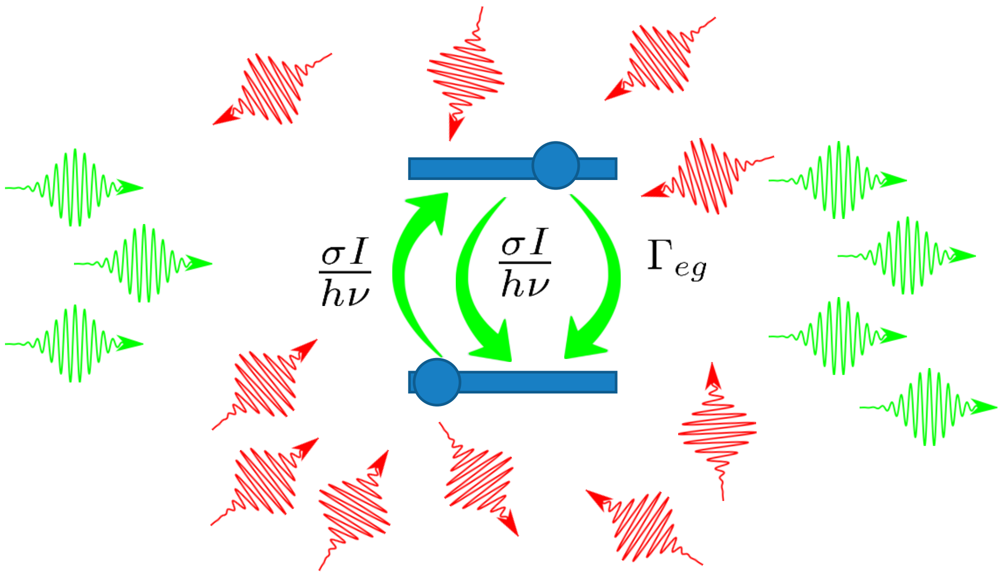
$$g = \sigma_{eg} \underbrace{(n_e - n_g)}_{\Delta n}$$

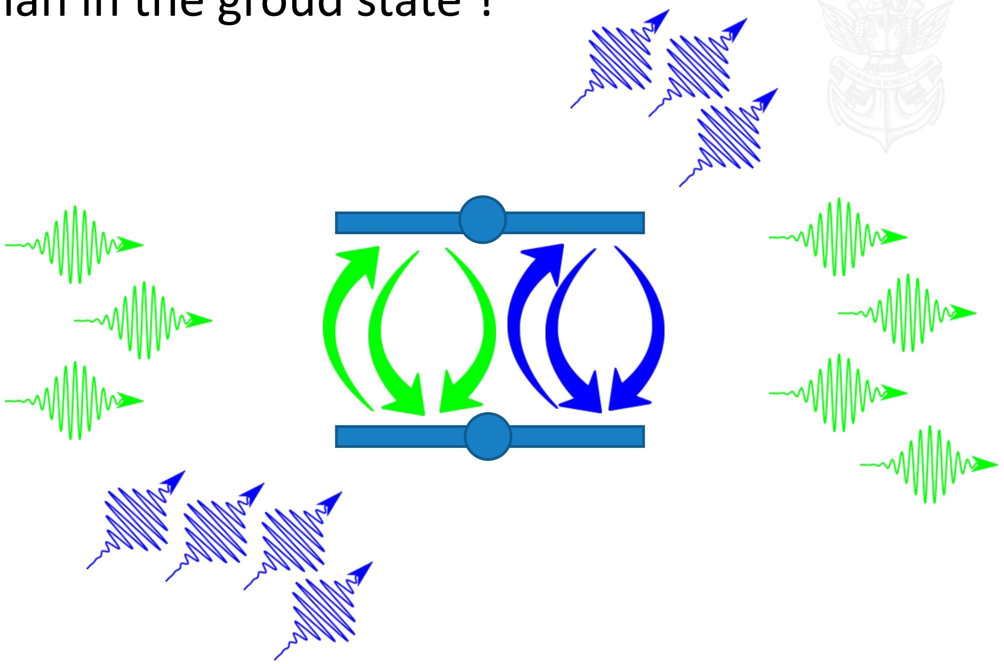
Gain depends on
- **Interaction cross section**
- **Population inversion**

To provide amplification, we need more atoms in the excited state than in the ground state.

II. Inversion schemes : 2 lvls system

How to get more atoms in the excited state than in the groud state ?





Option 1 : Increase temperature ?

Thermal equilibrium : $\frac{n_e}{n_g} = \exp\left(-\frac{E_e - E_g}{k_B T}\right) \leq 1$

Thermal equilibrium has to be broken

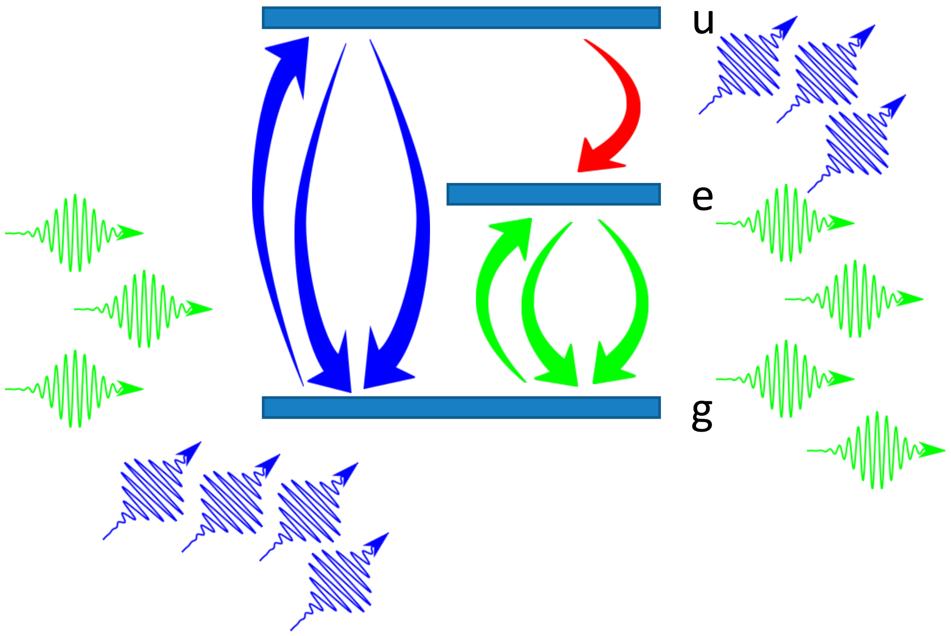
Option 2 : Add another « pump » beam

$$\frac{n_e}{n_g} = \frac{\frac{\sigma_L I_L}{h\nu_L} + \frac{\sigma_P I_P}{h\nu_P}}{\Gamma_{eg} + \frac{\sigma_L I_L}{h\nu_L} + \frac{\sigma_P I_P}{h\nu_P}} \leq 1$$

Can't pump a 2 level system to inversion

II. Inversion schemes : 3 lvls system

Trick : introduce a 3rd level



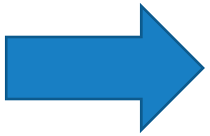
Lasing transition between excited and ground state.

Pump atoms from the ground state to the « up » state

(Spontaneous and stimulated emission to ground state also occur)

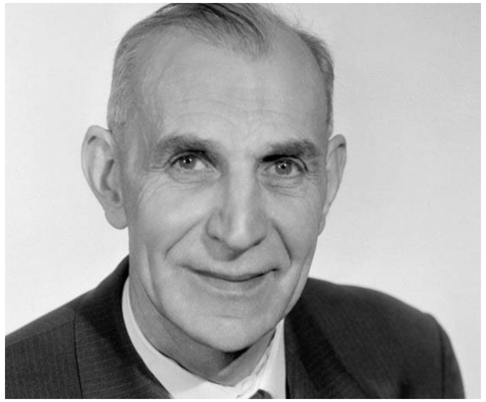
The « up » state can spontaneously decay to excited state.

If this decay is fast, atoms accumulate in the excited state



Optical pumping strategy

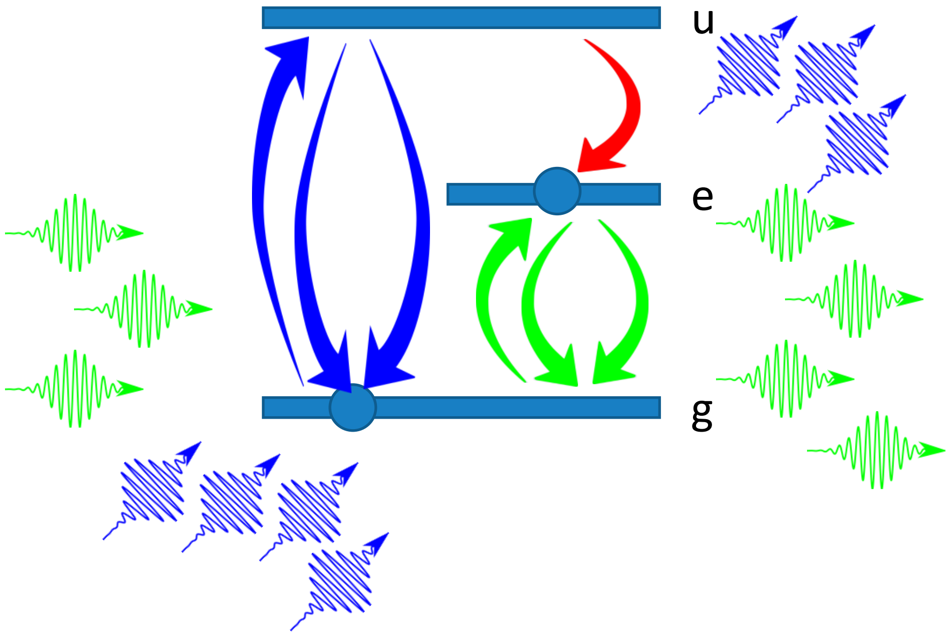
1966, "for the discovery and development of optical methods for studying Hertzian resonances in atoms."



Alfred Kastler

II. Inversion schemes : 3 lvls system

Trick : introduce a 3rd level



Pump atoms from the ground state to the « up » state

$$r_{abs}^{u-g} = \frac{\sigma_{ug} I_p}{h\nu_p} = W_p \qquad r_{stim}^{u-g} = W_p \qquad r_{spont}^{u-g} = \Gamma_{ug}$$

Pump rate, usual notation.

The « up » state can spontaneously decay to excited state.

$$r_{spont}^{u-e} = \Gamma_{ue}$$

Lasing transition between excited and ground state.

$$r_{abs}^{g-e} = \frac{\sigma_{eg} I_L}{h\nu_L} \qquad r_{stim}^{e-g} = \frac{\sigma_{eg} I_L}{h\nu_L} \qquad r_{spont}^{e-g} = \Gamma_{eg}$$

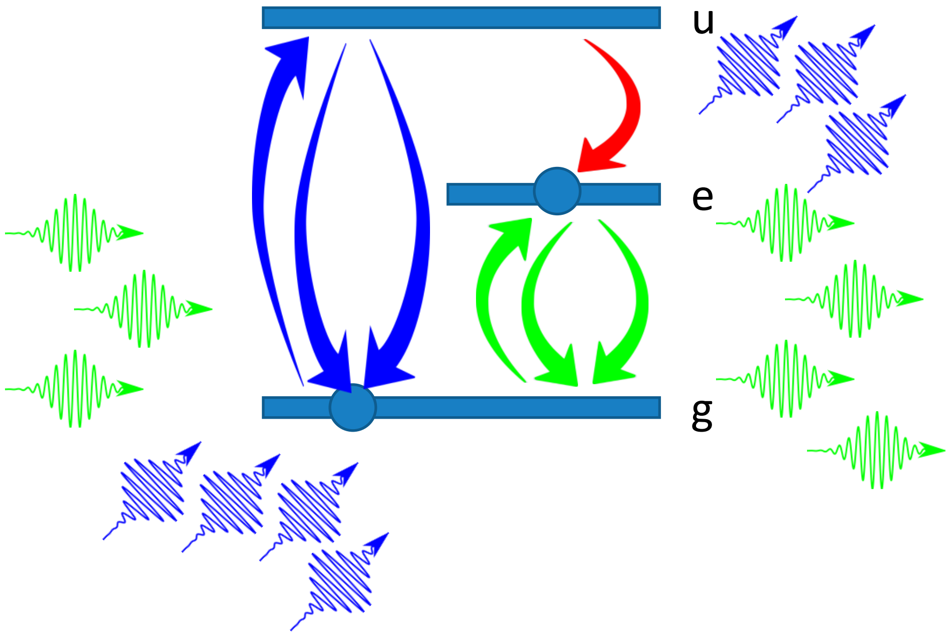
Balance of population on the up state :

$$n_u(t+dt) = n_u(t) + W_p\, n_g(t)dt - W_p\, n_u(t)dt - \Gamma_{ug} n_u(t)dt - \Gamma_{ue} n_u(t)dt$$

$$\Rightarrow \quad \frac{d}{dt} n_u = W_p\, n_g(t) - (W_p + \Gamma_{ug} + \Gamma_{ue})\, n_u(t)$$

II. Inversion schemes : 3 lvls system

Trick : introduce a 3rd level



Rate equations :

$$\frac{d}{dt}n_u = W_p\, n_g(t) - (W_p + \Gamma_{ug} + \Gamma_{ue})\, n_u(t)$$

$$\frac{d}{dt}n_e = \frac{\sigma_{eg} I_L}{h\nu_L} n_g(t) + \Gamma_{ue} n_u(t) - \left(\frac{\sigma_{eg} I_L}{h\nu_L} + \Gamma_{eg}\right) n_e(t)$$

$$\frac{d}{dt}n_g = \left(\frac{\sigma_{eg} I_L}{h\nu_L} + \Gamma_{eg}\right) n_e(t) + (W_p + \Gamma_{ug})\, n_u(t) - \left(\frac{\sigma_{eg} I_L}{h\nu_L} + W_p\right) n_g(t)$$

Hypothesis : fast decay u → e

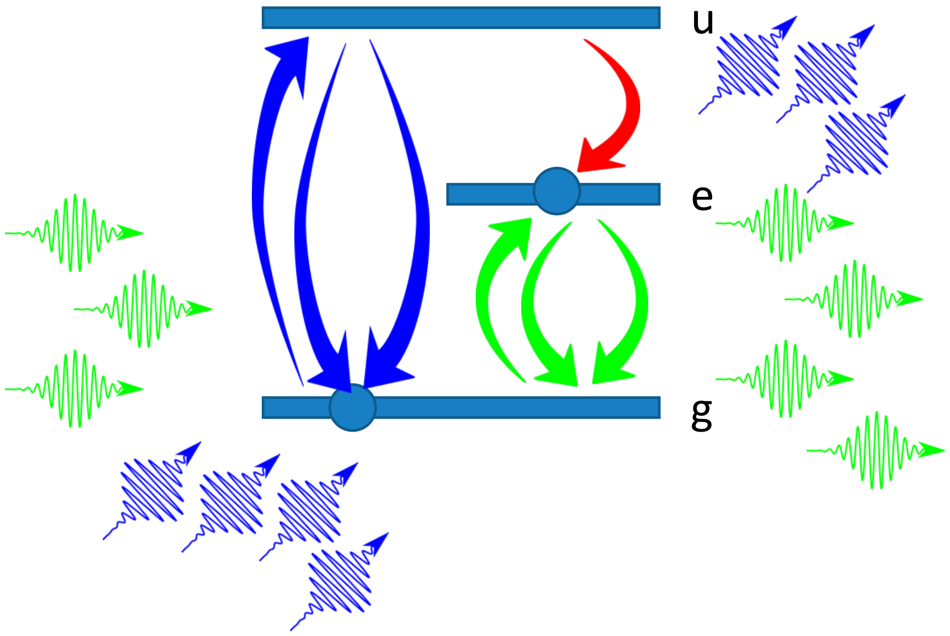
$$n_u = \frac{W_p}{W_p + \Gamma_{ug} + \Gamma_{ue}} n_g \simeq \frac{W_p}{\Gamma_{ue}} n_g$$

Population inversion dynamics:

$$\frac{d}{dt}\Delta n = (W_p - \Gamma_{eg})\, n_{tot} - \left(W_p + \Gamma_{eg} + 2\frac{\sigma_{eg} I}{h\nu}\right) \Delta n$$

II. Inversion schemes : 3 lvls system

Trick : introduce a 3rd level



Hypothesis : fast decay u → e

$$n_u = \frac{W_p}{W_p+\Gamma_{ug}+\Gamma_{ue}} n_g \simeq \frac{W_p}{\Gamma_{ue}} n_g$$

Rate equations :

$$\frac{d}{dt}n_u = W_p\, n_g(t) - (W_p + \Gamma_{ug} + \Gamma_{ue})\, n_u(t)$$

$$\frac{d}{dt}n_e = \frac{\sigma_{eg} I_L}{h\nu_L} n_g(t) + \Gamma_{ue} n_u(t) - \left(\frac{\sigma_{eg} I_L}{h\nu_L} + \Gamma_{eg}\right) n_e(t)$$

$$\frac{d}{dt}n_g = \left(\frac{\sigma_{eg} I_L}{h\nu_L} + \Gamma_{eg}\right) n_e(t) + (W_p + \Gamma_{ug})\, n_u(t)$$

$$- \left(\frac{\sigma_{eg} I_L}{h\nu_L} + W_p\right) n_g(t)$$

Stationary state :

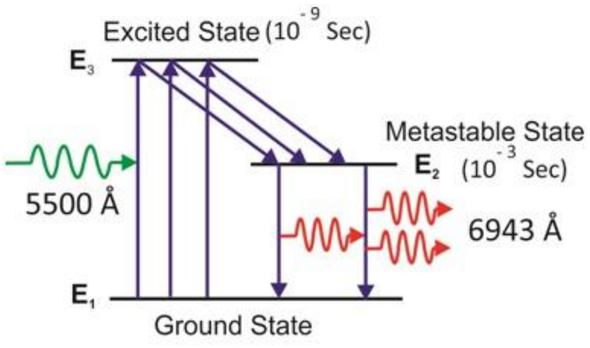
$$n_e - n_g = \frac{W_p - \Gamma_{eg}}{W_p + \Gamma_{eg}} \frac{1}{1 + \frac{2\sigma_{eg} I_L}{h\nu_L (W_p + \Gamma_{eg})}} n_{\mathrm{tot}}$$

If $W_p > \Gamma_{eg}$ ✓

Happy ?

II. Population inversion in practice

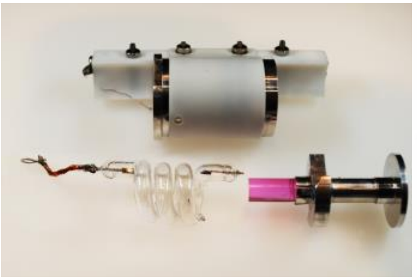
The historical 3 levels system *(Ruby laser)*

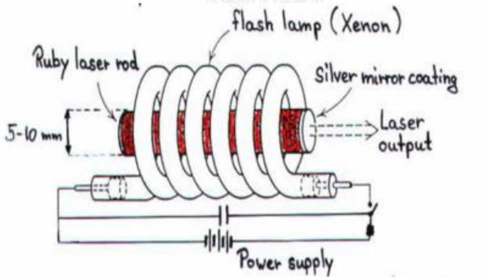


Energy Level Diagram of Ruby LASER

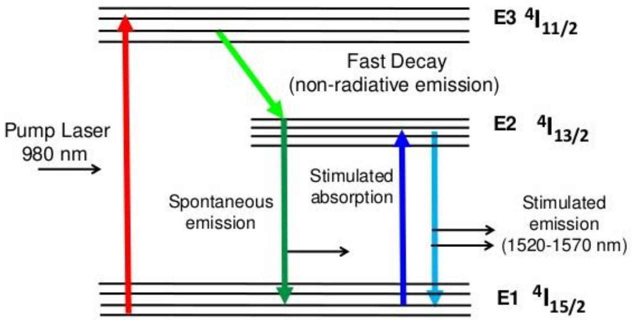
Pump with a flash light

Convert energy from incoherent light
into a laser beam



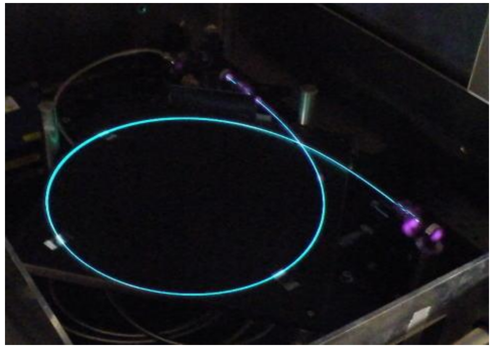


Side remark : Technological application *(Erbium doped fibered amplifier)*



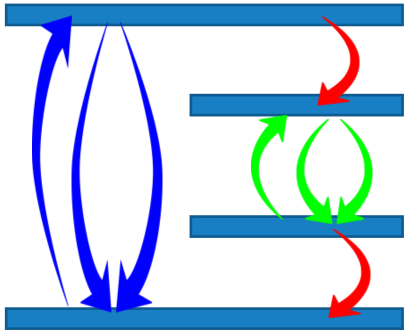
Pump with a laser light

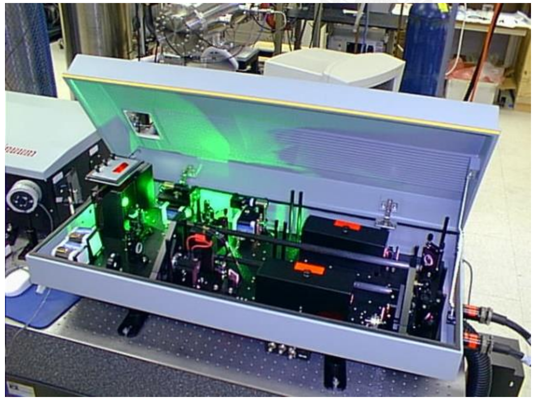
Convert energy from one laser beam
Into another beam



II. Laser culture : inversion strategies (1/2)

Improved scheme : 4 levels system *(YAG)*

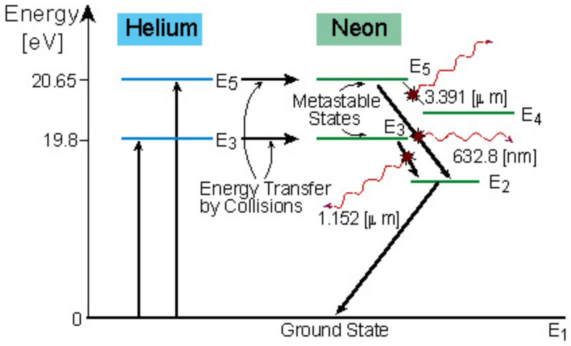


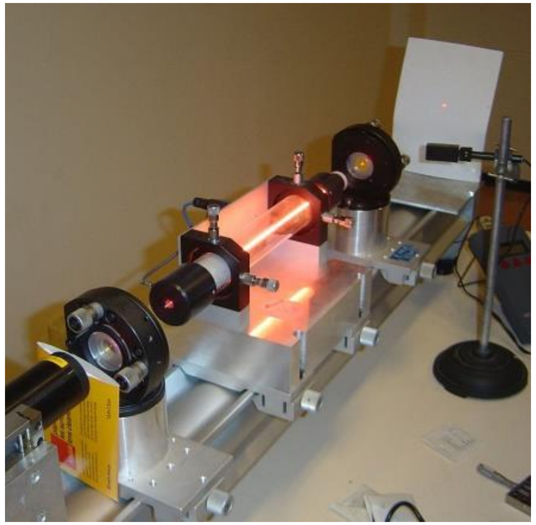


less population on the lasing ground state

less absorption of the laser beam

Gas laser : electrical discharge *(He-Ne)*



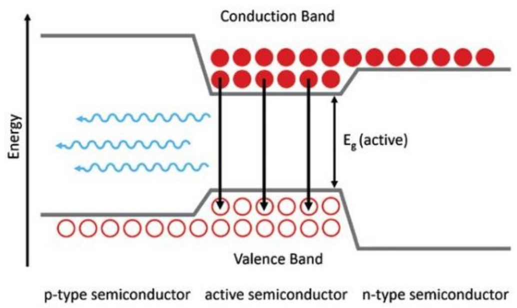


Accelerated electrons collide with atoms

Direct (e^- - atom) or indirect (e^- - atom – atom) energy transfer

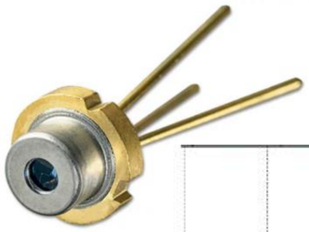
II. Laser culture : inversion strategies (2/2)

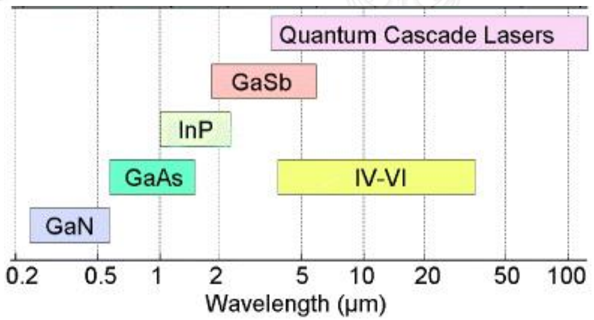
Laser diode : electrical injection



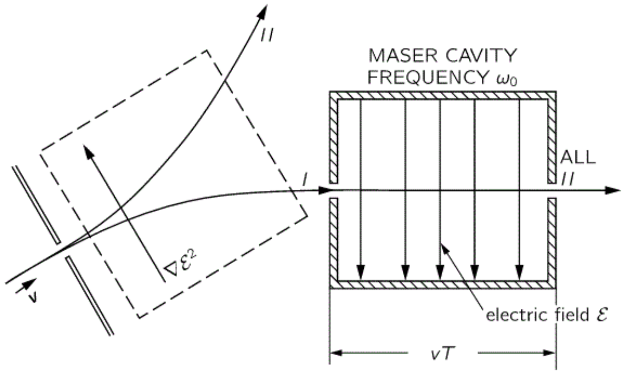
Extract electrons from low energy levels

Inject electrons in high energy levels





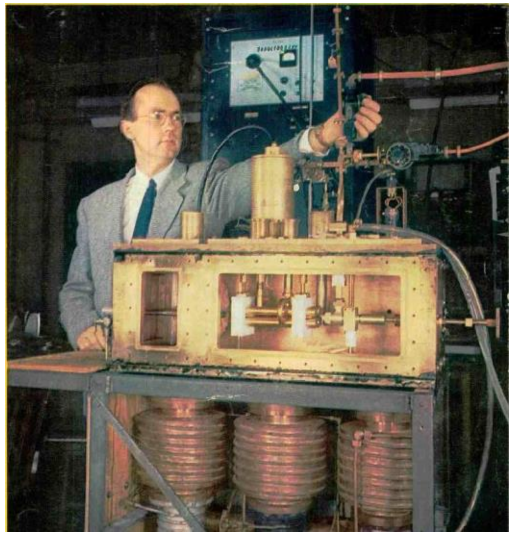
Ammonia Maser : state selectivity



Reminder (Phy205) : ammonia molecule in a field

$$E_{\pm} = E_0 \pm \sqrt{J^2 + q^2\delta^2\mathcal{E}^2}$$

Select only molecules in ψ_+ state



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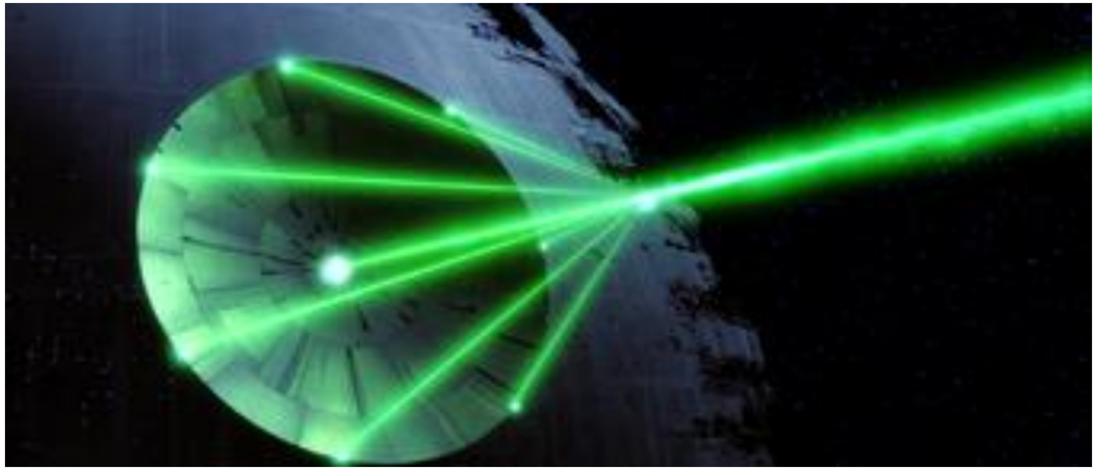
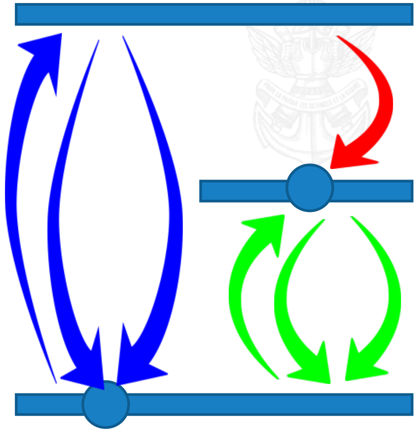
III. Laser operation (steady state)



IV. Laser operation (mode competition)

III. Back to optical gain



Population in a 3 level system :

$$n_e - n_g = \frac{W_p - \Gamma_{eg}}{W_p + \Gamma_{eg}} \frac{1}{1 + \frac{2\sigma_{eg} I_L}{h\nu_L (W_p + \Gamma_{eg})}} n_{\text{tot}}$$

Gain in a 3 level system :

(actually, very generic form)

$$g = \sigma \Delta n = \frac{g_0}{1 + I/I_{\text{sat}}}$$

Unsaturated gain *[m⁻¹]*

$$g_0 = \sigma \frac{W_p - \Gamma_{eg}}{W_p + \Gamma_{eg}} n_{\text{tot}}$$

Saturation intensity *[W.m⁻²]*

$$I_{\text{sat}} = \frac{h\nu_L}{2\sigma_{eg}} (W_p + \Gamma_{eg})$$

III. Back to optical gain

Gain in a 3 level system :

$$g = \sigma \Delta n = \frac{g_0}{1+I/I_{\rm sat}}$$

Unsaturated gain *[m^{-1}]*

$$g_0 = \sigma \frac{W_p - \Gamma_{eg}}{W_p + \Gamma_{eg}} n_{\rm tot}$$

Saturation intensity *[$W.m^{-2}$]*

$$I_{\rm sat} = \frac{h\nu_L}{2\sigma_{eg}} (W_p + \Gamma_{eg})$$

- Unsaturated gain increases with pumping rate (W_p), interaction cross section (σ), atomic density (n_{tot})
- Unsaturated gain decreases recombination rate (Γ_{eg})
- Amplification or absorption depending on W_p - Γ_{eg}

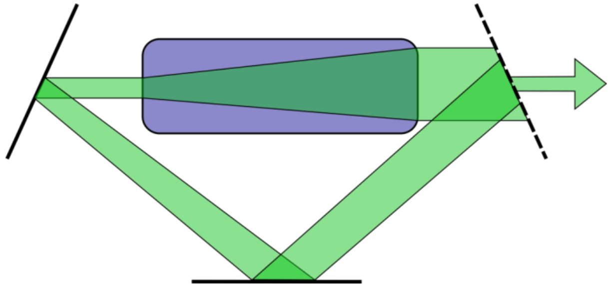
All atoms in g state $$-\sigma n_{\rm tot} \leq g_0 \leq \sigma n_{\rm tot}$$ *All atoms in e state*

- Gain decreases with laser intensity

At low laser intensity, $g = g_0$

At high laser intensity, $g \underset{I \gg I_{\rm sat}}{\sim} g_0 \frac{I_{sat}}{I} \rightarrow 0$

III. Laser steady-state



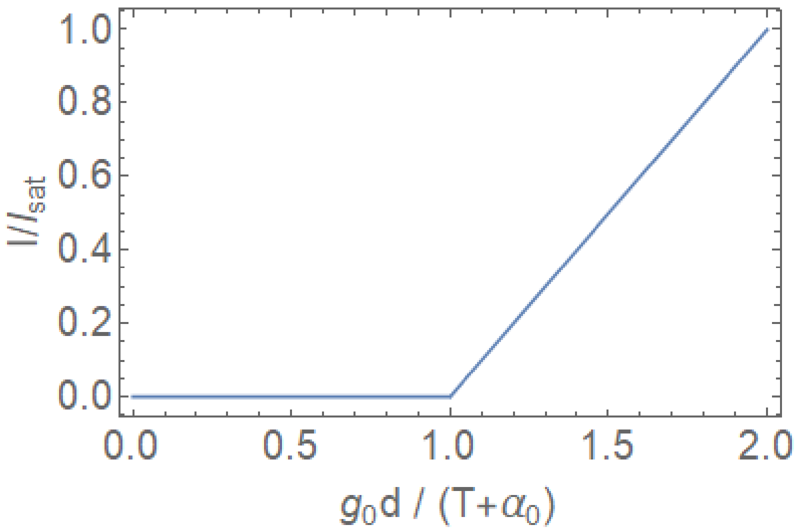
$$I(L) = I(0) \quad \Rightarrow \quad R\exp(g\,d)\,\exp(-\alpha_0 L) = 1$$

$$\Rightarrow \quad \frac{g_0 d}{1+I/I_{sat}} = \alpha_0 L + T$$

Optical gain   Optical losses

Laser intensity

$$I = I_{sat}\left(\frac{g_0 d}{T+\alpha_0 L} - 1\right)$$



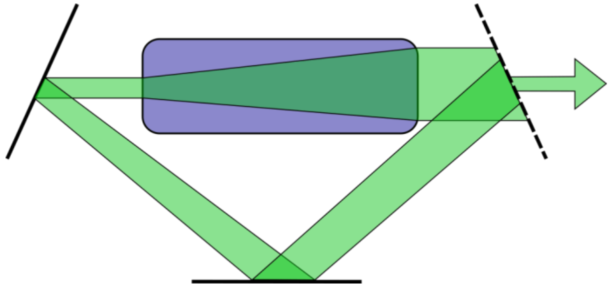
Necessary condition for I>0 :

Lasing threshold $\quad g_0 d \geq T + \alpha_0 L$

Laser intensity adjustes to unsaturated gain and losses,

so that total gain (including saturation) = total loss

III. Laser steady-state



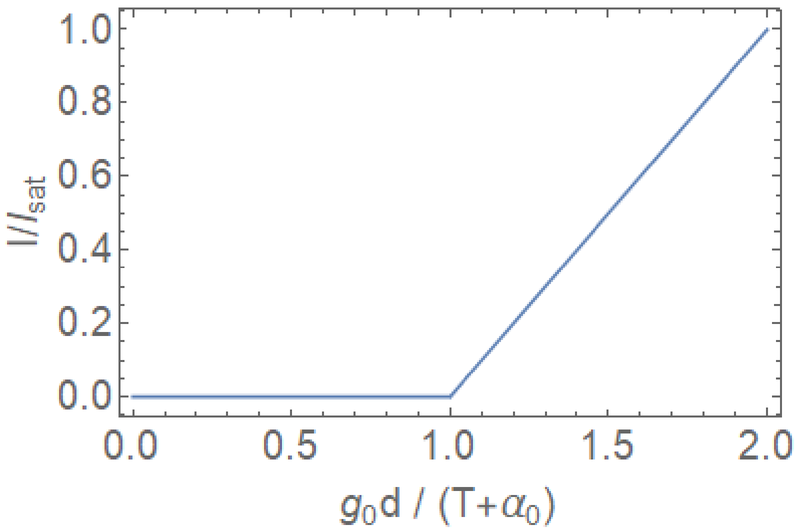
$$I(L) = I(0) \Rightarrow R\,\exp(g\,d)\,\exp(-\alpha_0 L) = 1$$

$$\Rightarrow \frac{g_0 d}{1+I/I_{sat}} = \alpha_0 L + T$$

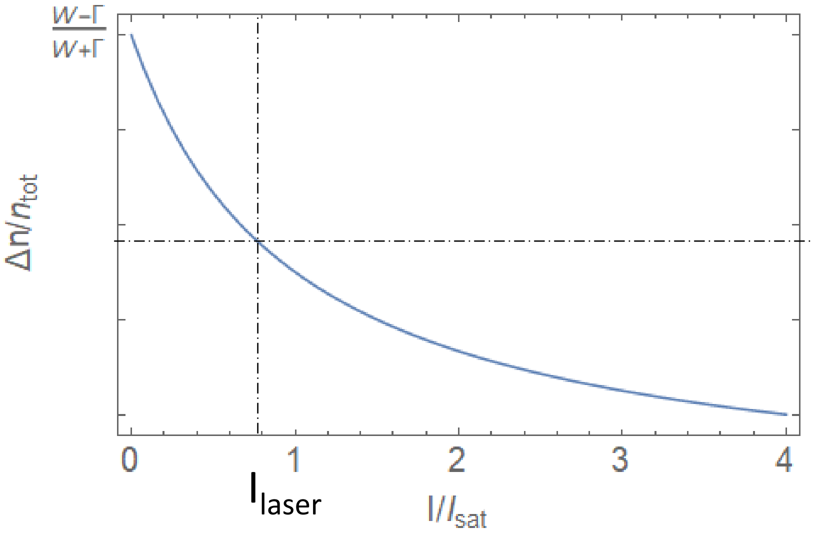
Optical gain Optical losses

Laser intensity

$$I = I_{sat}\left(\frac{g_0 d}{T+\alpha_0 L} - 1\right)$$



Population inversion



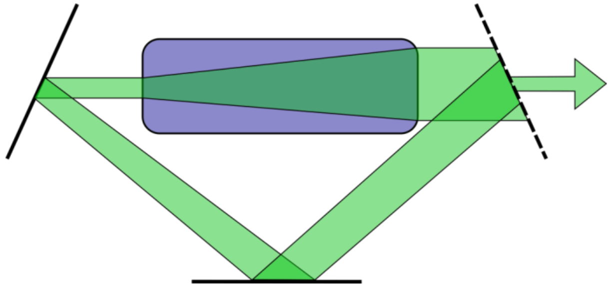
$$\Delta n = n_{tot}\frac{W-\Gamma}{W+\Gamma}\frac{1}{1+I/I_{sat}}$$

$$= \frac{T+\alpha_0 L}{\sigma d}$$

Such that gain = losses

No explicit dependence on I

III. Laser steady-state



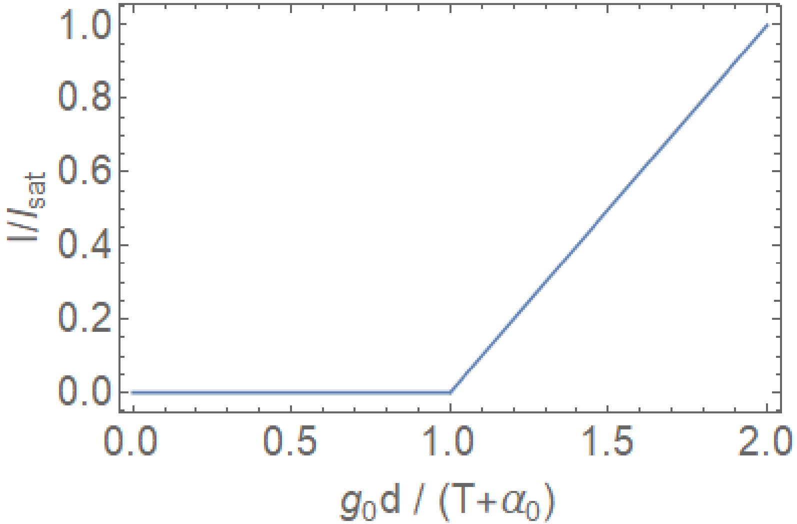
$$R\,\exp(g\,d)\,\exp(-\alpha_0 L) = 1$$

$$\frac{g_0 d}{1+I/I_{sat}} = \alpha_0 L + T$$

Optical gain Optical losses

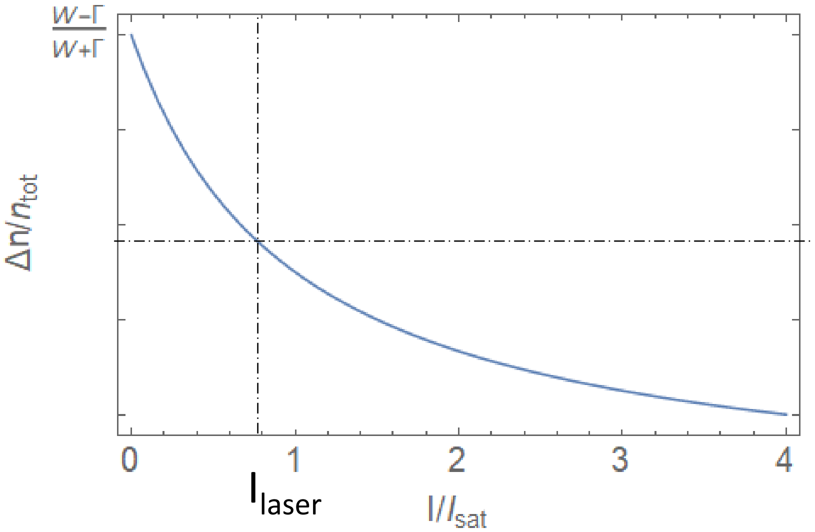
Laser intensity (inside the cavity)

$$I = I_{sat}\left(\frac{g_0 d}{T+\alpha_0 L} - 1\right)$$



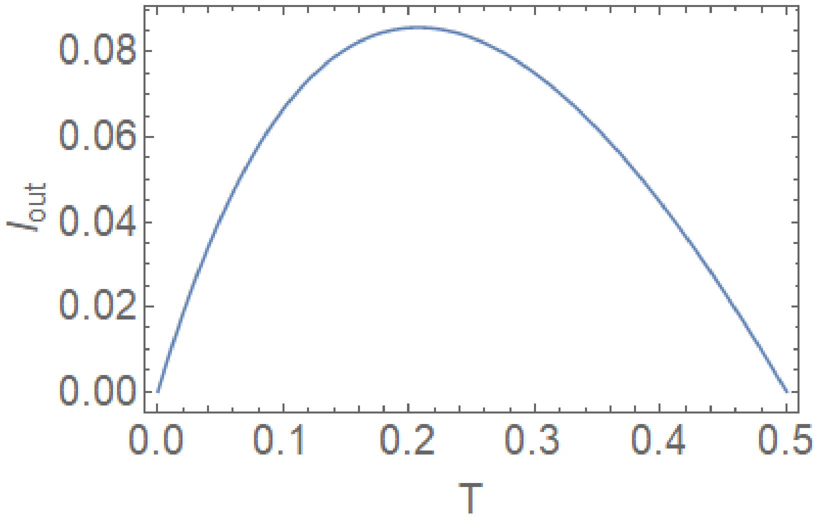
Population inversion

Such that gain = losses

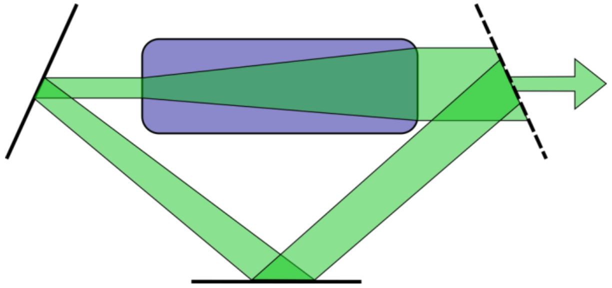


Laser intensity (emitted)

$$I_{out} = T \times I_{sat}\left(\frac{g_0 d}{T+\alpha_0 L} - 1\right)$$



III. What's next ?



Two conditions to have a laser :

Gain compensates losses → Requires population inversion

(What ? Why ? How ? How much ?)

Constructive interferences → Resonant cavity

(Not addressed with semi-quantum model)

Outline of lecture 2

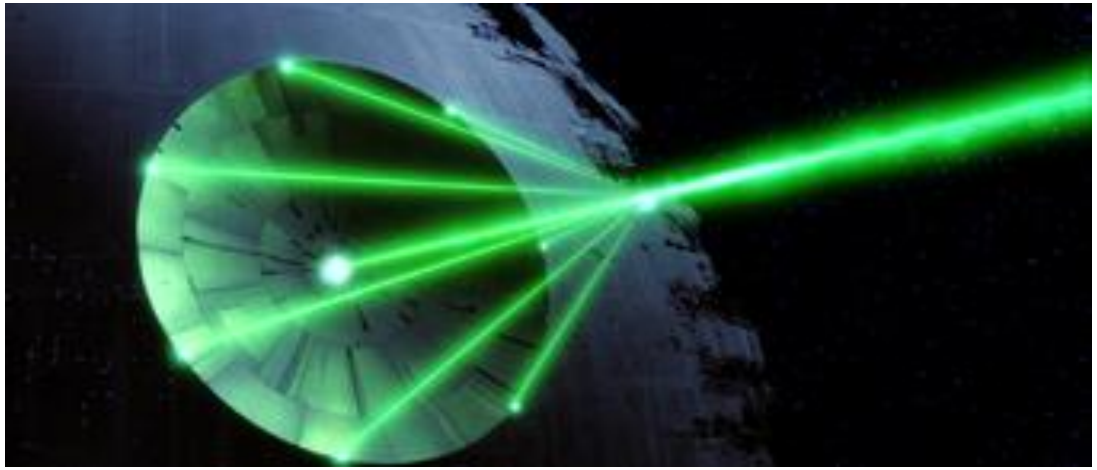
How does a (continuous monomode) laser actually work ?

I. A classical model for lasers: gain and phase conditions

II. Optical gain, population inversion
and saturation intensity

III. Laser operation (steady state)

IV. Laser operation (mode competition)

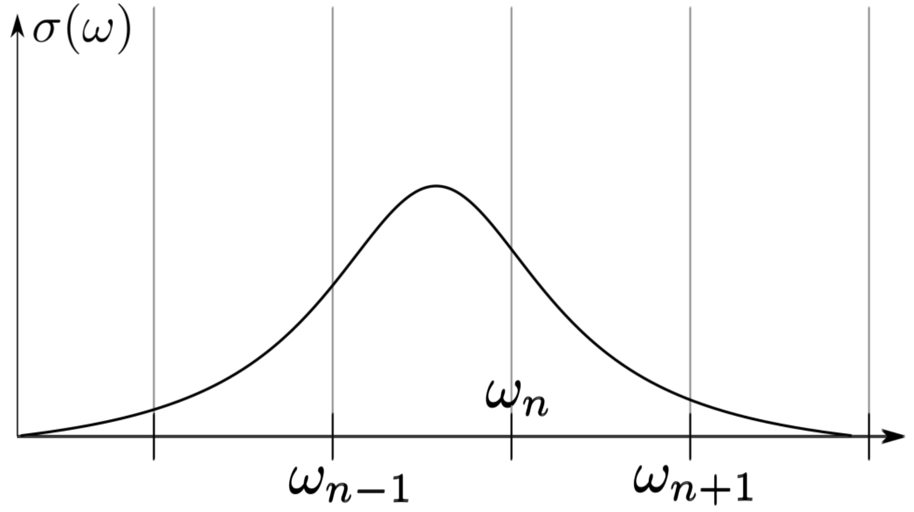


III. Lasing modes

So far, we have been working with only *one* frequency.

The cavity allows lasing at any frequency satisfying the phase condition $k'd + k_0 \left(L - d\right) + \theta = 2p\pi$

The interaction cross section is maximal for resonant frequency, but has finite width

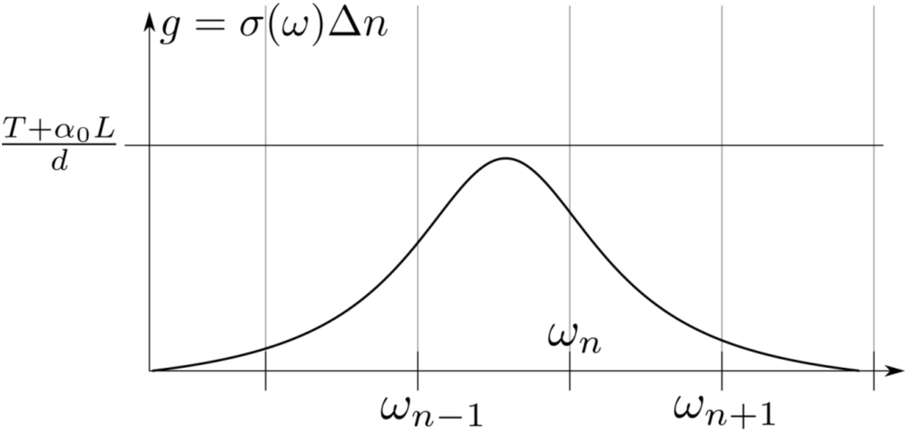


→ Several frequency could be considered

More generally : several **modes** (frequency, polarization, transverse profile…) can be considered.

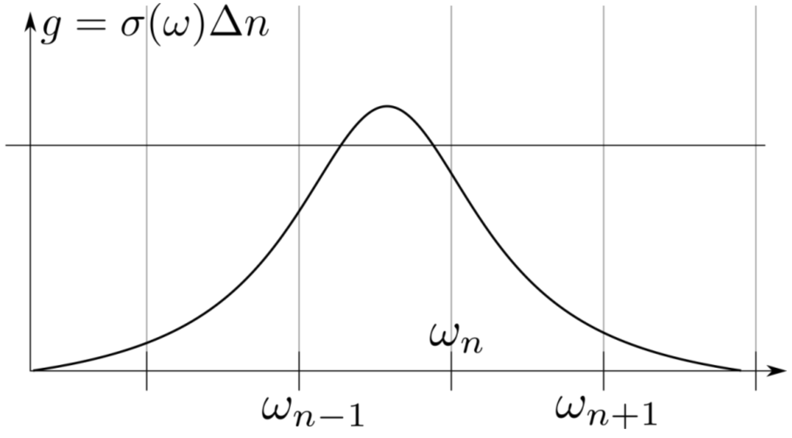
III. Will it lase?

Increasing pumping



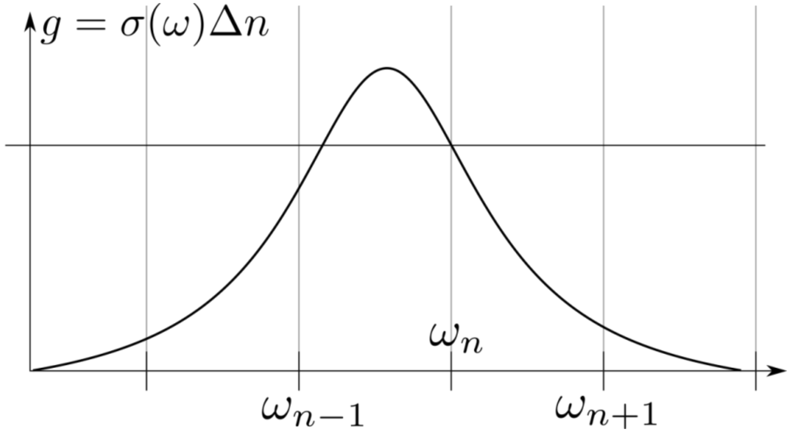
No

Because the gain is too small to compensate for losses



No

Because the gain is too small to compensate for losses at frequencies permitted by the cavity



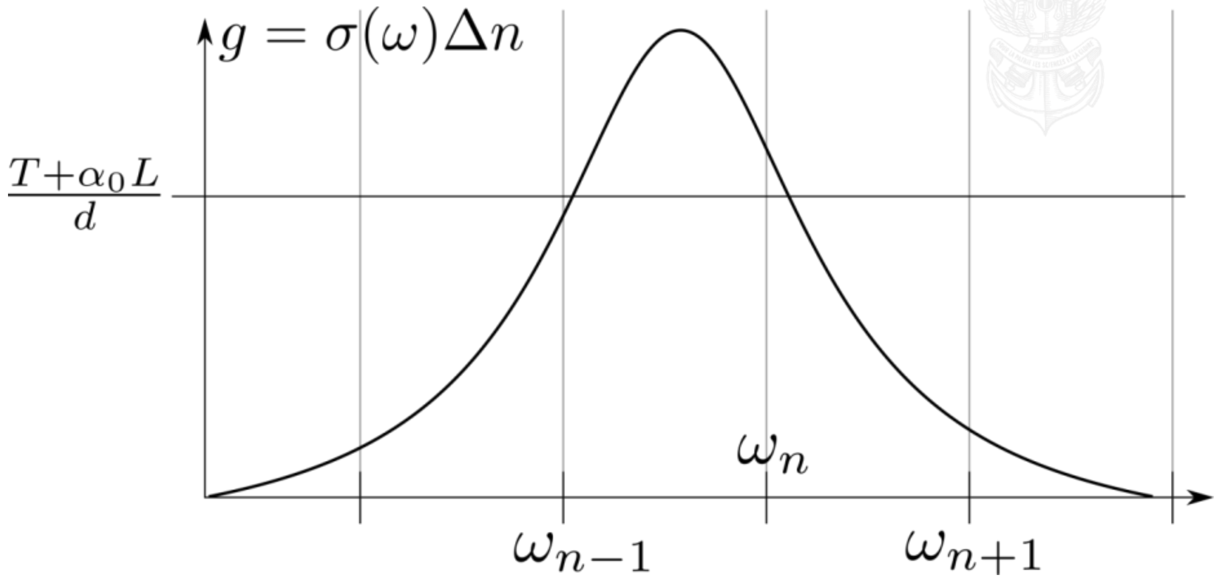
Yes

The gain is just sufficient to allow lasing at ω_n

III. Mode competition

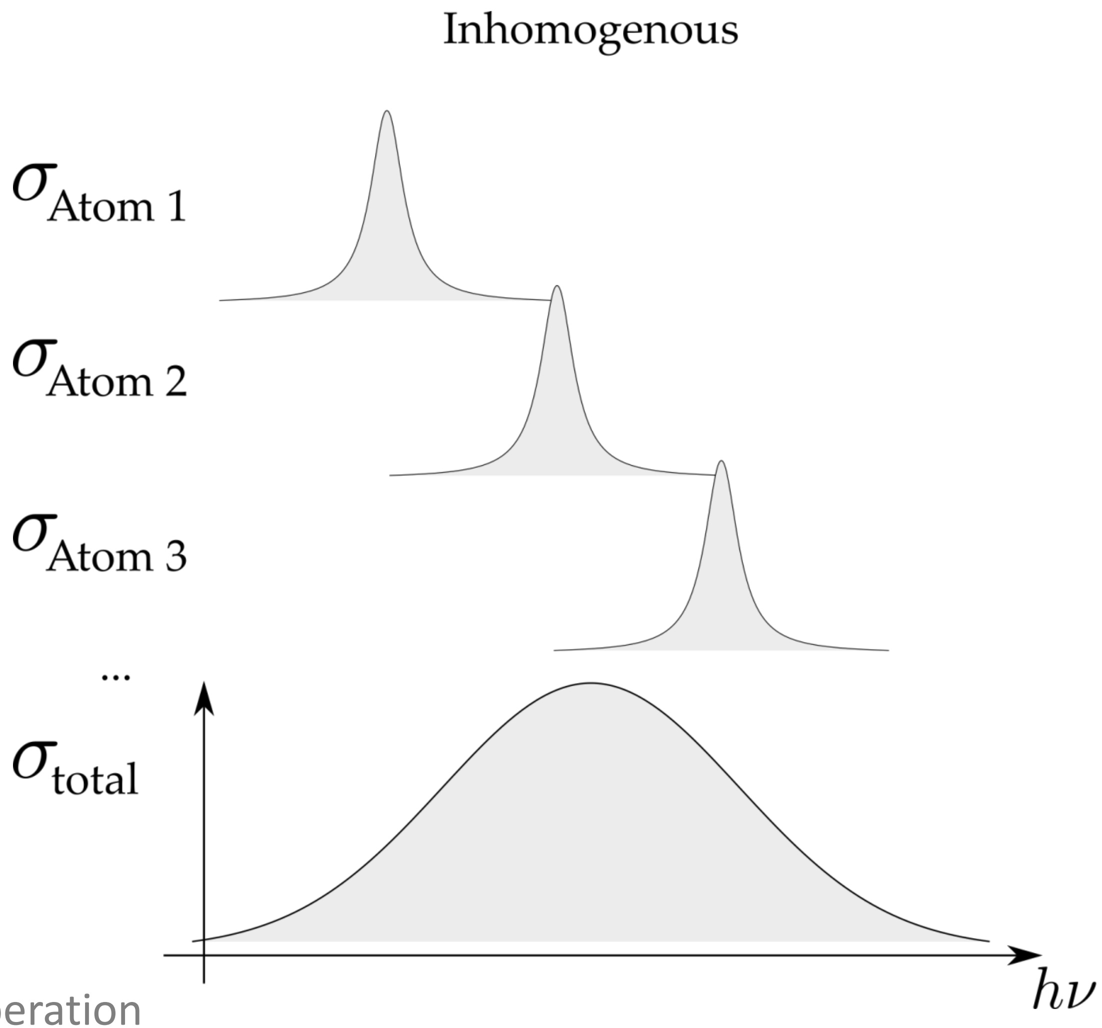
We further increasing puming, such that one mode is well above threshold

Population inversion adjusts so that total gain = total losses



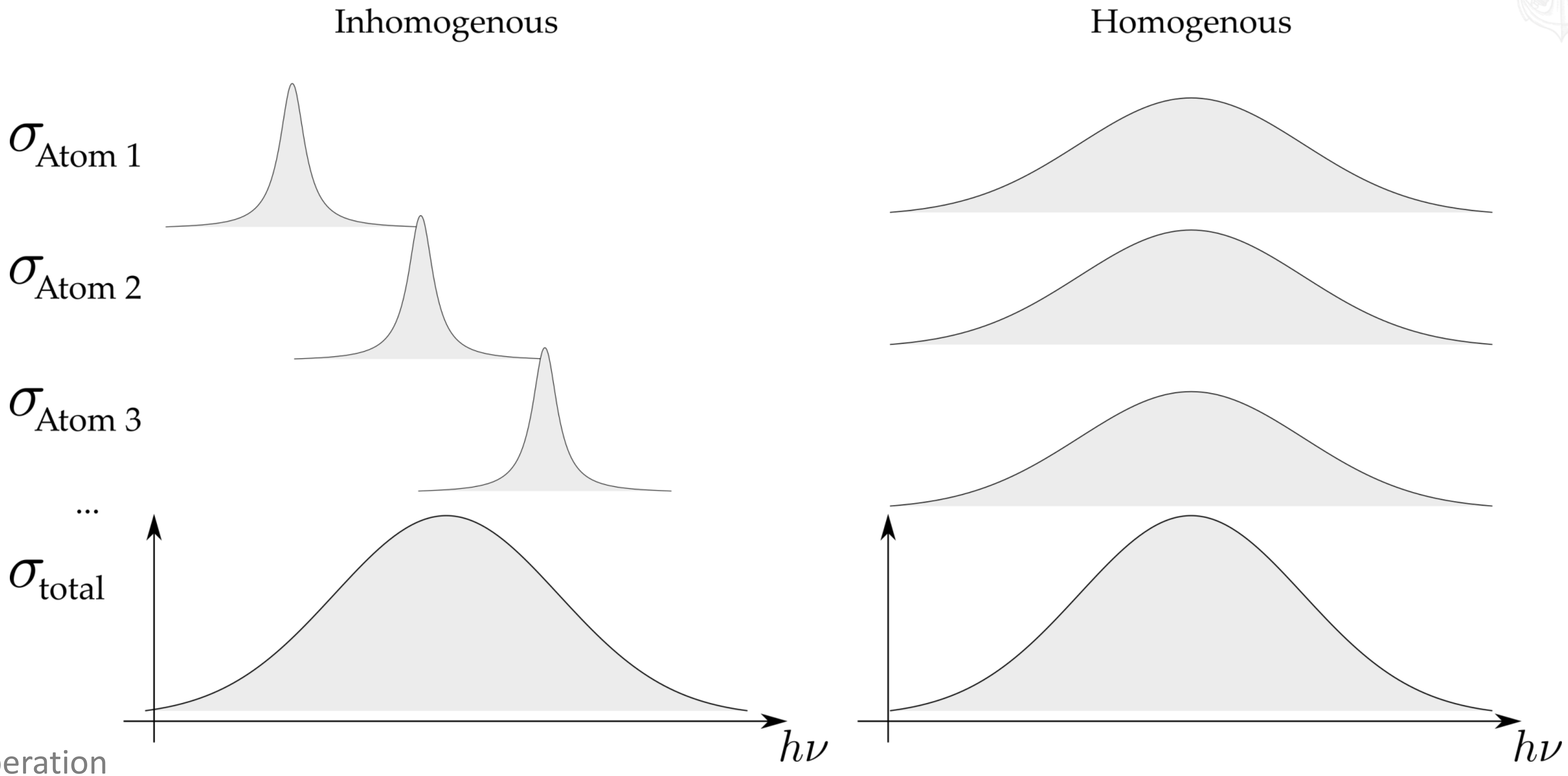
III. Mode competition

Which atoms are responsible for which gain ?



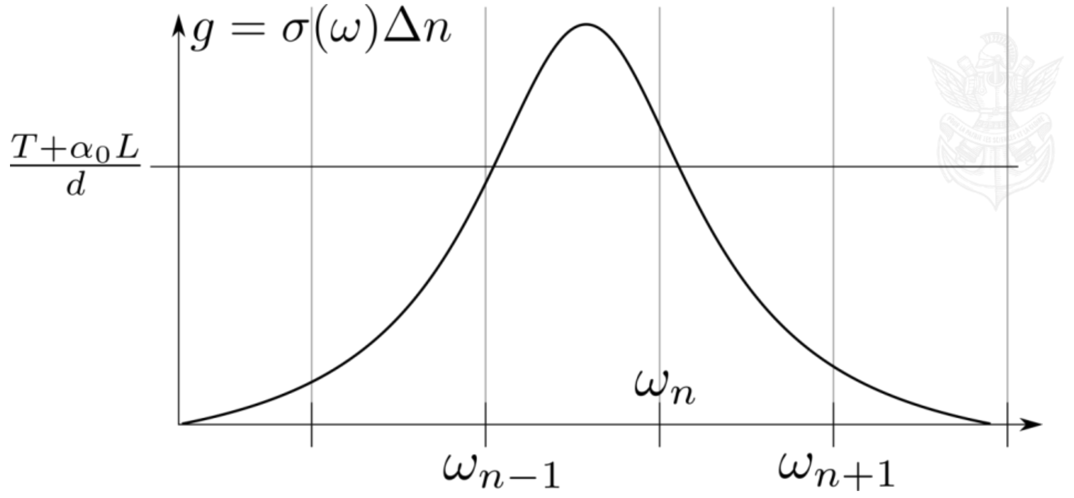
III. Mode competition

Which atoms are responsible for which gain ?



III. Mode competition

We further increasing puming, such that one mode is well above threshold



Population inversion adjusts so that total gain = total losses

Homogenous

Inhomogenous

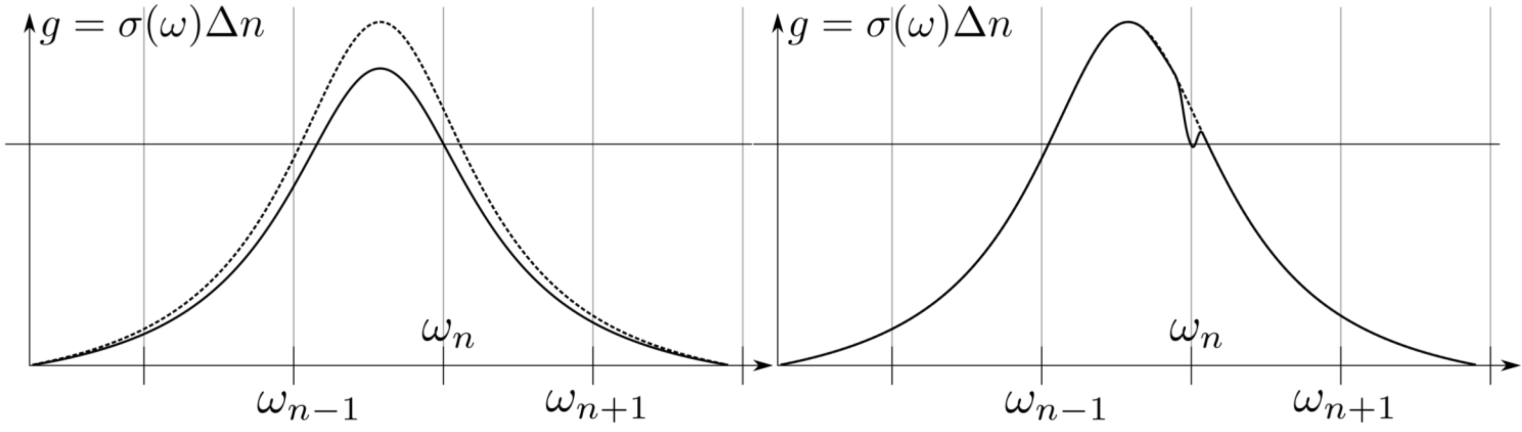
Homogeneous gain :

All atoms are addressed by all modes

Inhomogeneous gain :

Different atoms are addressed by different modes

$\frac{T+\alpha_0 L}{d}$



III. Mode competition

We further increasing puming, such that
two modes is well above threshold

Inhomogeneous gain :

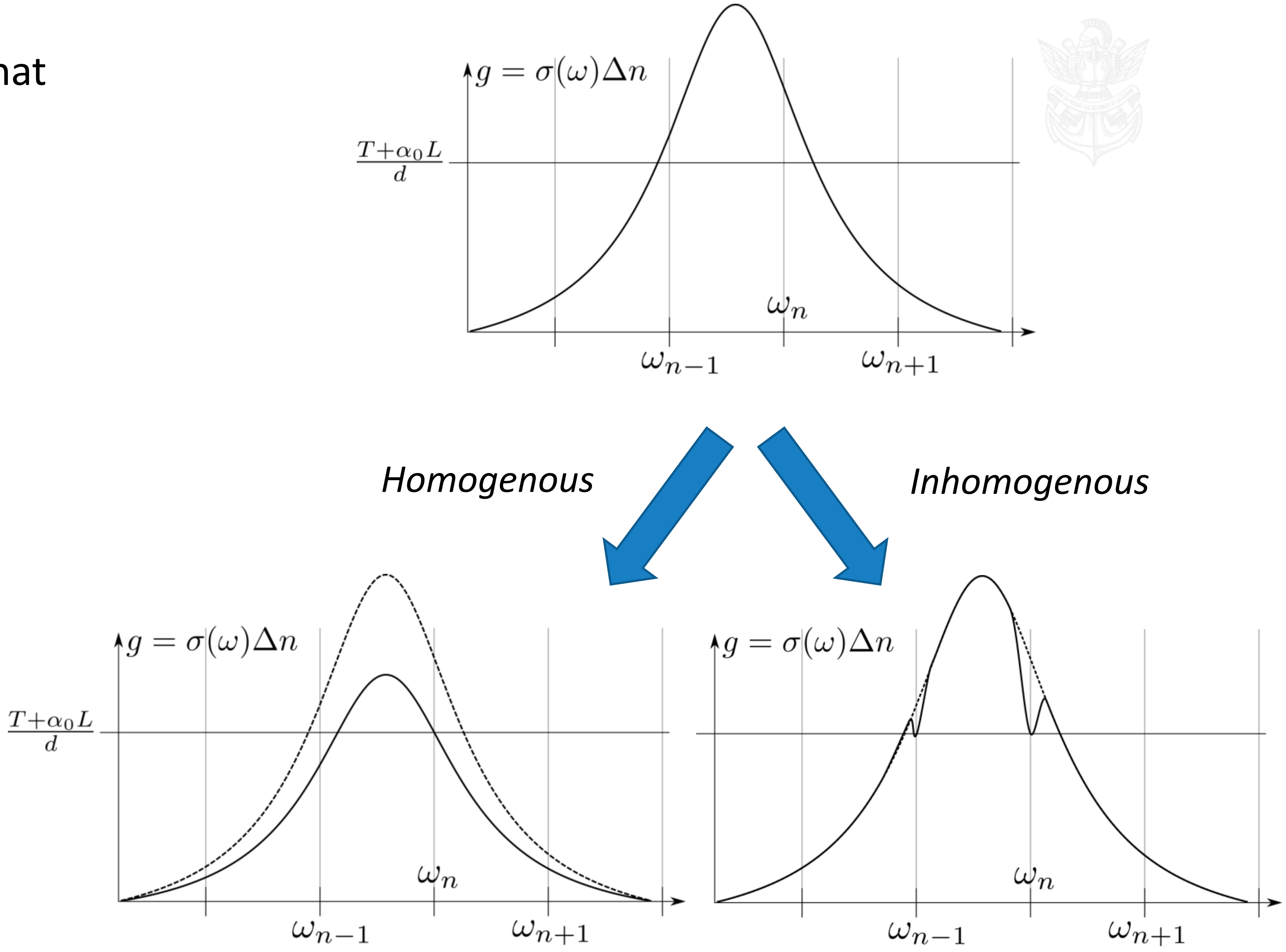
Laser intensity in 1 mode decreases gain in this mode

Modes are independent

Homogeneous gain :

Laser intensity in 1 mode decreases gain in all modes

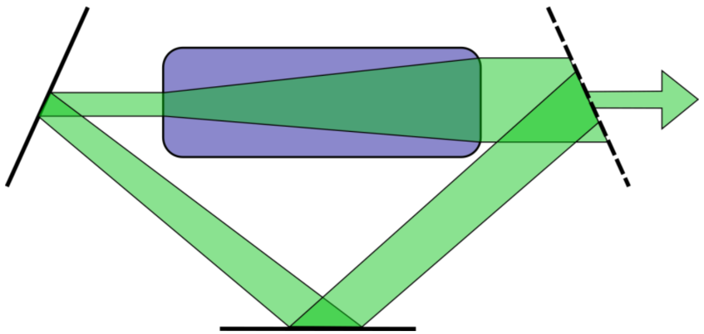
Mode competition !



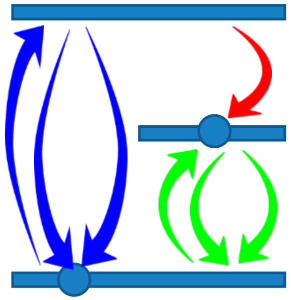
Take home message

An optical cavity (« oscillator ») → condition on phase

An amplifying medium (« gain ») → condition on ampl. / intensity



Introduced a 3 level systems.



$$r_{\text{abs}} = r_{\text{stim}} = \frac{\sigma I}{h\nu} = W$$

$$r_{\text{spont}} = \Gamma$$

Need gain to compensate losses (output + parasitic)

Impossible in Lorentz model,
requires population inversion.

$$g = \sigma_{eg} \underbrace{(n_e - n_g)}_{\Delta n}$$

Basic laser properties

$$g = \frac{g_0}{1+I/I_{\text{sat}}}$$

Laser threshold,
Gain saturation,
Steady state intensity,
Steady state population inversion