PHY208 – atoms and lasers
Lecture 1

Basic models for light-matter interactions

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Outline of lecture 1

Models !

A scientific theory should be as simple as possible, and as complex as needed.

I. Models for light

II. Models for atoms

III. Models for light-matter interactions

IV. Focus on the semi-quantum model

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Classical light : EM wave

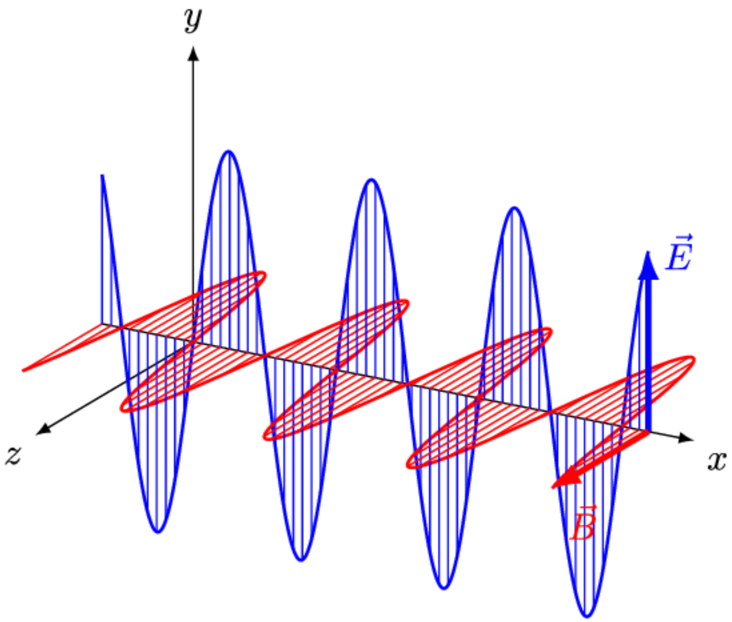
Light = (**E**,**B**) field following Maxwell equations.

Reminder from PHY104 etc.

$$\nabla . \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla . \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \qquad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \partial_t \mathbf{E} \right)$$

J. C. Maxwell



State of the field = (complex) amplitude in each mode i.e.

Wavevector modulus (frequency)

Wavevector direction

Polarization

Classical light : EM wave

Light = (**E**,**B**) field following Maxwell equations.

Reminder from PHY104 etc.

J. C. Maxwell

Plane waves

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\mathcal{E}} e^{i(\mathbf{k}.\mathbf{r}-\omega t)}\right)$$

Dispersion relation

$$\omega = kc/n$$

Energy

$$w(\mathbf{r},t) = \tfrac{1}{2}\epsilon_0 \mathbf{E}^2(\mathbf{r},t) + \tfrac{1}{2\mu_0}\mathbf{B}^2(\mathbf{r},t)$$

[J.m^{-3}]

Intensity

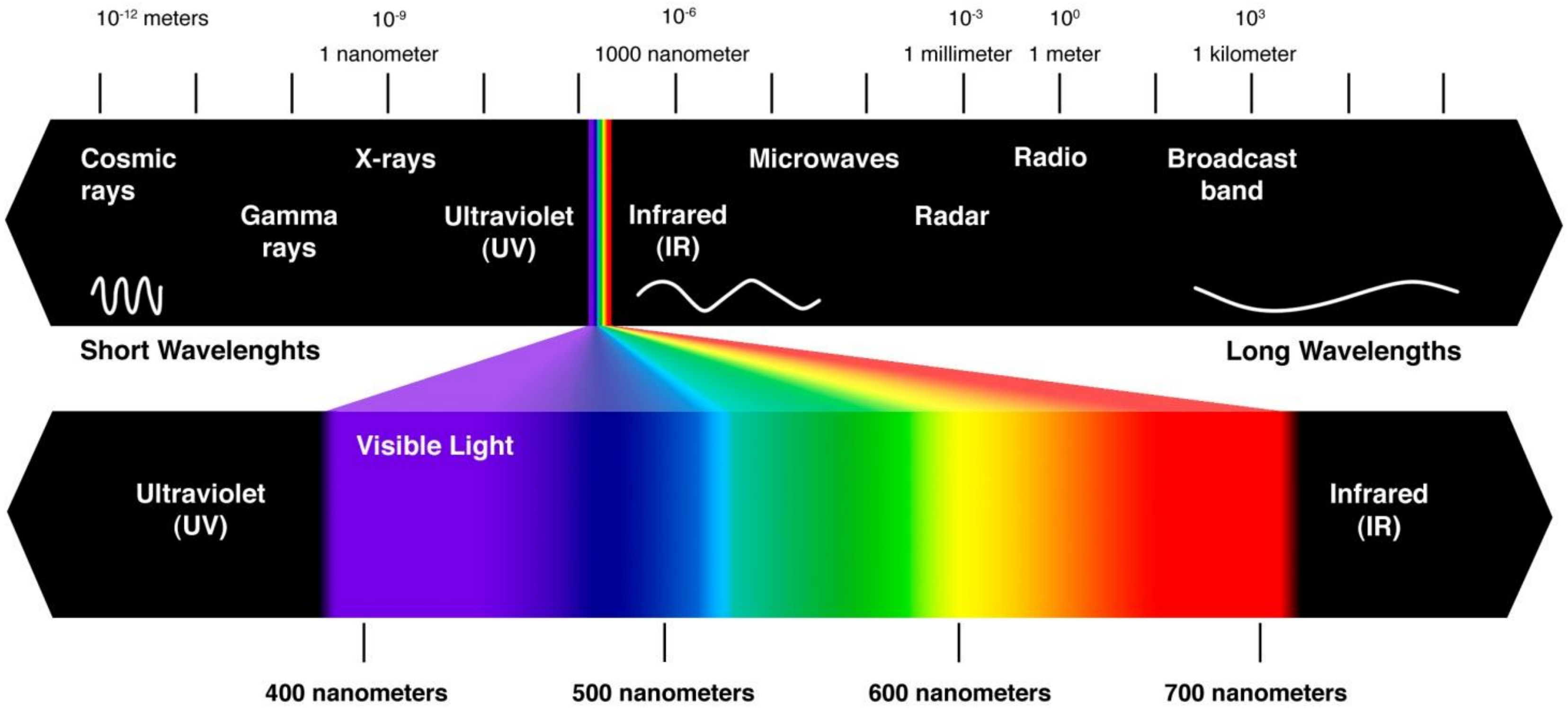
$$I = \left\langle \frac{\mathbf{E}(t)\times\mathbf{B}(t)}{\mu_0} \right\rangle = \frac{1}{2\mu_0 c}\left|\mathcal{E}\right|^2$$

[W.m^{-2}]

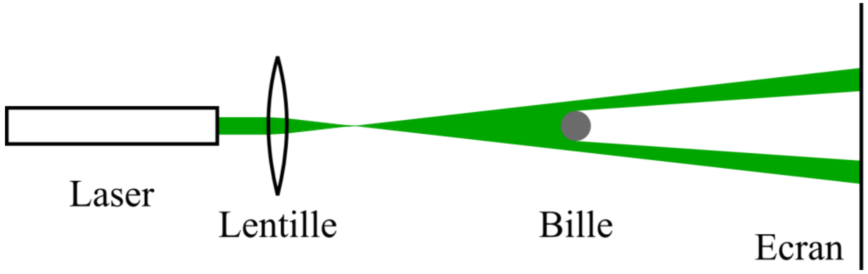
Classical light : EM wave - applications

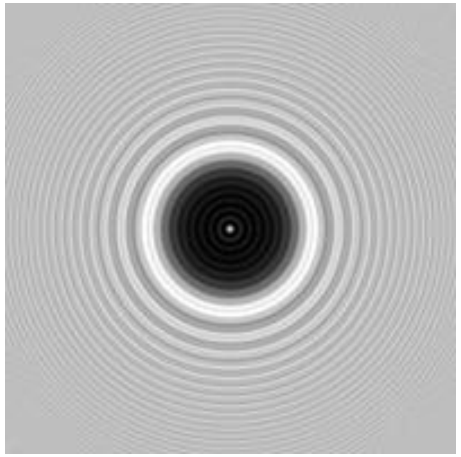
Light = (**E,B**) field following Maxwell equations.

Electromagnetic spectrum



Poisson spot





Quantum light : photons

Light = « wavefunction » for a given Hamiltonian

Reminder from PHY205

$$i\hbar\partial_t\psi_{\mathrm{light}} = \hat{H}_R\psi_{\mathrm{light}}$$

Remember « Chef Schrödinger's recipe » ?

$$U = \int d^3r \left(\frac{1}{2}\epsilon_0\mathbf{E}^2(\mathbf{r},t) + \frac{1}{2\mu_0}\mathbf{B}^2(\mathbf{r},t)\right)$$

$$\hat{H}_R = \int d^3r \left(\frac{1}{2}\epsilon_0\hat{\mathbf{E}}^2(\mathbf{r},t) + \frac{1}{2\mu_0}\hat{\mathbf{B}}^2(\mathbf{r},t)\right)$$

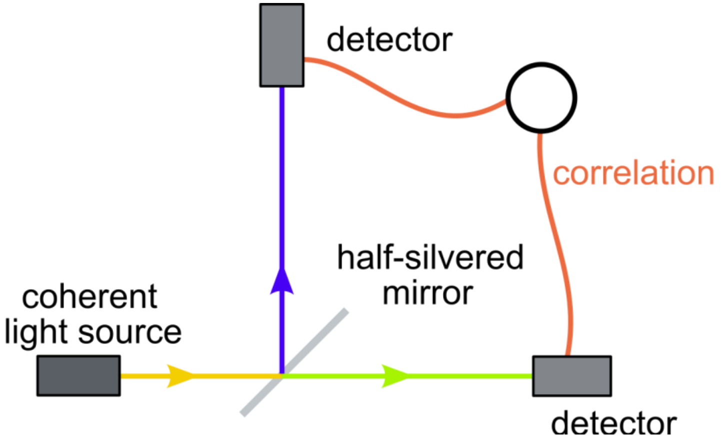
$\hat{\mathbf{E}} =$ 

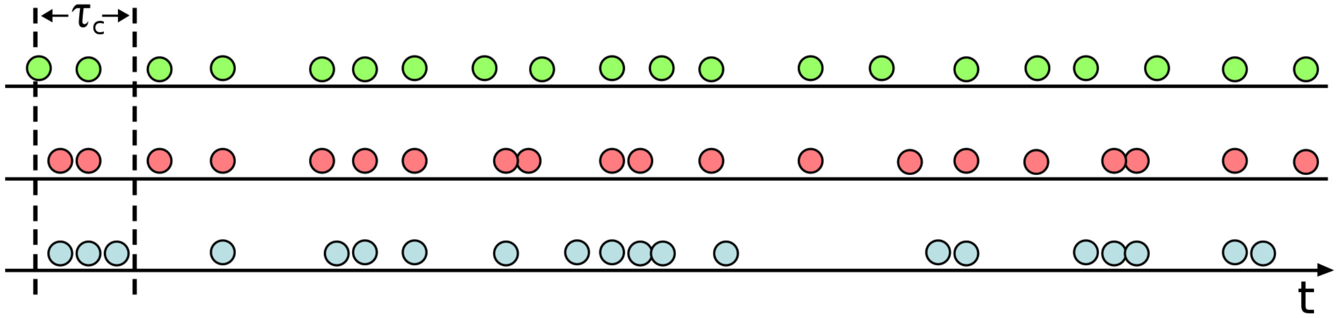
State of the field = linear combination of H eingenstates.

Won’t be addressed in this course

Quantum light : photons - applications

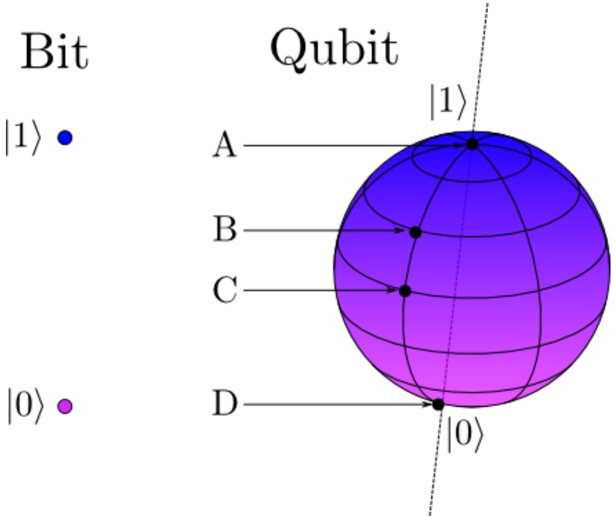
Statistical behavior of light





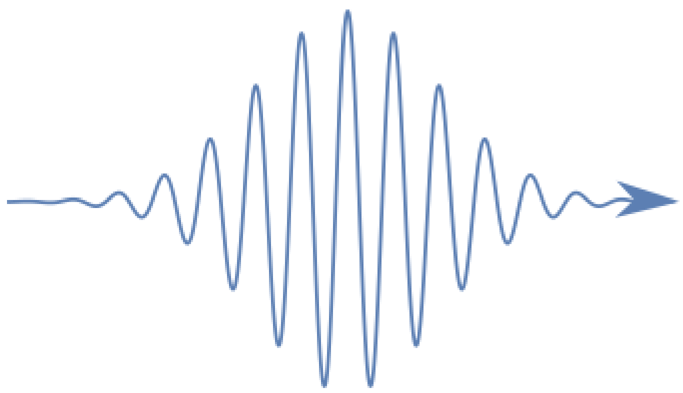
Photon detections as function of time for a) antibunched, b) random, and c) bunched light

Quantum computing



Intermediate light : « photons »

Light = massless particles with a specific energy-momentum relation



Momentum

$$\mathbf{p} = \hbar \mathbf{k}$$

Energy

$$E = \hbar \omega = |p| c$$

« Classical » marble-like particle
but with some properties
from the quantum world.

State of the field = number of « *photons* » in each mode.

Energy

Direction

(Polarization)

→ No notion of phase here !

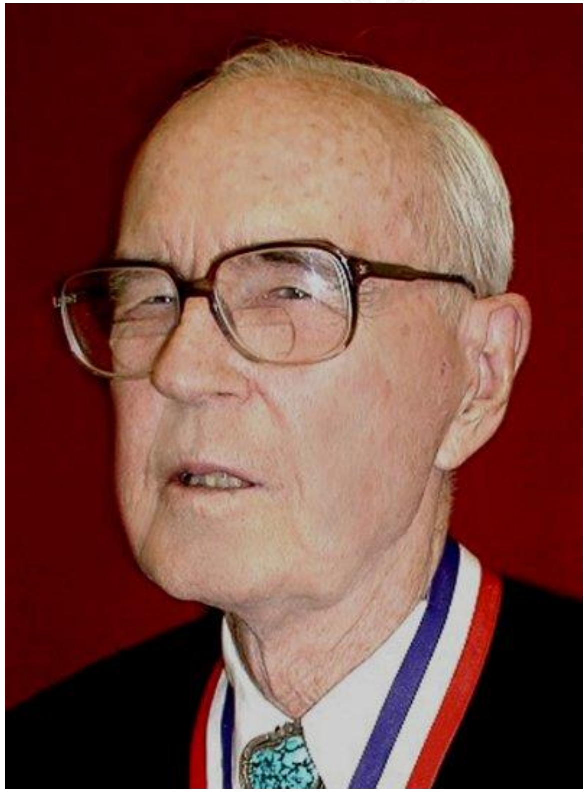
Intermediate light : « photons »

W. E. Lamb, **Anti-photon**, Appl. Phys. B 60, 77-84 (1995).

It should be apparent from the title of this article that the author does not like the use of the word "photon", which dates from 1926. In his view, there is no such thing as a photon. Only a comedy of errors and historical accidents led to its popularity among physicists and optical scientists.

I suggested that a license be required for use of the word photon, and offered to give such license to properly qualified people.

My records show that nobody working [here] in Rochester, and very few other people else where, ever took out a license to use the word "photon".



W. Lamb

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A scientific theory should be as simple as possible, and as complex as needed.

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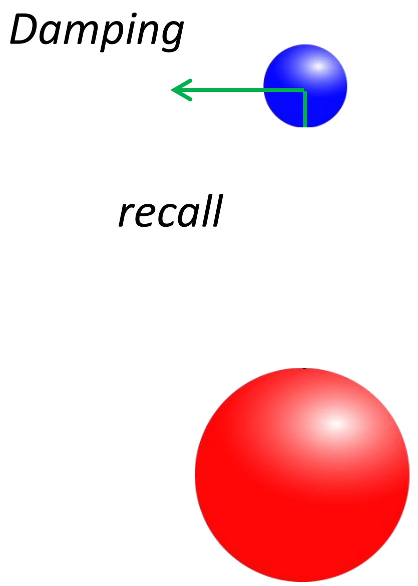
III. Models for light-matter interactions

IV. Focus on the semi-quantum model

III. Classical atoms : Lorentz model

Atom = elastically bound electron

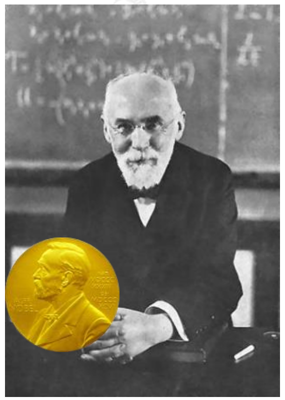
Reminder from PHY101, 201 etc.



State of the atom

=

position and velocity of the electron



H. Lorentz

Nucleus is usually assumed still (because $m_p \gg m_e$)

Coulomb force between atom and electron → elliptic orbit

For small perturbations → sprint-like restoring force

+ damping (purely phenomenological for now)

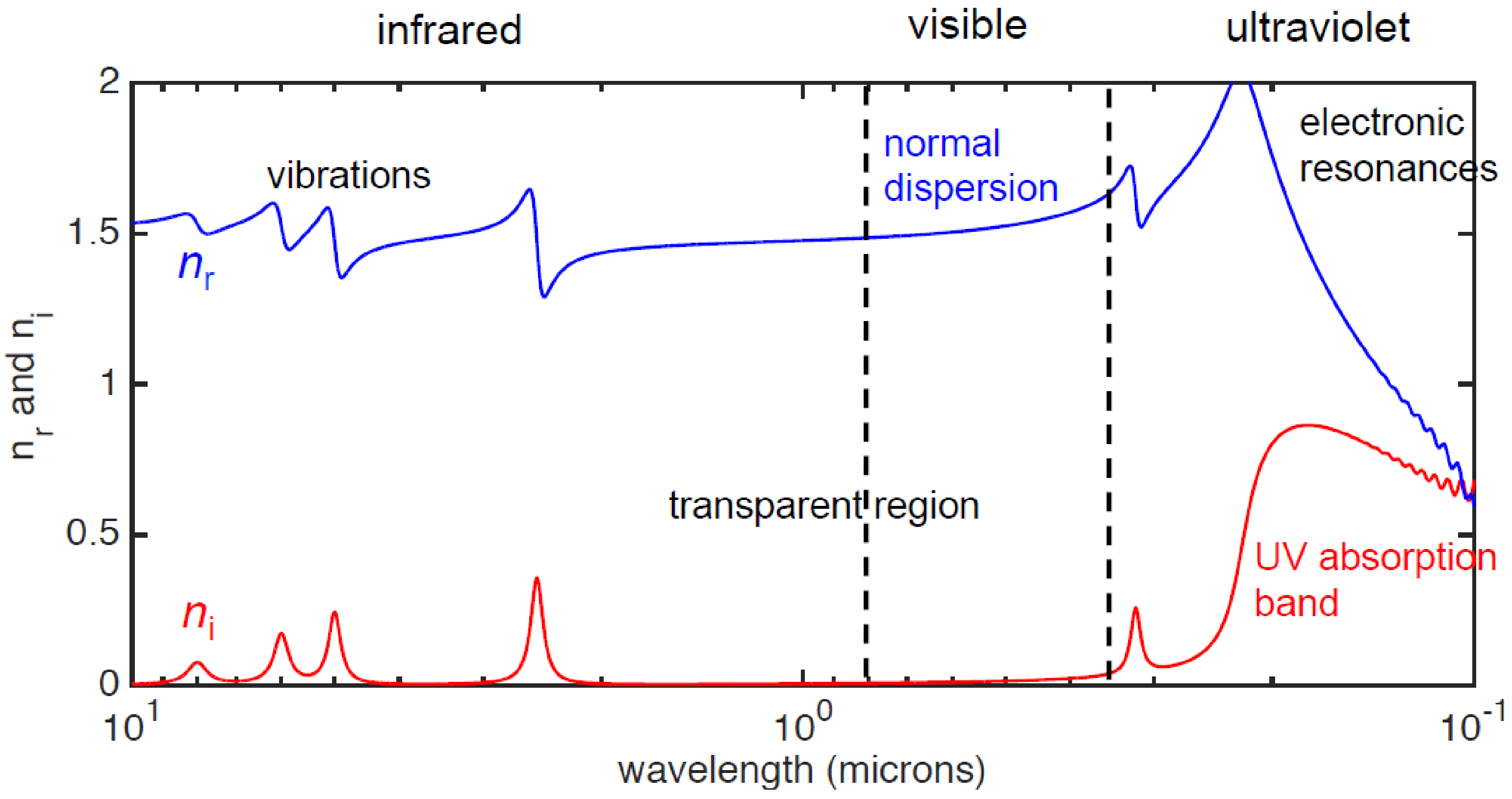
$$m_e \frac{d^2}{dt^2}\mathbf{r} = -m_e \omega_0^2 \mathbf{r} - m\Gamma \frac{d}{dt}\mathbf{r}$$

III. Classical atoms : Lorentz model - applications

Atom = elastically bound electron

Reminder from PHY202

Optical index



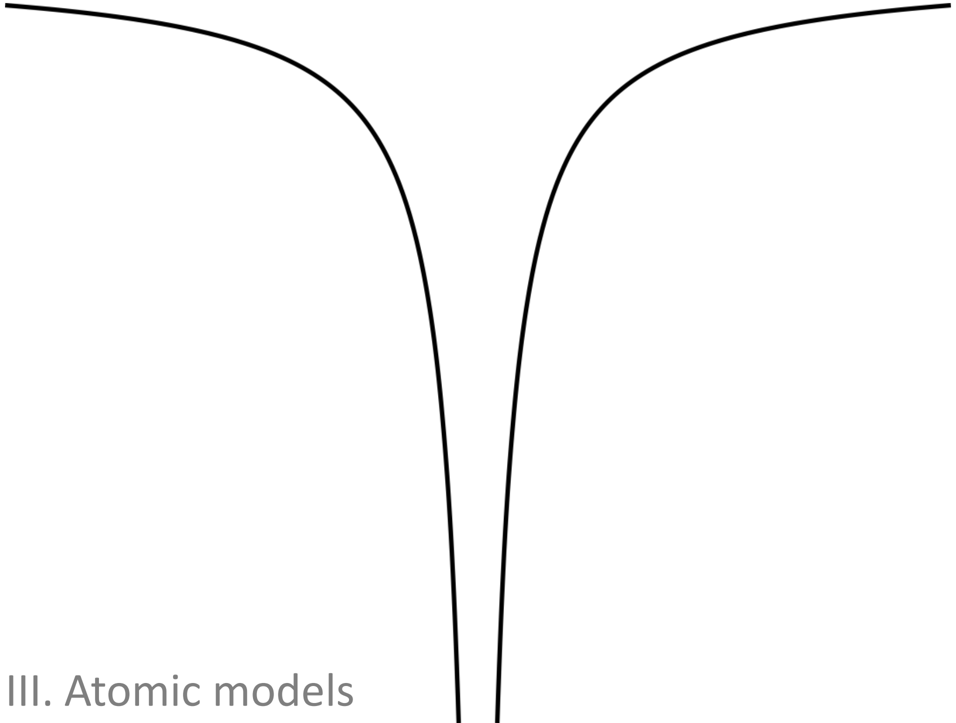
III. Quantum atoms : Schrodinger model

Atom = wavefunction for a given Hamiltonian

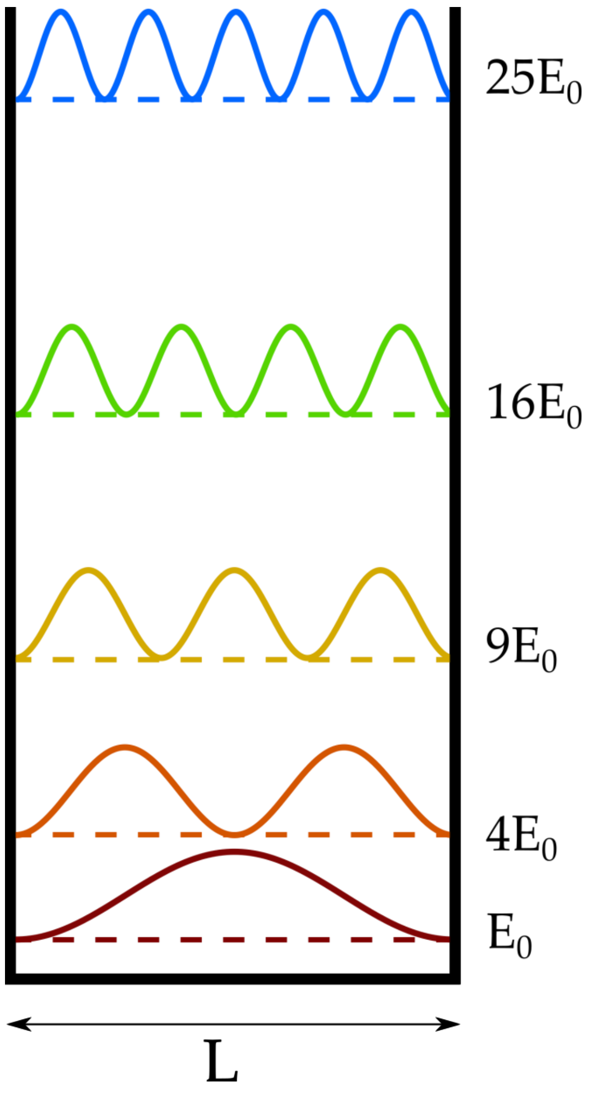
Reminder from PHY205

$$i\hbar\partial_t\psi_{\mathrm{e}} = \hat{H}\psi_{\mathrm{e}}$$

$$E = \frac{p^2}{2m} - \frac{e^2}{r} \quad \Rightarrow \quad \hat{H} = \frac{\hat{p}^2}{2m} - \frac{e^2}{\hat{r}}$$



III. Reminder : potential well



Key idea : Confinement → discrete energy levels

Reminder from PHY205

Reminder from PHY205 : flat potentiel well

Eigenstates

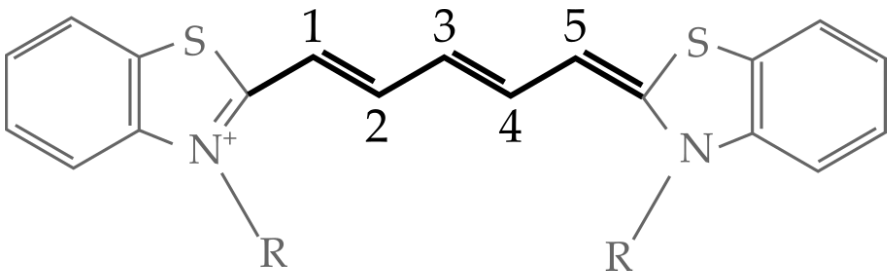
Energy spectrum

$$E_n = n^2 \frac{h^2}{8mL^2}$$

Applications for this model :

Quantum dots

Molecules



III. Quantum atoms : Schrodinger model

Atom = wavefunction for a given Hamiltonian

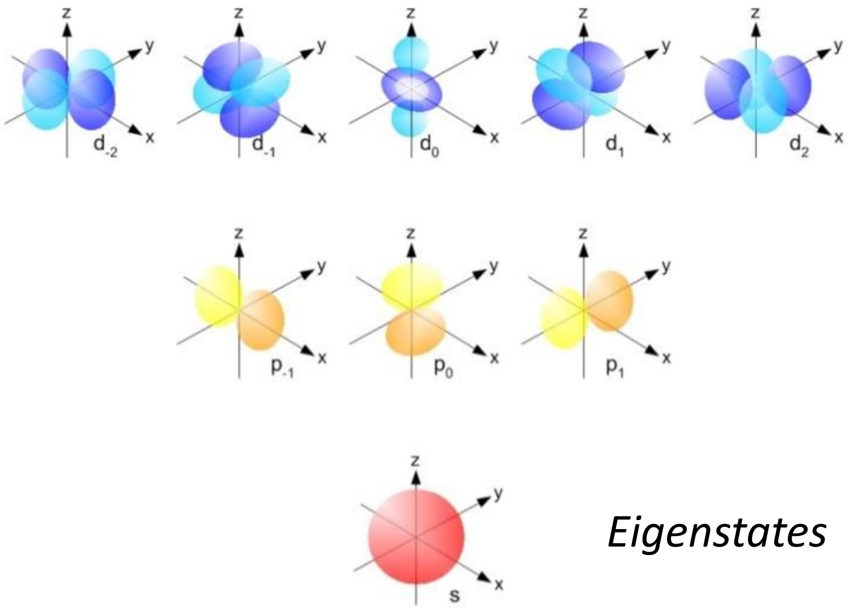
Out of scope for now…

$$i\hbar\partial_t\psi_{\rm e} = \hat{H}\psi_{\rm e}$$

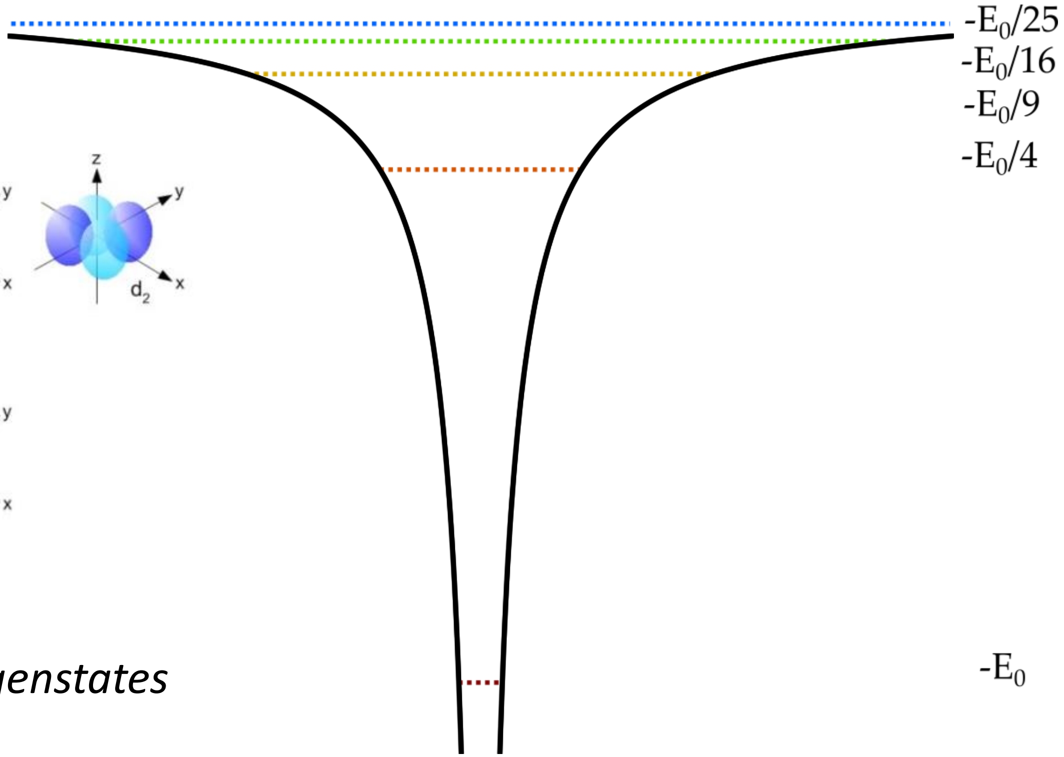
Will be fully treated in Lecture 6

$$\hat{H} = \frac{\hat{p}^2}{2m} - \frac{e^2}{\hat{r}}$$

$$\hat{H}\psi_{\rm n} = E_n\psi_{\rm n}$$



Eigenstates



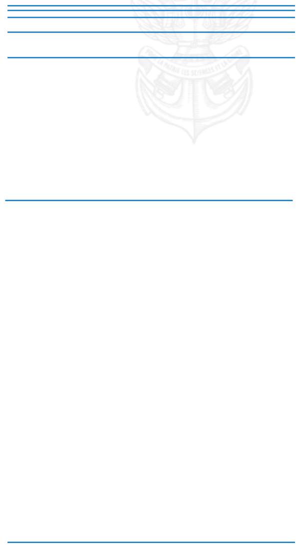
Energy spectrum

$$E_n = -\frac{1}{n^2}\frac{m_e e^4}{2\hbar^2}$$

State of the atom = wavefunction = sum of eigenstates

III. Intermediate atoms : Bohr model

Atom = electron with a Coulomb force + wave consideration



Bohr assumption :

$$L = n\hbar$$

« Classical » atom but

with some properties

from the quantum world.

De Broglie interpretation :

$$2\pi r = n\lambda$$

and $\lambda = \frac{h}{p}$

$$r \times p = n\frac{h}{2\pi}$$

$$E_n = -\frac{1}{n^2}\frac{m_e e^4}{2\hbar^2}$$

Same spectrum as

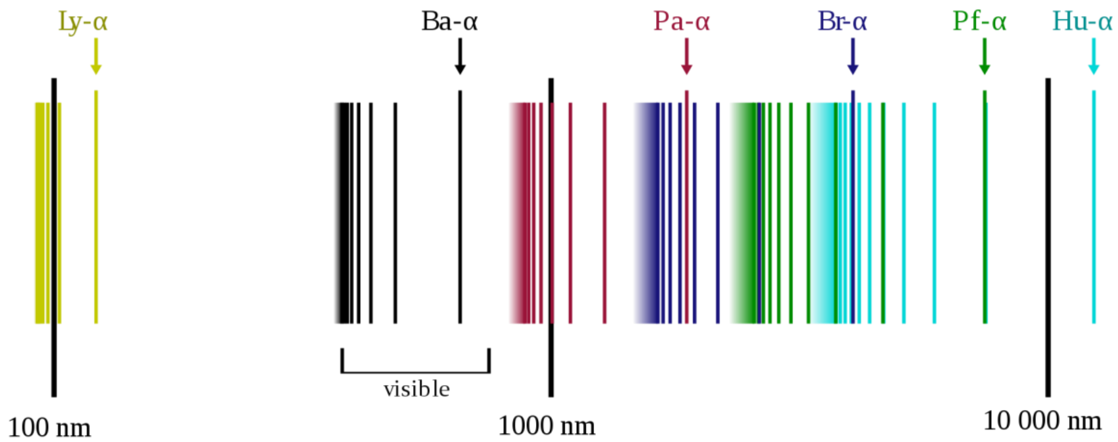
Schrodinger model !

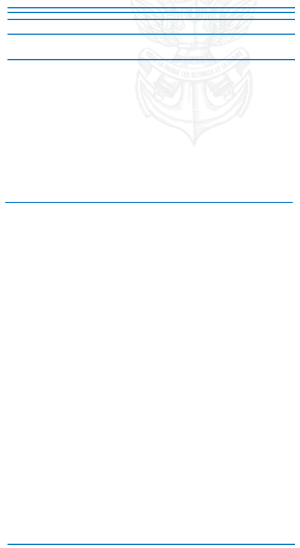
III. Intermediate atoms : Bohr model - applications

Atom = electron with a Coulomb force + wave consideration

Main application : hydrogen spectrum

Reminder from PHY207





$$E_n = -\frac{1}{n^2}\frac{m_e e^4}{2\hbar^2}$$

Rule 1 : Possible transitions correspond to $h\nu = E_m - E_n$

Rule 2 : If more than 1 electron, store up to 2 electrons / level, starting from low energy states.

Same spectrum as Schrodinger model !

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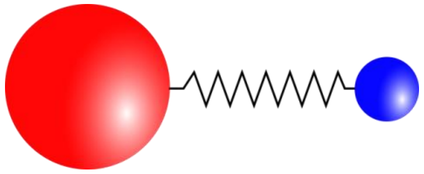
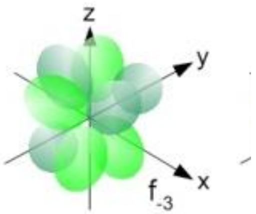
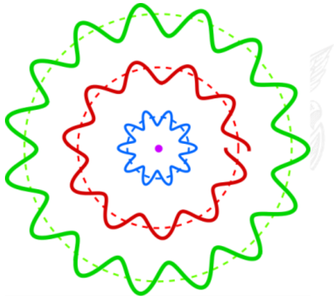
IV. Model map for light-matter interactions



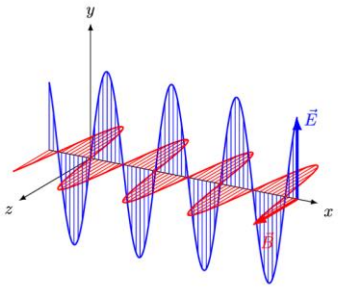




Classical

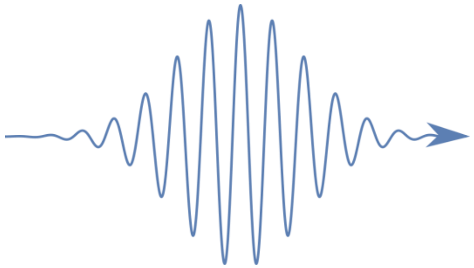
Semi classical

Classical +



Susceptiblity χ

Semi quantum



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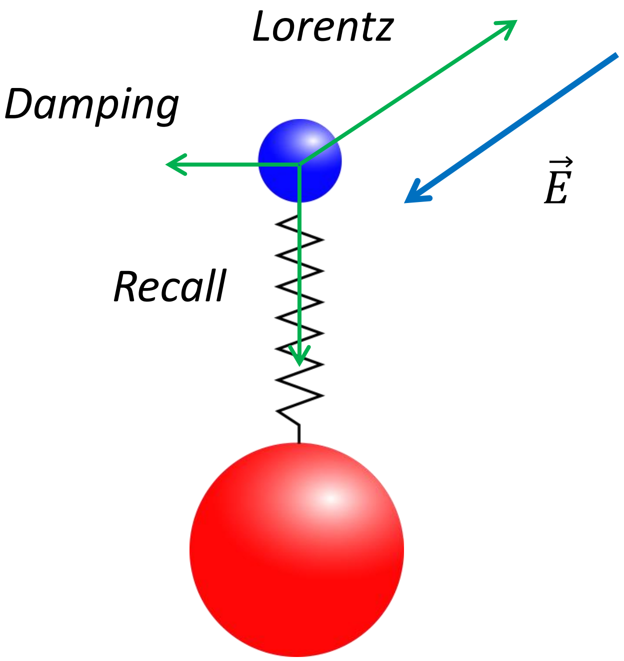
III. Classical atoms : Lorentz model

Atom = elastically bound electron

Reminder from PHY101, 201 etc.

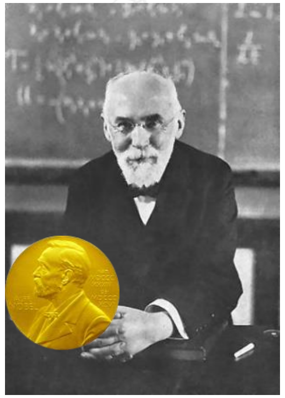


State of the atom

=

position and velocity of the electron

Nucleus is usually assumed still (because $m_p \gg m_e$)



H. Lorentz

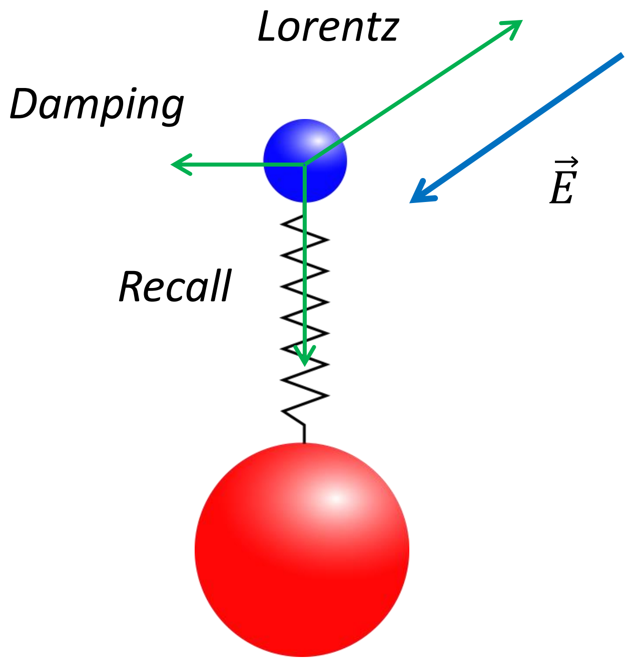
$$m_e \frac{d^2}{dt^2}\mathbf{r} = -m_e\omega_0^2\mathbf{r} - m\Gamma\frac{d}{dt}\mathbf{r} - q\boldsymbol{\mathcal{E}}$$

Under a planewave excitation, we take $\mathbf{r} = \mathbf{r}_0 e^{-i\omega t}$

$$-m_e\omega^2\mathbf{r}_0 = -m_e\omega_0^2\mathbf{r}_0 + im\omega\Gamma\mathbf{r}_0 - q\boldsymbol{\mathcal{E}}$$

III. Classical atoms : Lorentz model

Atom = elastically bound electron



Reminder from PHY101, 201 etc.

Dipole = dielectric moment per atom

$$\mathbf{p} = -q\mathbf{r}$$

$$E = -\mathbf{p}.\mathbf{E}$$

Polarization (of matter) = dielectric moment per volume

$$\mathbf{P} = -nq\langle\mathbf{r}\rangle = \epsilon_0\chi\mathbf{E}$$

III. Classical atoms : Lorentz model

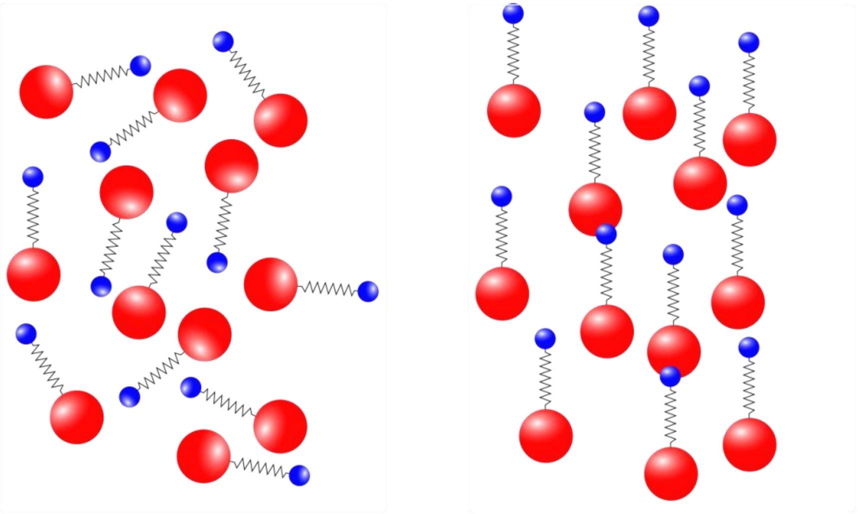
Atom = elastically bound electron

Reminder from PHY101, 201 etc.

Dipole = dielectric moment per atom

$$\mathbf{p} = -q\mathbf{r}$$

$$E = -\mathbf{p}.\mathbf{E}$$



No field

With field

Polarization (of matter) = dielectric moment per volume

$$\mathbf{P} = -nq\langle\mathbf{r}\rangle$$

with

$$\chi_{\mathrm{Lorentz}} = \frac{nq^2}{m\epsilon_0}\frac{1}{\omega_0^2-\omega^2-i\omega\Gamma}$$

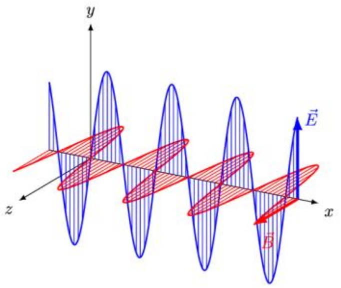
Reminder from PHY202 & 204

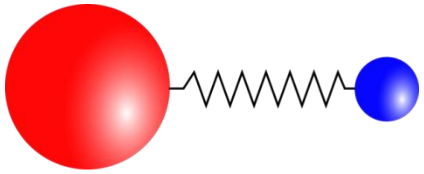
Related notions :

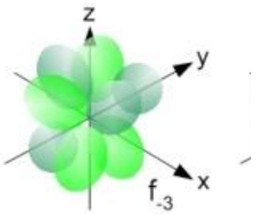
Optical index , permitivity, susceptibility,…

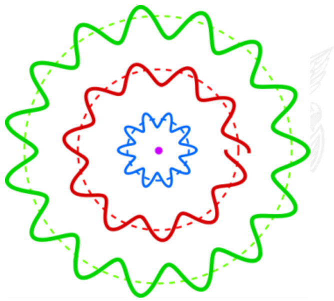
$$n^2 = 1 + \chi = \frac{\epsilon}{\epsilon_0}$$

IV. Model map for light-matter interactions









Classical

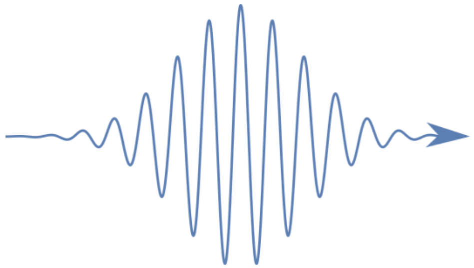
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$
$$n^2 = 1 + \chi$$
$$\mathbf{P} = nq\left\langle \mathbf{r} \right\rangle = \epsilon_0 \chi \mathbf{E}$$

Semi classical

$$H = -e\hat{\mathbf{r}}.\mathbf{E}$$
$$n^2 = 1 + \chi$$
$$\mathbf{P} = nq\left\langle \psi \right| \hat{\mathbf{r}} \left| \psi \right\rangle = \epsilon_0 \chi \mathbf{E}$$

Classical +

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$
$$n^2 = 1 + \chi$$
$$\mathbf{P} = nq\left\langle \mathbf{r} \right\rangle = \epsilon_0 \chi \mathbf{E}$$



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Semi quantum

$$\mathbf{F}_{\text{atom}} = \frac{\Delta \mathbf{p}_{\text{photon}}}{\Delta t}$$

Rate equations

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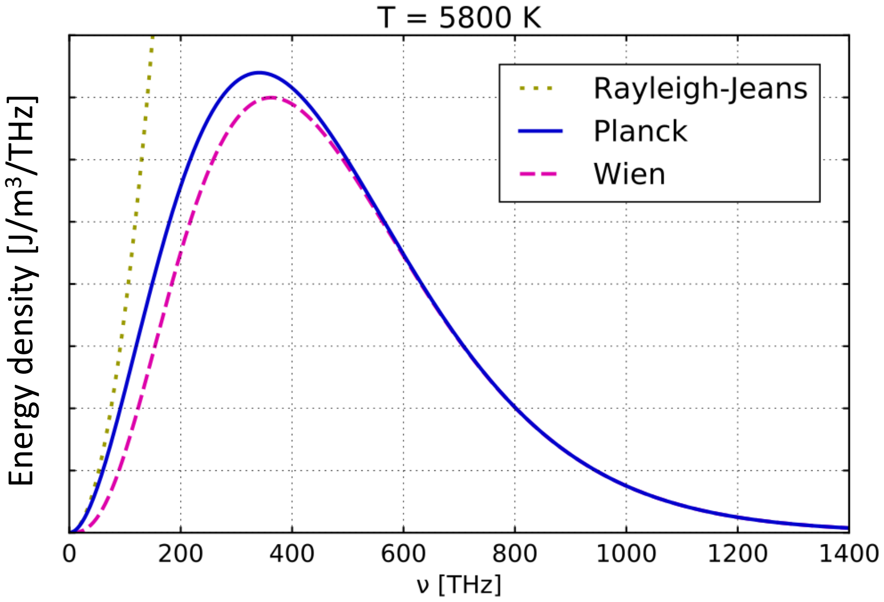
V. Blackbody radiation

Radiation universally radiated by all bodies at the same temperature

One of the hottest topics a 100 years ago !

Intensity spectrum :



Low energy :
Rayleigh-Jeans law

$$u(\nu) = \frac{8\pi\nu^2}{c^3} k_B T$$

High energy :
Wien law

$$u(\nu) = \nu^3 f\left(\frac{\nu}{T}\right) \propto \nu^3 \exp\left(-\frac{h\nu}{k_B T}\right)$$

$$u(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

Ad hoc formula :
Planck law

How to model the photon density in a blackbody ?

V. Be careful with densities !

Energy density per frequency, wavelength or energy ?

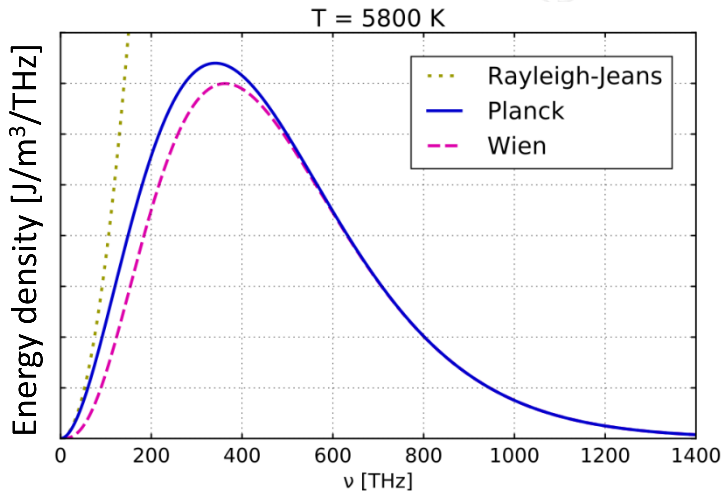
$$u(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

Units: J/m²/Hz

per nm ? per eV ?

$$u(\nu)d\nu = u(\lambda)d\lambda = u(E)dE$$

$$u(\lambda) = \frac{2hc}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1} \qquad u(E) = \frac{E^3}{4\pi^3\hbar^3 c^3} \frac{1}{\exp\left(\frac{E}{k_B T}\right) - 1}$$

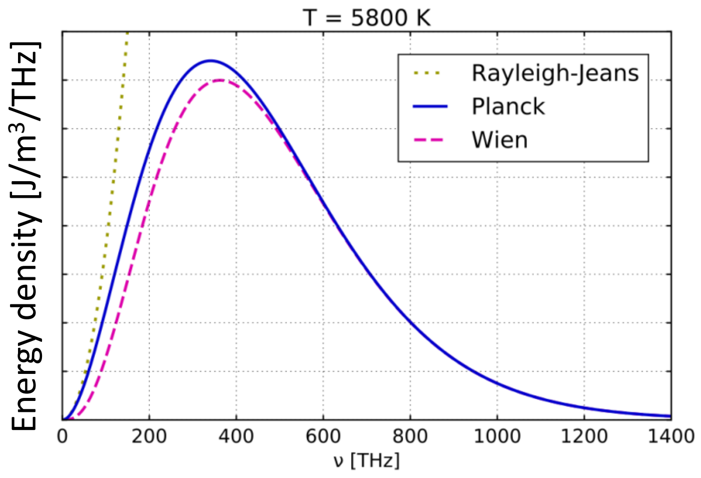


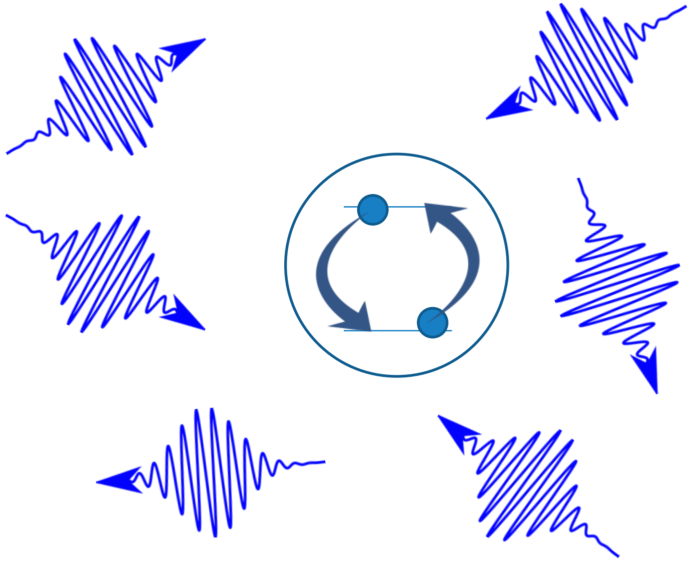
Energy density or photon density ?

Each photon carries an energy $h\nu$

$$u(\nu) = h\nu \times n(\nu)$$

V. Einstein rate equations





System: two levels atom

at thermal equilibrium

with a radiation of frequency ν

Hypothesis : thermal distribution

$$\frac{N_e}{N_g} = \exp\left(-\frac{E_e - E_g}{k_B T}\right)$$

Numerics : At 300K, for 400 THz, $\frac{N_e}{N_g} \leq 10^{-25}$

Hypothesis : transition rate

For an atom in the ground state,

$$r_{\text{abs}} = B_{ge} n(\nu)\ [s^{-1}]$$

For an atom in the excited state,

$$r_{\text{em}} = A_{eg}\ [s^{-1}]$$

What is the corresponding steady state density for the radiation ?

V. Toolbox : basic accounting

*1/ Write explicitly the balanced considered (**What ? Where ? When ?**)*

Balance of atoms in the ground state
in the whole system
between time t and time t+dt

2/ Write the balance as **Quantity in the system at time t+dt**

= quantity in the system at time t

+ quantity entering the system during dt

- quantity leaving the system during dt

+ quantity created during dt

- quantity destroyed during dt

$$N_g(t+dt) = N_g(t) + r_{\text{em}} dt\, N_e(t) - r_{\text{abs}} dt\, N_g(t)$$

3/ Simplify infinitesimal terms

$$\frac{dN_g}{dt} = r_{\text{em}} N_e(t) - r_{\text{abs}} N_g(t)$$

V. Equation analysis

$$\frac{dN_g}{dt} = r_{\rm em} N_e(t) - r_{\rm abs} N_g(t) \qquad \frac{dN_e}{dt} = -r_{\rm em} N_e(t) + r_{\rm abs} N_g(t)$$

Comments :

1/ The total number of atoms N_g+N_e is conserved

2/ If no absorption, exponential decay of excited population

3/ Steady state : $$\frac{dN_g}{dt} = 0 \Rightarrow r_{\rm em} N_e(t) = r_{\rm abs} N_g(t)$$

Population in state i
x
Transition rate from i to j

=

Population in state j
x
Transition rate from j to i

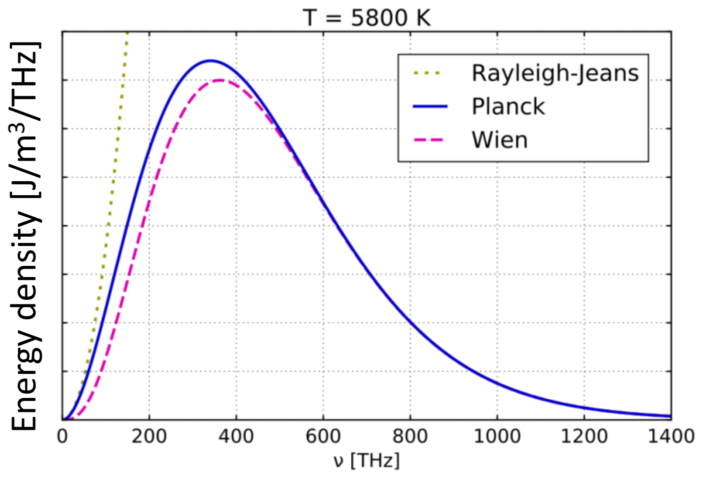
= Nb of jumps ***from i*** *per second*

= Nb of jumps ***to i*** *per second*

Detailed Balance

$$n_i \sum_{j \neq i} r_{i \to j} = \sum_{j \neq i} n_j r_{j \to i}$$

V. Einstein rate equations



Hypothesis : thermal distribution

$$\frac{N_e}{N_g} = \exp\left(-\frac{E_e - E_g}{k_B T}\right)$$

Numerics : At 300K, for 400 THz, $\frac{N_e}{N_g} \leq 10^{-25}$

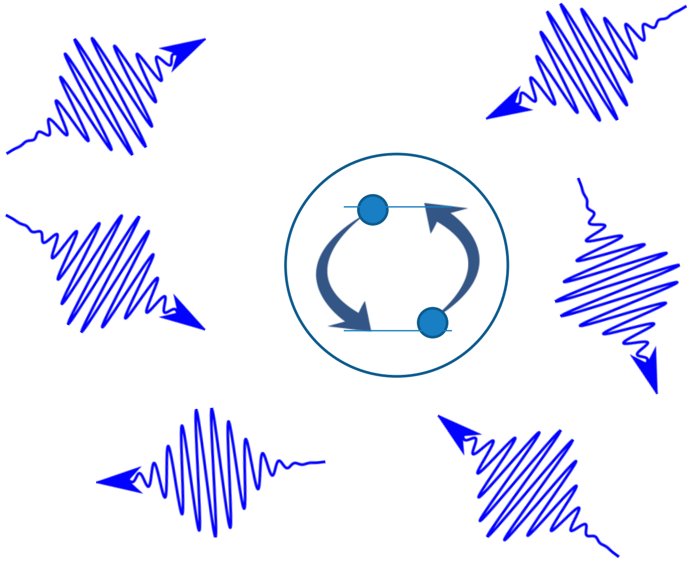
Hypothesis : transition rate

For an atom in the ground state,

$$r_{\text{abs}} = B_{ge} n(\nu)\ [s^{-1}]$$

For an atom in the excited state,

$$r_{\text{em}} = A_{eg}\ [s^{-1}]$$



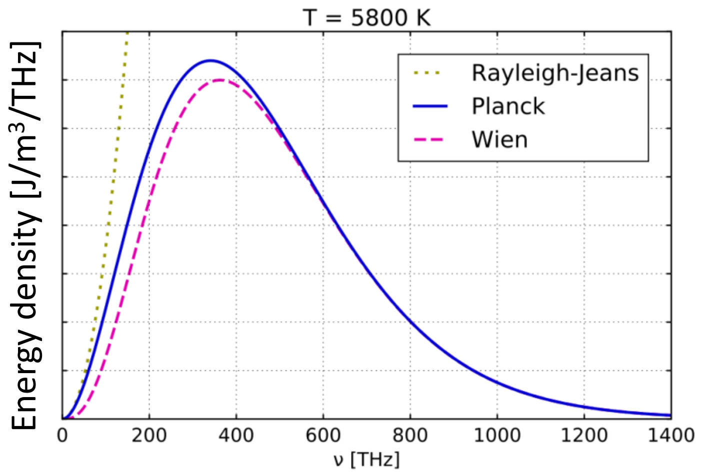
Steady state: $$\frac{dN_g}{dt} = 0 \Rightarrow r_{\text{em}} N_e(t) = r_{\text{abs}} N_g(t)$$

$$A_{eg} N_e(t) = B_{ge} n(\nu)\, N_g(t)$$

$$n(\nu) = \frac{A_{eg}}{B_{ge}} \frac{N_e}{N_g} = \frac{A_{eg}}{B_{ge}} \exp\left(-\frac{E_e - E_g}{k_B T}\right)$$

Happy ?

V. Einstein rate equations



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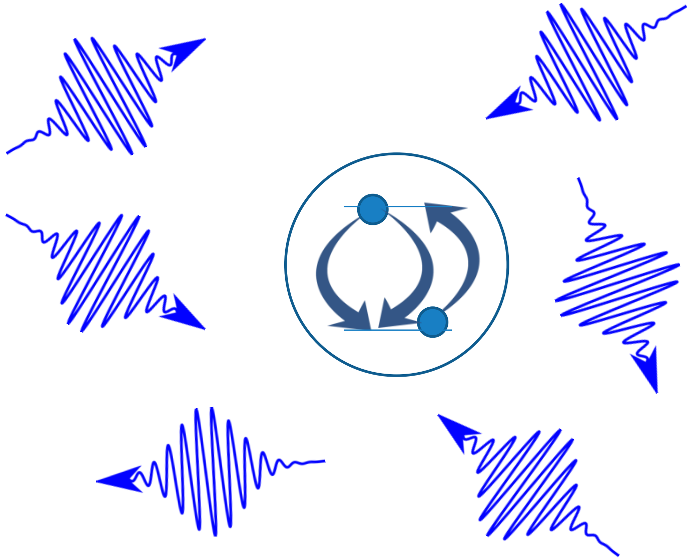


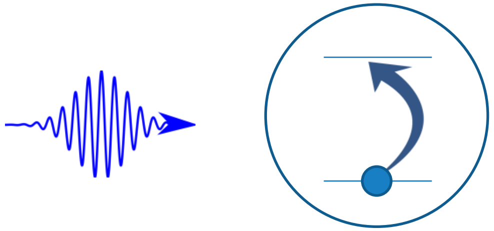
Hypothesis : stimulated emission

For an atom in the excited state,

$$r_{\text{stim}} = B_{eg} n(\nu) \; [s^{-1}]$$

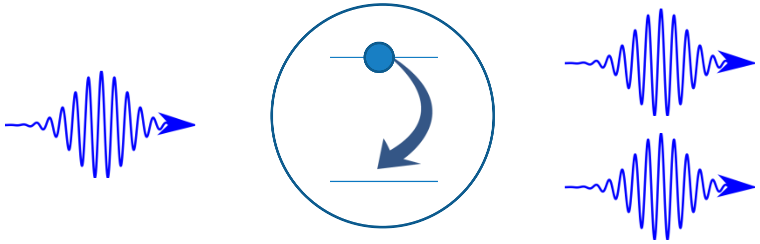
V. Fundamental processes

Absorption



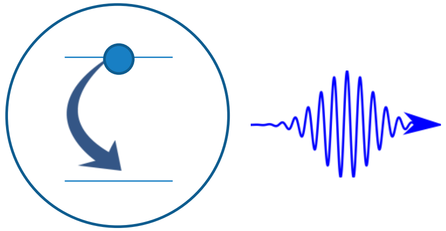
$$r_{\text{abs}} = B_{ge} n(\nu) \; [s^{-1}]$$

Stimulated emission



$$r_{\text{stim}} = B_{eg} n(\nu) \; [s^{-1}]$$

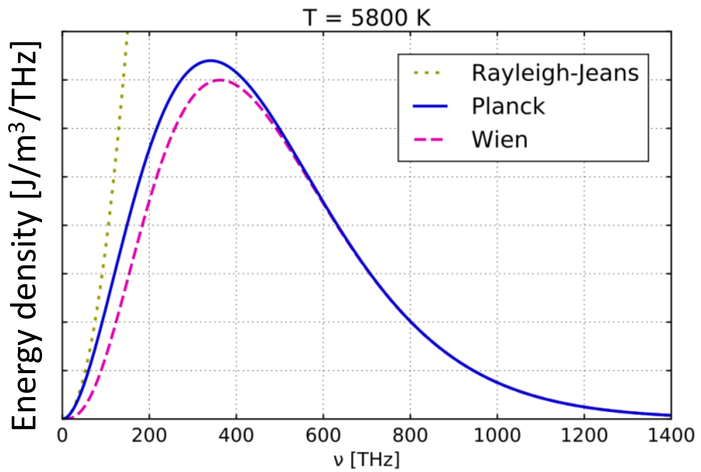
Spontaneous emission



$$r_{\text{em}} = A_{eg} \; [s^{-1}]$$

/!\ usually written with energy density, not photon density

V. Einstein rate equations

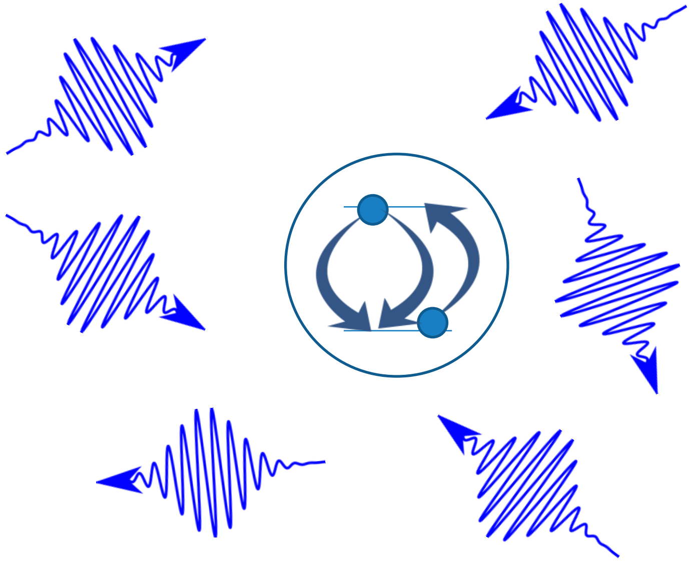


Hypothesis : thermal distribution

$$\frac{N_e}{N_g} = \exp\left(-\frac{E_e - E_g}{k_B T}\right)$$

Hypothesis : transition rates + stimulated emission

$$r_{\text{abs}} = B_{ge} n(\nu)\ [s^{-1}] \qquad r_{\text{em}} = A_{eg}\ [s^{-1}] \qquad r_{\text{stim}} = B_{eg} n(\nu)\ [s^{-1}]$$

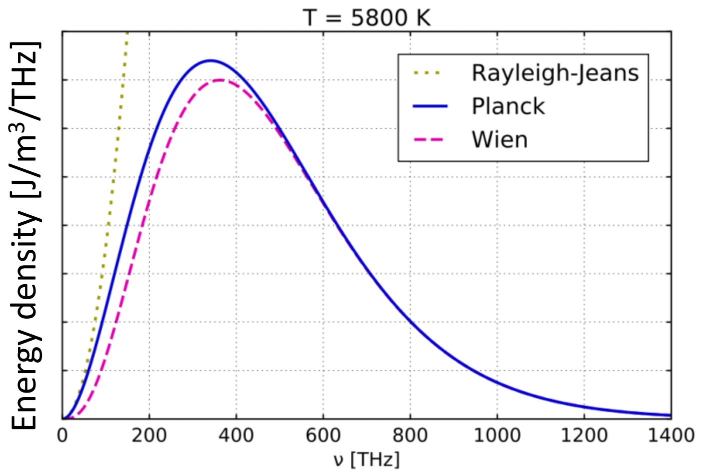


Population balance : $$\frac{d}{dt} N_g = (A_{eg} + B_{ge} n(\nu))\, N_e(t) - B_{eg} n(\nu) N_g(t)$$

Steady state : $$n(\nu) = \frac{A_{eg}}{B_{eg} \exp\left(\frac{E_e - E_g}{k_B T}\right) - B_{ge}}$$

Happy ?

V. Einstein rate equations



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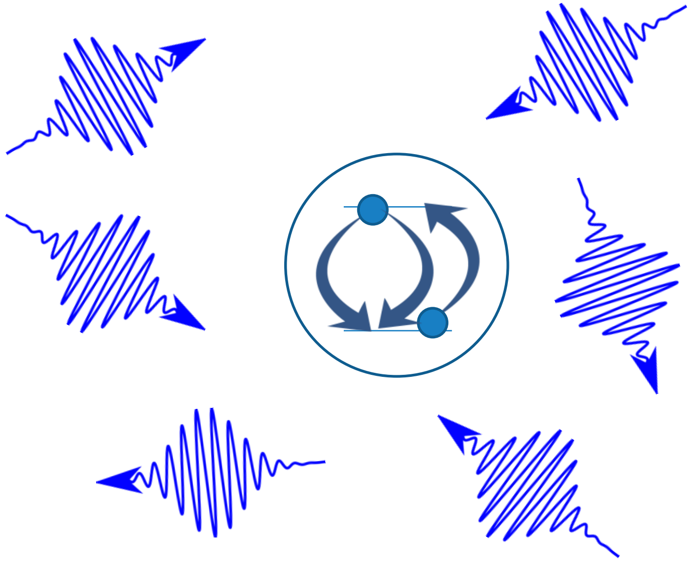


Steady state :

$$n(\nu) = \frac{A_{eg}}{B_{eg}\exp\left(\frac{E_e - E_g}{k_B T}\right) - B_{ge}}$$

Three consequences :

$$B_{eg} = B_{ge}$$

Probability absorption
=
Probability stim. emission

$$h\nu = E_e - E_g$$

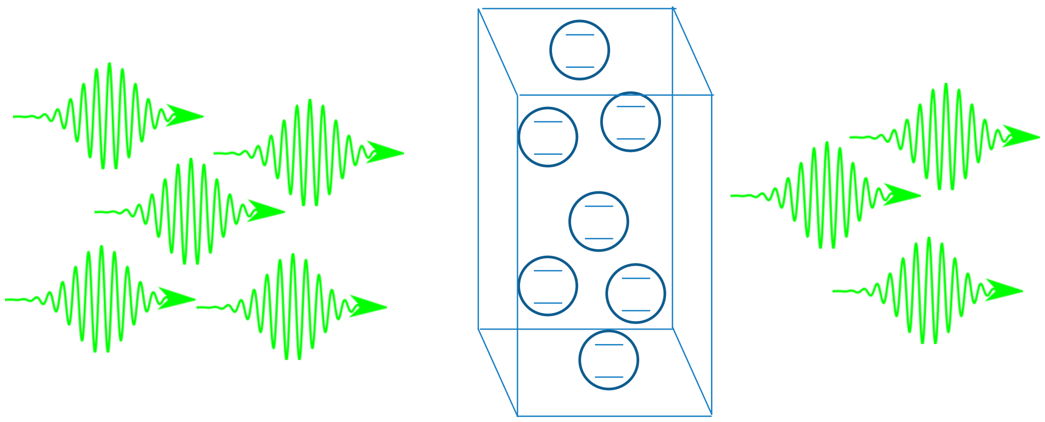
Equilibrium radiation

$$\frac{A_{eg}}{B_{ge}} = \frac{8\pi\nu^2}{c^3}$$

Relation between abs. and spont. recombination

V. From the light perspective

Application : propagation of a light beam of frequency span Δν (semi quantum model)



$$I(x,t) = n(\nu,x,t)\Delta\nu \times h\nu \times c$$

Balance of energy for photons of the beam

inside a slab of thickness dx and surface dS

between time t and t+dt

$$h\nu\, n(\nu,x,t+dt)\Delta\nu\, dS dx = h\nu\, n(\nu,x,t)\Delta\nu\, dS dx + I(x,t)\, dS dt - I(x+dx,t)\, dS dt$$

$$+ h\nu\, r_{\mathrm{stim}} N_e\, dt - h\nu\, r_{\mathrm{abs}} N_g\, dt$$

$$\frac{1}{c}\frac{dI}{dt} + \frac{dI}{dx} = \frac{B_{\mathrm{eg}}}{c\Delta\nu}\left(n_e - n_g\right) I$$

Steady state : $\frac{d}{dx} I = \sigma\,\left(n_e - n_g\right) I$

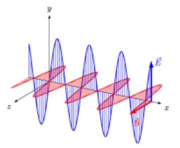
Interaction cross section :

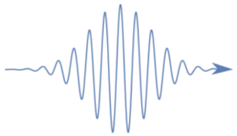
$$r_{\mathrm{abs}} = r_{\mathrm{stim}} = \frac{\sigma\, I}{h\nu}$$

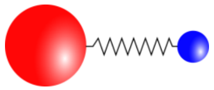
Beer lambert law

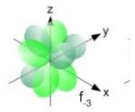
Take home message

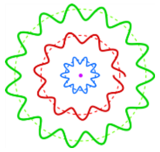
Basic models for light, matter and interactions









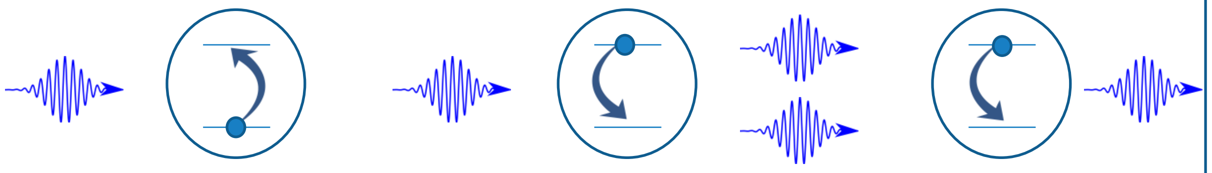


	Classical	*Semi classical*	*Classical +*
	-	-	*Semi quantum*

Fundamental processes

$$B_{eg} = B_{ge} \qquad \frac{A_{eg}}{B_{ge}} = \frac{8\pi\nu^2}{c^3}$$

Absorption | Stimulated emission | Spontaneous emission



$$r_{\rm abs} = B_{ge} n(\nu)\ [s^{-1}] \qquad r_{\rm stim} = B_{eg} n(\nu)\ [s^{-1}] \qquad r_{\rm em} = A_{eg}\ [s^{-1}]$$

Light propagation : Beer-Lambert law

$$\frac{d}{dx} I = \sigma\ (n_e - n_g)\ I$$

Interaction cross section

$$r_{\rm abs} = r_{\rm stim} = \frac{\sigma\, I}{h\nu}$$